

# tUPL Parallel Programming Paradigm

# Data Flow Computing

Traditionally, compilers analyze program source code for data dependencies between instructions in order to better organize the instruction sequences in the binary output files.

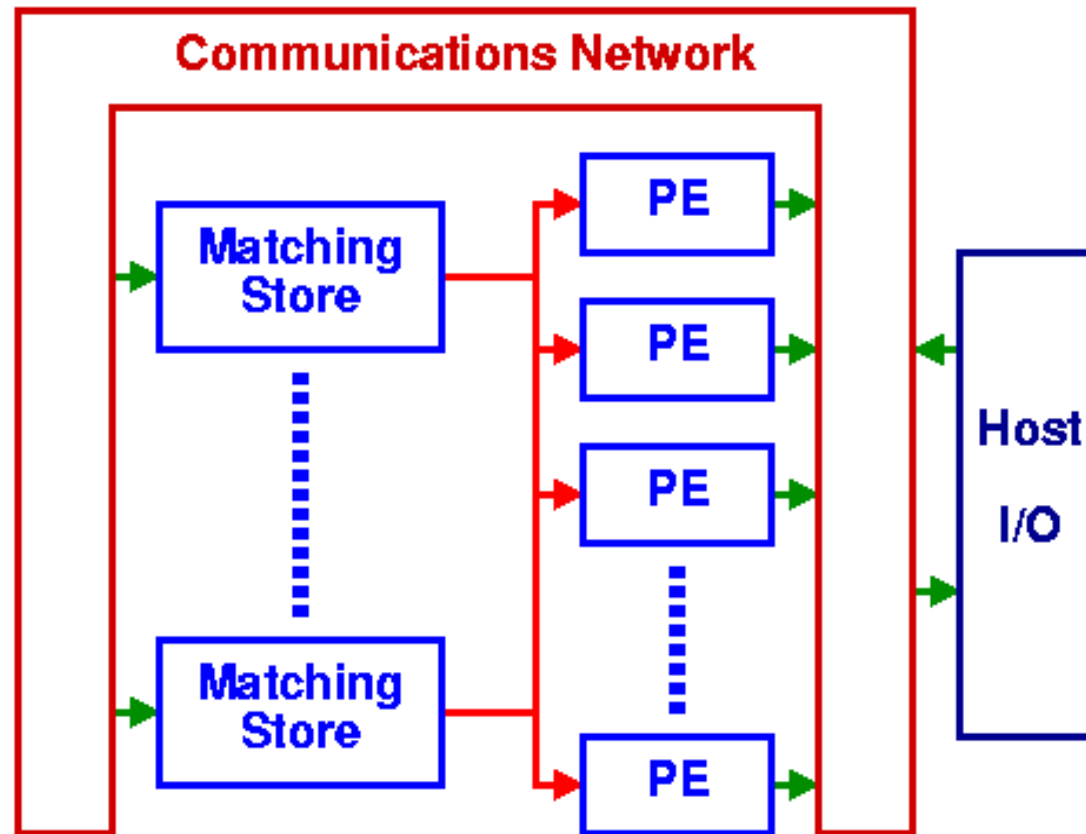
A dataflow compiler records these dependencies by creating **unique tags** for each dependency instead of using variable names. By giving each dependency a unique tag, it allows the non-dependent code segments in the binary to be executed *out of order* and in parallel.

# Dataflow Execution

- Programs are loaded into the **Content Addressable Memory (CAM)** of a dynamic dataflow computer.
- When **all of the tagged operands** of an instruction become available (that is, output from previous instructions and/or user input), **the instruction is marked as ready** for execution by an execution unit. This is known as *activating* or *firing* the instruction.
- Once an instruction is completed by an execution unit, **its output data is stored (with its tag) in the CAM**. Any instructions that are dependent upon this particular datum (identified by its tag value) are then marked as ready for execution.

# In a Picture

## Manchester Data Flow Machine



# Dataflow in Practice

However, in practice the following problems occurred:

- Efficiently **broadcasting data tokens** in a massively parallel system.
- Efficiently **dispatching instruction tokens** in a massively parallel system.
- Building Content Addressable Memory (Tag Memory) **large enough** to hold all of the dependencies of a real program.

# Linda Coordination Language

- Main usage: in combination with other existing languages, e.g. C/Fortran, provide a mean to link less expensive desktop computers together and combine their power so they can jointly tackle problems.
- A logically **global associative memory**, called a **tuplespace**, in which processes store and retrieve tuples.
- This model is implemented as a "coordination language" in which **several primitives** operating on ordered sequence of typed data objects, "tuples"
  - **in** atomically reads and removes—consumes—a tuple from tuplespace
  - **rd** non-destructively reads a tuplespace
  - **out** produces a tuple, writing it into tuplespace
  - **eval** creates new processes to evaluate tuples, writing the result into tuplespace

# tUPL

- Free Computer Programming from common artifacts like data structures, data dependencies, explicit parallelism constructs
- Harness a compilation framework such that
  - Data structures are generated automatically
  - Data dependencies are turned into opportunities to optimize performance
  - Parallel execution is guaranteed

# Basic tUPL Data Type

< token, data >

Formally, this basic data type is even further stripped down to

< token ><sub>(A, F<sub>A</sub>)</sub>

With  $A$  the “shared” space in which  $data$  is stored, and with  $F_A$  an address function on  $A$ , s.t.  $data$  is represented as:

$A [F_A(<token>)]$

So  $data == A [F_A(<token>)]$



# Address function $F_A$

$F_A$  can be **any function**, but mostly it is an affine mapping/projection:

$$\mathbb{Z}^n \rightarrow \mathbb{Z}^k$$



With  $n$  being the number of fields in token and  $k$  the dimensionality of  $A$ . So  $F_A$  can be represented as

$$\text{Addr}(t) = \vec{m} + Mt^T = \begin{pmatrix} m_{10} \\ \dots \\ m_{k0} \end{pmatrix} + \begin{pmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ \dots & \dots & \dots & \dots \\ m_{k1} & m_{k2} & \dots & m_{kn} \end{pmatrix} t^T$$

# NOTE!!!!

$$A [ I, J ] = 5.0$$

does **NOT** mean that element [ I, J ] of Matrix A, or of a 2-Dimensional Array A is assigned the value 5.0.

## **BUT:**

5.0 is stored in A at [  $F_A(I, J)$  ], with  $F_A = Id$ , or that the **data** value of  $\langle I, J \rangle_{(A, F_A)}$  becomes 5.0, or that  $\langle I, J, \text{data} \rangle = \langle I, J, 5.0 \rangle^*$

\*Note that tokens can be more dimensional: **token tuples t**  
In case tuples have more than one field, then **t.i** represents the  $i^{\text{th}}$  field of t

# Multiple Shared Spaces and Associated Address Function per Shared Space

Consider the following **tUPL** code fragment:

$$A[I, J] = A[I-1, 2*J] + B[J]$$

Then in this code fragment we have **2 shared** spaces:

**A** and **B**

and **3 address functions**:

$$F_A^1 = \text{Id} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \langle I, J \rangle$$

$$F_A^2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \langle I, J \rangle$$

$$F_B = \begin{pmatrix} 0 & 1 \end{pmatrix} \langle I, J \rangle$$

So, for token  $t = \langle I, J \rangle$  perform:

$$A[F_A^1(t)] \leftarrow A[F_A^2(t)] + B[F_B(t)]$$

SO, **data structures** as we know them do not exist in **tUPL**, only

**single storage locations for each data item,  
represented by token tuples**

We need a mean to express a **collection** or **set** of these single storage locations

**→ (Token) Tuple Reservoirs**

# Examples of Tuple Reservoirs (I)

A **Digraph G(V,E)**:

$$\mathbf{T} = \{ \langle \mathbf{u}, \mathbf{v} \rangle \mid \mathbf{u}, \mathbf{v} \in V \text{ and } (\mathbf{u}, \mathbf{v}) \in E \}$$

with address function **Weight** [  $\mathbf{u}, \mathbf{v}$  ] representing the address at which the weight of edge  $(\mathbf{u}, \mathbf{v})$  is stored

A **Sparse Matrix A**:

$$\mathbf{T} = \{ \langle \mathbf{i}, \mathbf{j} \rangle \mid \text{at row } i \text{ and column } j \\ \text{there is a nnz element} \}$$

with address function **Value** [  $\mathbf{i}, \mathbf{j}$  ] representing the address at which the value of matrix A [  $\mathbf{i}, \mathbf{j}$  ] is stored

# Examples of Tuple Reservoirs (II)

A **Linked List** (of single storage locations):

$$\mathbf{T} = \{ \langle \mathbf{i}_k, \mathbf{j}_k \rangle \mid 1 \leq k \leq n, \\ \text{for every } \mathbf{j}_k, 1 \leq k < n, \\ \text{there exists exactly one } \mathbf{i}_m, \\ \text{such that } \mathbf{j}_k = \mathbf{i}_m, \text{ and} \\ \text{for all } \mathbf{j}_k, 1 \leq k \leq n, \\ \text{the values are different} \}$$

Together with an address function **Value** [  $\mathbf{i}_k, \mathbf{j}_k$  ] representing the value at the  $k^{\text{th}}$  position in the list.

**OR** address function **Value** [  $\mathbf{i}_k$  ] ! (**tUPL** allows both)

# Examples of Tuple Reservoirs (III)

## Relational Database Tables

$\mathbf{T} = \{ \langle \mathbf{i} \rangle \mid 1 \leq \mathbf{i} \leq n, \text{ with } \mathbf{i} \text{ representing the } \mathbf{i}^{\text{th}} \text{ record in the database table} \}$

and associated address functions:

$\text{field}_1 [ \mathbf{i} ], \text{field}_2 [ \mathbf{i} ], \dots, \text{field}_t [ \mathbf{i} ]$

# tUPL Loop Structures

Two **BASIC** Loop Structures:

**forelem** (  $t; t \in T$  )

**whilelem** (  $t; t \in T$  )

Both structures are inherently  
**parallel** and **non-deterministic**

This means that any tuple of  $T$  can be taken at any time!!

In the **forelem** structure every tuple is taken **exactly once**, while in the **whilelem** every tuple can be taken an **arbitrary number of times** (details later)



# Example I

## Sparse Matrix-Vector Multiplication

```
forelem ( t; t  $\in$  T )  
{  
    Value_C[t.i] += Value_A[t.i, t.j]  
                  * Value_B[t.j]  
}
```

# Example II (LU factorization)

```
for (k; k ∈ N)
{
  pivot = IDX_A<i,j>[(k,k)] ();
  forelem (t; t ∈ A.<i,j>[<(k,∞),k>])
  {
    mult = Value[t.i,t.j]/Value[t.pivot,t.pivot];
    Value[t.i,t.j] = mult;
    forelem (r; r ∈ A.<i,j>[<t.j,(t.j,∞)>])
    {
      cand = NULL
      forelem (q; q ∈ A.<i,j>[<t.i,t.j>])
      cand = q;
      if (cand == NULL)
      {
        cand = <t,i,t.j>
        A = A U cand;
        Value[cand.i,cand.j] = 0
      }
      Value[cand.i,cand.j] -= mult*Value[r.i,r.j]
    }
  }
}
```

# Example III

## SORTING

```
whilelem ( t; t  $\in$  T )  
{  
    if ( X[t.i] > X[t.j] )  
        swap ( X[t.i], X[t.j] )  
}
```

# Example IV: MaxFlow

$T = \{ \langle u, v, w \rangle \mid (u, v) \text{ and } (v, w) \text{ (back)edges of } G \text{ and } w \neq u \}^*$

```
whilelem ( t; t ∈ T )
{
  if (Delta[t.u,t.v] > 0 && Remainder[t.v,t.w] > 0)
  {
    delta_change = min(Remainder[t.v,t.w],Delta[t.u,t.v]);
    Delta[t.v,t.w]+= delta_change;
    Remainder[t.v,t.w] -= delta_change;
    Remainder[t.w,t.v] += delta_change;
    F[t.u,t.v] += delta_change;
    Delta[t.u,t.v] -= delta_change
  }
  if (Delta[t.u,t.v] > 0 && Remainder[t.v,t.w] == 0)
  {
    if (t.v == 's' || t.v == 't')
    {
      F[t.u,t.v] += Delta[t.u,t.v];
      Delta[t.u,t.v] = 0
    }
    else
    {
      # Reverse Flow
      Delta[t.v,t.u] += Delta[t.u,t.v];
      Remainder[t.v,t.u]-= Delta[t.u,t.v];
      Delta[t.u,t.v] = 0
    }
  }
}
*|T| ≈ (aver_out+aver_in)*(aver_out+aver_in-1)*|V|
≈ aver_out^4*|V|
```

# tUPL Loop Body

One or more conditionally executed serial codes operating on data items which are defined by the tokens from the Tuple Reservoir and their associated address functions\*, i.e.

## tUPL Loop Body:

```
if ( Cond_1 )
{
    Serial_Code_1 (< t >)
}
if ( Cond_2 )
{
    Serial_Code_2 (< t >)
}
...
if ( Cond_n )
{
    Serial_Code_n (< t >)
}
```

- All Cond\_i's are **exclusive for forelem**. For **whilelem** multiple conditions can be **true** at the same time for a tuple.
- n can be 1 and Cond\_1 can be **true**.

\*Except for local/temporary variables with respect to the Loop Body

# Scheduling `whilelem` ( $t; t \in T$ )

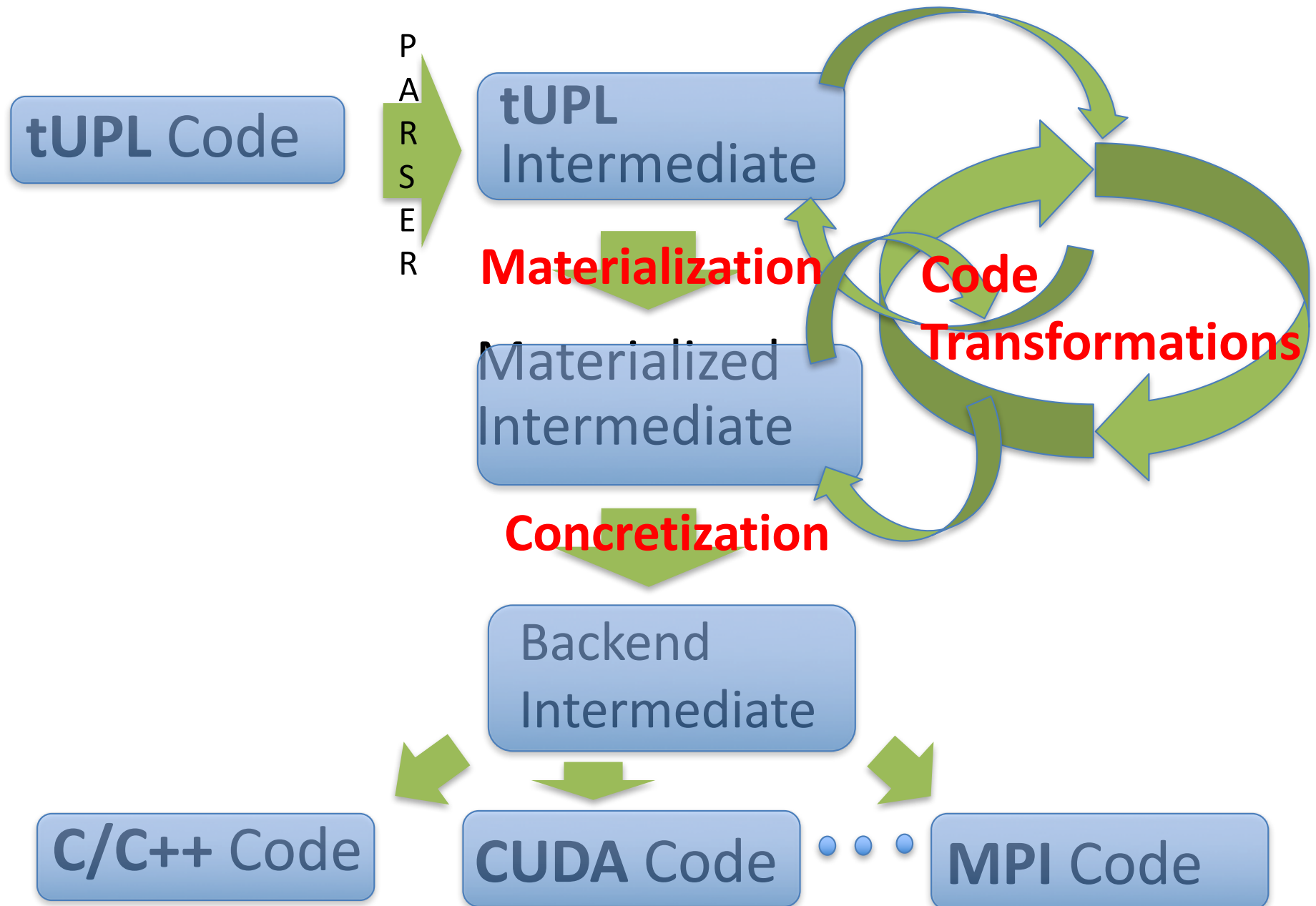
- For each execution of a tuple exactly one of the tuples with a valid conditional serial code is chosen.
- If there are no tuples left with a valid conditional serial code, then the `whilelem` loop terminates.
- Any loop scheduling for a `whilelem` loop must guarantee that every tuple with a valid conditional serial code that is continuously enabled beyond a certain point is taken infinitely many times (cf. **just computation**).

## Scheduling `forelem` ( $t; t \in T$ )

- For each execution of a tuple exactly one of the tuples is chosen with a valid conditional serial code **and which has not been executed so far.**
- If there are no tuples left with a valid conditional serial code, then the `forelem` loop terminates.

Note that if the conditions are not carefully chosen it can happen that the `forelem` loop terminates before all tuples have been executed.

# Automatic Data Structure Generation in tUPL





# tUPL Intermediate

```
forelem ( t; t  $\in$  T )  
{  
    ... t ...  
}  
whilelem ( t; t  $\in$  T )  
{  
    ... t ...  
}
```



```
forelem ( i; i  $\in$  pT )  
{  
    ... T[i] ...  
}  
whilelem ( i; i  $\in$  pT )  
{  
    ... T[i] ...  
}
```

- pT and T[i] notation allows for a more clear expression of the materialization and concretization phase
- tUPL allows mix use of **tUPL** notation and intermediate notation

# Some Code Transformations\*

## Orthogonalization

```
forelem (i; i  $\in$  pA)  
  ... A[i]...
```



```
forelem (ii; ii  $\in$  A.field1)  
  forelem (i; i  $\in$  pA.field1[ii])  
    ... A[i]...
```

A.field1 is the set of all possible field1 values of tuples in A: { i.field1 | i  $\in$  A }

## Encapsulation

```
forelem (i; i  $\in$  pA.field1)  
  ... ..
```



```
forelem (i; i  $\in$  N10)  
  ... ..
```

If A.field1 would be { 0, 1, 3, 4, 7, 9, 10 }, for instance. This transformation only makes sense, if the execution of the inner loop for the other i-value's results into a NOP. i.e. C[i] = C[i] + B[i], and B[i] == 0 for 2, 5, 6 and 8.

\***forelem** is used in the examples but the trafo's equally apply to **whilelem**

# Some Code Transformations (2)

## Loop Collapse

```
forelem (i; i  $\in$  pA)  
  forelem (j; j  $\in$  pB.field_b[A[i].field_a])  
    ... A[i].field_c ... B[j].field_d ...
```



```
forelem (i; i  $\in$  pAxB.field_b[field_a])  
  ... AxB[i].field_c ... AxB[i].field_d ...
```

AxB is the cross product of the two tuple sets A and B:  $\{ \langle i, j \rangle \mid i \in A \text{ and } j \in B \}$

# Some Code Transformations (3)

## Loop Interchange

```
forelem (i; i  $\in$  pA)  
  forelem (j; j  $\in$  pB)  
    ... A[i] ... B[j] ...
```



```
forelem (j; j  $\in$  pB)  
  forelem (i; i  $\in$  pA)  
    ... A[i] ... B[j] ...
```

## Horizontal Iteration Space Reduction

```
forelem (i; i  $\in$  pA)  
  ... A[i].field2 ... A[i].field3 ...
```



```
forelem (i; i  $\in$  pA')  
  ... A'[i].field2 ... A'[i].field3 ...
```

With  $A' = \{ \langle \text{field2}, \text{field3} \rangle \mid \langle \text{field1}, \text{field2}, \text{field3} \rangle \in A \}$

# Materialization

```
forelem (i; i  $\in$  pA.field[X])  
... A[i]...
```



```
forelem (i; i  $\in$  N*)  
... PA[i]...
```

**N\*** represents the set  $\{ 1, 2, \dots, |PA| \}$ , with PA an enumeration of the set:


$$\{ i \mid i \in A \text{ and } i.\text{field} == X \}$$

**DO NOT CONFUSE** PA with a linear array data structure

# Some more code transformations



## Tuple Splitting

```
forelem (i; i  $\varepsilon$  A.field)
  forelem (k; k  $\varepsilon$  pB.field[i])
    ... B[k].value ...
```



```
forelem (i; i  $\varepsilon$   $\mathbf{N}_{10}$ )
  forelem (k; k  $\varepsilon$  pB.field[i])
    ... B[k].value ...
```

```
forelem (i; i  $\varepsilon$   $\mathbf{N}_{10}$ )
  forelem (k; k  $\varepsilon$   $\mathbf{N}^*$ )
    ... B[i][k].value ...
```



```
forelem (i; i  $\varepsilon$   $\mathbf{N}_{10}$ )
  forelem (k; k  $\varepsilon$   $\mathbf{N}^*$ )
    ... B[i].value[k] ...
```

2 dimensional materialization into B[][] necessary because of outerloop dependence.

# Some more code transformations (2)

## N\* Materialization

```
forelem (i; i  $\in$   $\mathbf{N}_{10}$ )  
  forelem (k; k  $\in$   $\mathbf{N}^*$ )  
    ... A[i][k] ...
```



```
forelem (i; i  $\in$   $\mathbf{N}_{10}$ )  
  forelem (k; k  $\in$  PA_len[i])  
    ... A[i][k] ...
```

# Some more code transformations (3)

## Data Localization

```
forelem (i; i  $\in$  pA)  
... B [ A[i] ] ...
```

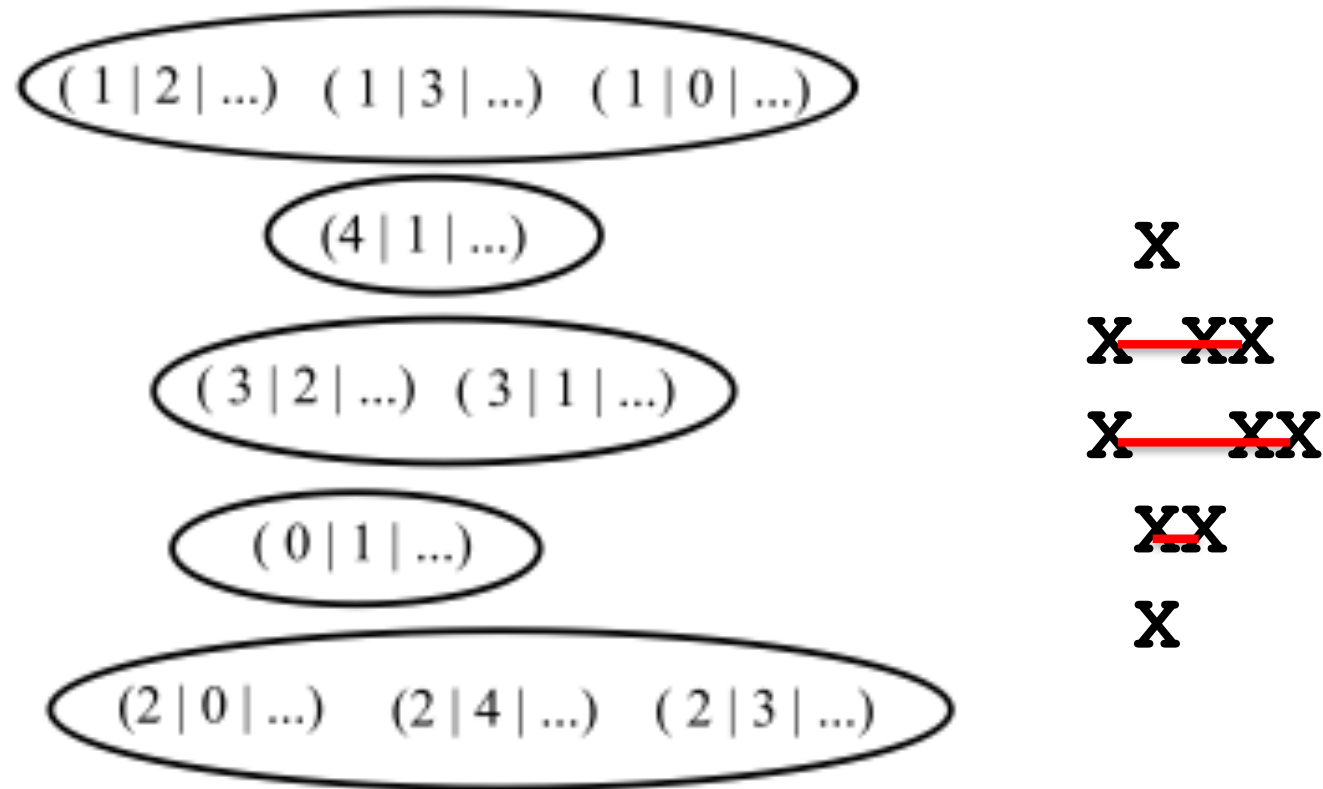


```
forelem (i; i  $\in$  pA')  
... A'[i].field_B ...
```

Here the tuples in reservoir A are being extended to include the data at address `@B[A[i].field_k]`. So  $A' = \{ \langle t, B[t] \rangle \mid t \in A \}$ . By default, this transformation is only allowed for read only data at B.

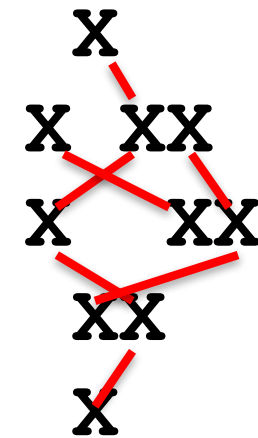
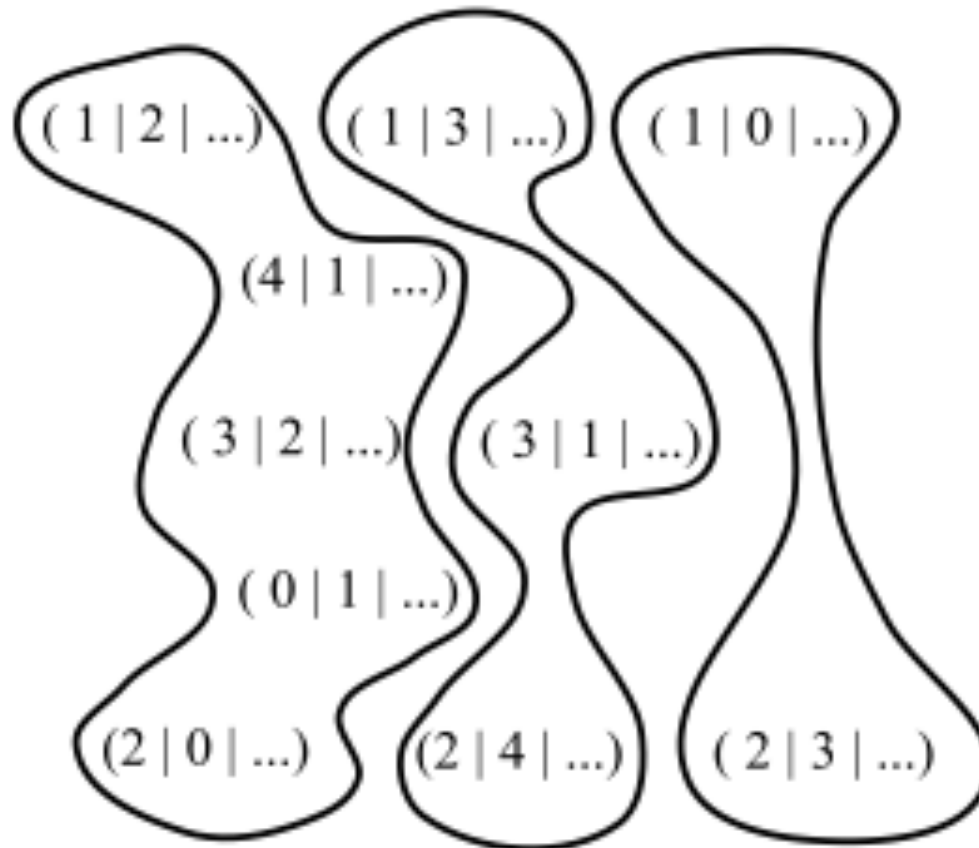


# Regrouping of Single Storage Locations (Tuples)

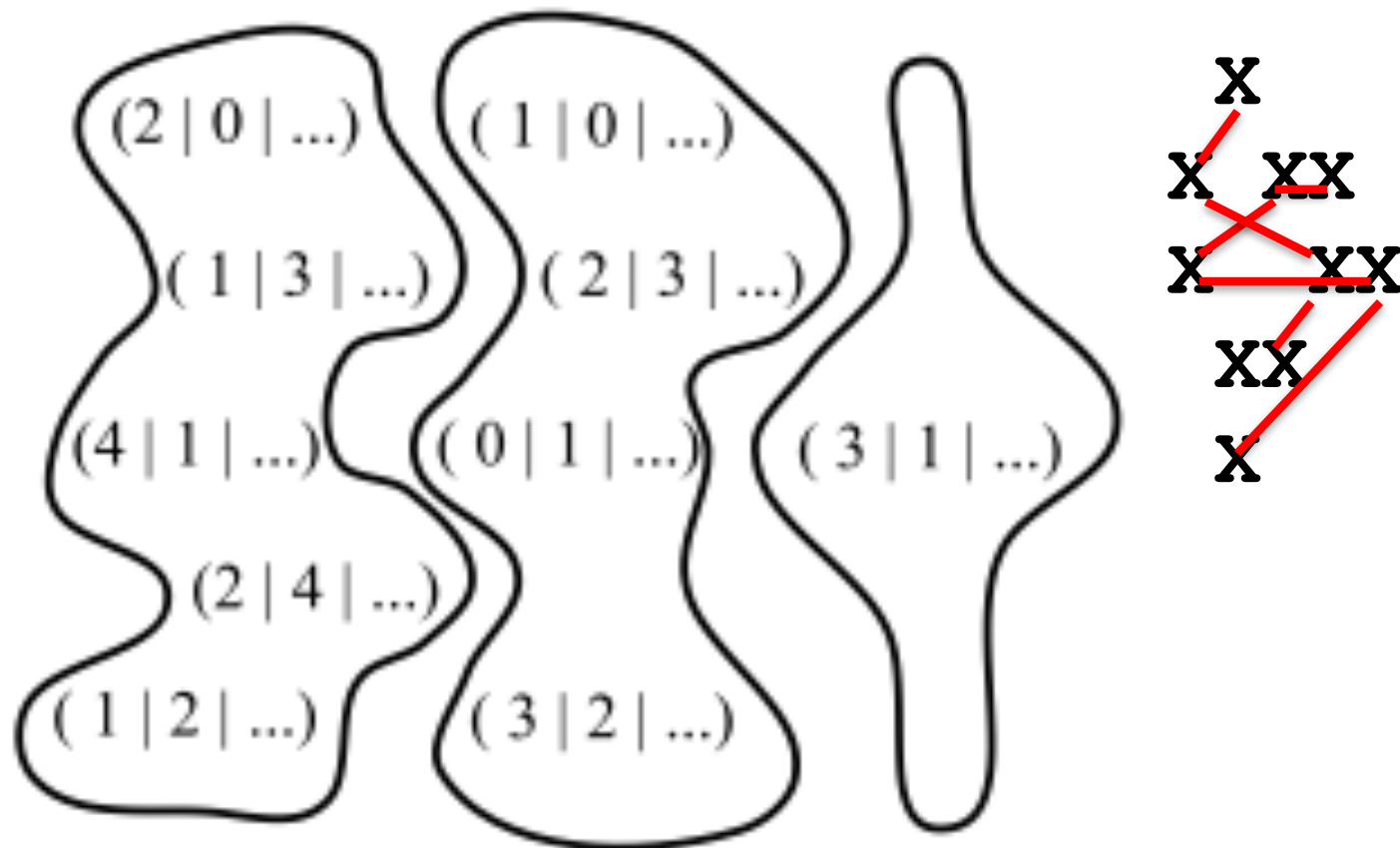


Regrouping as a result of **orthogonalization** on the first field

# Regrouping after **Materialization** and **Loop Interchange**



Regrouping after **orthogonalization** on the second field followed by **materialization** and **loop interchange**



# Concretization

```
forelem (i; i  $\in$  N*)
```

```
... PA[i]...
```



```
forelem (i; i  $\in$  PA_len[i])
```

```
... PA[i] ...
```



```
for (i = 0; i < PA_len; i++)
```

```
... PA[i] ...
```

# Some Concretization Steps

tUPLE loop construct	Concretization
<b>forelem</b> (i; i $\varepsilon$ pA) ... A[i]...	Linked list of struct's
<b>forelem</b> (i; i $\varepsilon$ $\mathbf{N}_{10}$ ) ... A[i]...	An array of struct's
<b>forelem</b> (i; i $\varepsilon$ $\mathbf{N}_{10}$ ) <b>forelem</b> (k; k $\varepsilon$ PA_len[i]) ... A[i][k] ...	An array of arrays of struct's
<b>forelem</b> (i; i $\varepsilon$ $\mathbf{N}_{10}$ ) <b>forelem</b> (k; k $\varepsilon$ PA_len[i]) ... A[i][k].value ...	An array of arrays of struct's
<b>forelem</b> (i; i $\varepsilon$ $\mathbf{N}_{10}$ ) <b>forelem</b> (k; k $\varepsilon$ PA_len[i]) ... A[i].value[k] ...	An array of arrays of values

# Example

```
forelem (i; i ∈ pA)  
  ... B[A[i]]...
```

**A linked list of struct's: A +  
A multidimensional array: B**

**Data Localization**

```
forelem (i; i ∈ pA')  
  ... A'[i].field_B ...
```

**An linked list of struct's: A**

**Materialization**

```
forelem (i; i ∈ pA'_len)  
  ... PA'[i].field_B ...
```

**An array of struct's A'**

**Tuple Splitting**

```
forelem (i; i ∈ pA'_len)  
  ... PA'.field_B[i]...
```

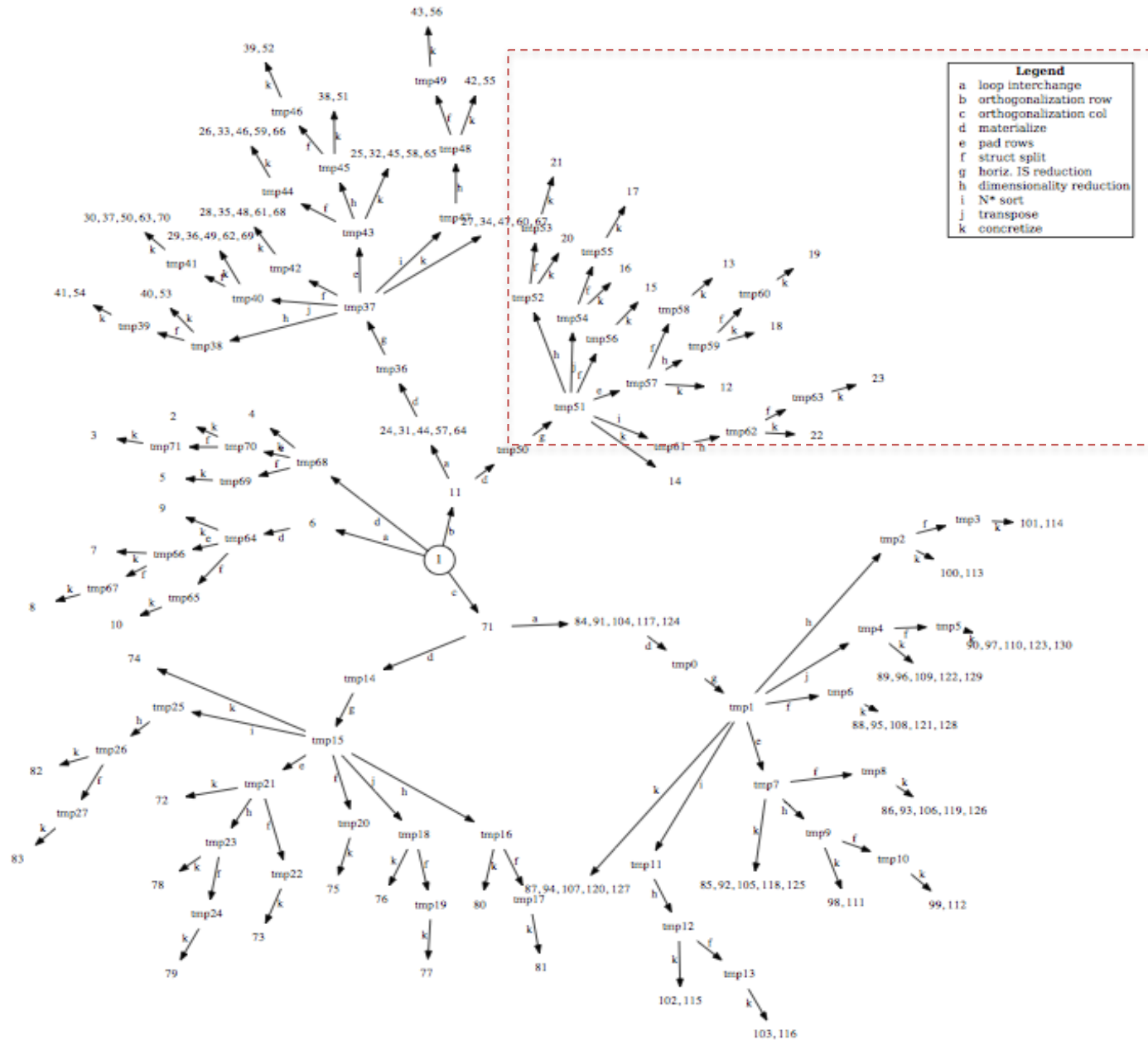
**Several Arrays for each field  
of A'**

**Horizontal Iteration Space Reduction**

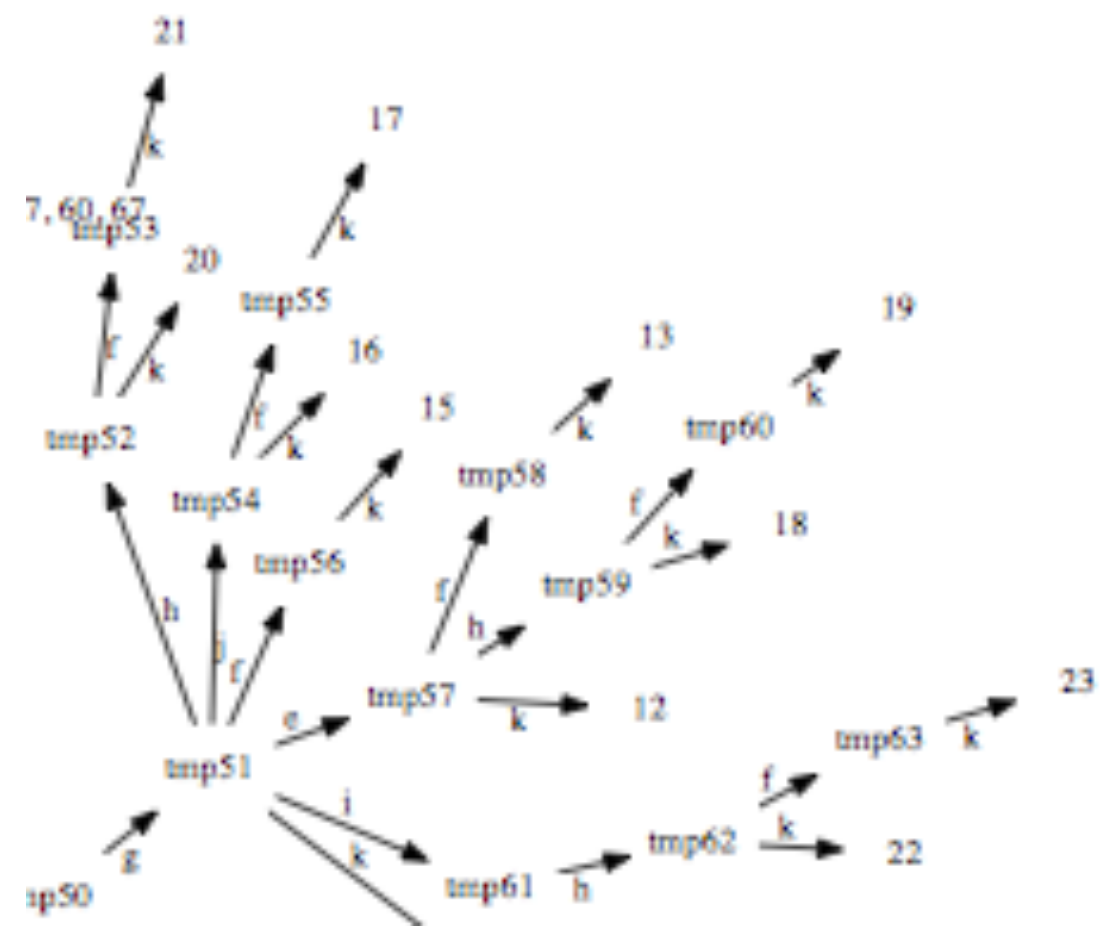
```
forelem (i; i ∈ pA'_len)  
  ... PA'.field_B[i]...
```

**Just one array of field\_B  
values**

# The Transformation Search Space for SpMxM



Legend	
a	loop interchange
b	orthogonalization row
c	orthogonalization col
d	materialize
e	pad rows
f	struct split
g	horiz. IS reduction
h	dimensionality reduction
i	N* sort
j	transpose
k	concretize





# Algorithmic Optimization

- **tUPL** will automatically choose sequences of valid serial codes to be executed one after the other, so that their execution is being optimized.
- So, next to the automatic generation of data structures **tUPL** will also automatically optimize and change the order in which operations are performed and by doing so will change the actual algorithm being used to compute the results.
- These sequences are being identified as **chains** of pairs of tuples and serial codes:

`( tk, Serial_Code_i )*`

representing

`Serial_Code_i (< tk >)`

\***Note that Cond\_i has to evaluate to true for every t<sub>k</sub>**

# Recap

## tUPL Loop Body:

```
if ( Cond_1 )
{
    Serial_Code_1 (< t >)
}
if ( Cond_2 )
{
    Serial_Code_2 (< t >)
}
...
if ( Cond_n )
{
    Serial_Code_n (< t >)
}
```

# Different kind of chains

- **Mono Chains (MC)**, every element in the chain has the same serial code:

$(t_1, \text{Serial\_Code}_i), (t_2, \text{Serial\_Code}_i), \dots$

- **Two Typed Chains:**

- **Alternating Chains (AC)**, consecutive elements in the chain alternate between  $\text{Serial\_Code}_i$  and  $\text{Serial\_Code}_j$

- **Cascading Chains (CC)**, first part of the chain uses  $\text{Serial\_Code}_i$  the second part of the chain uses  $\text{Serial\_Code}_j$

$(t_1, \text{Serial\_Code}_i), (t_2, \text{Serial\_Code}_i), \dots,$   
 $(t_k, \text{Serial\_Code}_j), (t_{k+1}, \text{Serial\_Code}_j), \dots$

- **Hybrid Chains (HC)**

# Profitable Chain

A chain  $C$  is **profitable\*** iff

- The consecutive execution of the elements in  $C$  can be optimized such that **the execution time of the whole chain is less than the sum of the execution times of the individual elements**
- AND the chain is **minimal** in such a way that the chain  $C$  cannot be broken into smaller chains  $C_1$  and  $C_2$  such that  $C = C_1 \parallel C_2$  and
$$\text{Exec}(C) = \text{Exec}(C_1) + \text{Exec}(C_2)$$

\*  $C$  is being referred to as a profit chain

# Main Theorem I

For every profit chain C:

all consecutive elements in C:

$(t_1, \text{Serial\_Code\_i}), (t_2, \text{Serial\_Code\_j})$

have a data dependence on an address function

A used in both serial codes: `Serial_Code_i`,  
`Serial_Code_j`, i.e.

$$@A[t_1] == @A[t_2]$$

# Profit Chains in SpMxV

```
forelem ( t; t  $\in$  T )  
  {  
    Value_C[t.i] += Value_A[t.i,t.j]  
                  * Value_B[t.j]  
  }
```

( $\langle 1, 1 \rangle$ , Serial\_Code\_1), ( $\langle 1, 2 \rangle$ , Serial\_Code\_1), ...  
can be optimized such that subsequent reads of  
Value\_C[t.i] are eliminated. So these chains are  
identified as profit chains.

In fact, the orthogonalization code optimization is a  
direct result of this chaining

# Covering Chain Set

A **covering chain set CCS** is a set of Chains  $C_i$  such that for every tuple  $(t_k, \text{Serial\_Code\_}i)$  there is an  $i$  such that

$$(t_k, \text{Serial\_Code\_}i) \in C_i$$

Note that if the possible set of profit chains is not covering then this set can be completed with **single (non-profit) chains**, consisting out of the  $(t_k, \text{Serial\_Code\_}i)$  pairs which were not covered, to obtain a covering chain set.

# Main Theorem II

If

**whilelem** (  $t; t \in T$  )

is just scheduled, then if

**whilelem** (  $C; C \in CCS$  )

**forelem** (  $t; t \in C$  )

is also just scheduled, then both loop structures are semantically equivalent.



**forelem** (  $t$ ;  $t \varepsilon T$  )

and

**forelem** (  $C$ ;  $C \varepsilon CCS$  )

**forelem** (  $t$ ;  $t \varepsilon C$  )

are semantically equivalent just based on the covering property of CCS.

# Examples of profit chains I

```
whilelem ( t; t  $\in$  T )  
  {  
    if ( X[t.i] > X[t.j] )  
      swap ( X[t.i], X[t.j] )  
  }
```

(<1,2>, Serial\_Code\_1),

(<2,3>, Serial\_Code\_1),

(<3,4>, Serial\_Code\_1),..., (<n-1,n>, Serial\_Code\_1)

with  $X[1]>X[2]$ ,  $X[2]>X[3]$ , etc, results in a sequence of  $n$  swaps,  
whereas it can be optimized by executing just one insert!!!

# Examples of profit chains II

```
whilelem ( t; t ∈ T )
{   if (Delta[t.u,t.v] > 0 && Remainder[t.v,t.w] > 0)
    {
        delta_change = min(Remainder[t.v,t.w],Delta[t.u,t.v]);
        Delta[t.v,t.w] += delta_change;
        Remainder[t.v,t.w] -= delta_change;
        Remainder[t.w,t.v] += delta_change;
        F[t.u,t.v] += delta_change;
        Delta[t.u,t.v] -= delta_change
    }
    if (Delta[t.u,t.v] > 0 && Remainder[t.v,t.w] == 0)
    {
        ...
        else
        {   # Reverse Flow
            Delta[t.v,t.u] += Delta[t.u,t.v];
            Remainder[t.v,t.u] -= Delta[t.u,t.v];
            Delta[t.u,t.v] = 0
        }
    }
}
```

Serial\_Code\_1

Serial\_Code\_2

Then  $(\langle s, 4, 6 \rangle, \text{Serial\_Code\_1}), (\langle 4, 6, 52 \rangle, \text{Serial\_Code\_1}), \dots, (\langle 100, 105, 107 \rangle, \text{Serial\_Code\_1}), (\langle 105, 107, 111 \rangle, \text{Serial\_Code\_2}), (\langle 111, 107, 105 \rangle, \text{Serial\_Code\_1}), \dots (\langle 6, 4, s \rangle, \text{Serial\_Code\_1})$  with  $\text{Remainder}[4, 6] > 0$ , with  $\text{Remainder}[6, 52] > 0, \dots$  etc., and  $\text{Remainder}[107, 111] = 0$  is a profit chain.

As well as

$(\langle s, 4, 6 \rangle, \text{Serial\_Code\_1}), (\langle 4, 6, 52 \rangle, \text{Serial\_Code\_1}), \dots, (\langle 100, 105, 107 \rangle, \text{Serial\_Code\_1}), (\langle 105, 107, t \rangle, \text{Serial\_Code\_1}),$  with  $\text{Remainder}[4, 6] > 0$ , with  $\text{Remainder}[6, 52] > 0, \dots$  etc.

Note that the latter profit chain is in fact the **augmented path** as defined by Ford and Fulkerson!!!

# Parallel Programming II (this spring)

- **tUPL** will automatically choose sequences of valid serial codes to be executed one after the other, so that their execution is being optimized.
- So, next to the automatic generation of data structures **tUPL** will also **automatically optimize and change the order in which operations are performed** and by doing so will change the actual algorithm being used to compute the results.
- In fact within **tUPL new algorithms can be automatically generated** which will not only execute in parallel but will also be adaptive to the underlying problem to be solved.

**END OF COURSE**