Voorbereiding Programmeerwedstrijden

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`http://www.liacs.leidenuniv.nl/~vlietrvan1/vbpw/`

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Geometry

Computational Geometry

13.4. Faster Than a Speeding Bullet

- given:
  - obstacles do not overlap
  - start and target positions lie outside of obstacles

- algorithm. . .

Faster Than a Speeding Bullet

$$\text{travel} = \text{distance}(s,t) + \sum_{\substack{\text{intersecting}\\\text{circles}}} (\text{arclength} - \text{linesegmentlength})$$

Representation Point

```
typedef double point[2];
const int X = 0;
const int Y = 1;

int main ()
{ point p;

  p[X] = ...;
  p[Y] = ...;
}
```

Representation Circle

```
typedef struct
{ point center;    // center of circle
  double r;        // radius of circle
} circle;

int main ()
{ circle c;

  c.center[X] = ...;
  c.center[Y] = ...;
  c.r = ...;
}
```

Representation Line

Representation Line

```
typedef struct
{ double a,   // a, b and c are coefficients in equation ax + by + c = 0,
         b,   // describing the line.
         c;
} line;

int main ()
{ line l;

  l.a = ...;
  l.b = ...;
  l.c = ...;
}
```

b (or a) is normalized to 1

Closest Point On Line

```
void closest_point (point p_in, line l, point p_c)
{ line perp;      // perpendicular to line l through point p

  if (fabs(l.b) <= EPSILON)  // vertical line
  { p_c[X] = -l.c;
    p_c[Y] = p_in[Y];
    return;
  }

  if (fabs(l.a) <= EPSILON)  // horizontal line
    ...   // analogous


  // Otherwise ...

}  // closest_point
```

Closest Point On Line

- find perpendicular line (how?)

- find intersection point (how?)

Perpendicular Line

$y = mx + k1$
is perpendicular to
$y = -(1/m)x + k2$

```
void point_and_slope_to_line (point p, double m, line *l)
{ ...
}
```

Point and Slope To Line

```
void point_and_slope_to_line (point p, double m, line *l)
{
  l->a = -m;
  l->b = 1;
  l->c = - ((l->a)*p[X] + (l->b)*p[Y]);

}
```

Closest Point On Line

```
void closest_point (point p_in, line l, point p_c)
{ line perp;      // perpendicular to line l through point p

  if (fabs(l.b) <= EPSILON)  // vertical line
  { p_c[X] = -l.c;
    p_c[Y] = p_in[Y];
    return;
  }

  if (fabs(l.a) <= EPSILON)  // horizontal line
    ...   // analogous

  point_and_slope_to_line (p_in, 1/(l.a), &perp);
  intersection_point (l, perp, p_c);

}  // closest_point
```

Intersection Point

(general case)

$$
\begin{aligned}
a_1 x + b_1 y + c_1 &= 0 \\
a_2 x + b_2 y + c_2 &= 0
\end{aligned}
$$

Intersection Point

(general case)

$$
\begin{aligned}
a_1 b_2 x + b_1 b_2 y + c_1 b_2 &= 0 \\
b_1 a_2 x + b_1 b_2 y + b_1 c_2 &= 0
\end{aligned}
$$

Intersection Point

(general case)

$$\begin{aligned} a_1 b_2 x + b_1 b_2 y + c_1 b_2 &= 0 \\ b_1 a_2 x + b_1 b_2 y + b_1 c_2 &= 0 \end{aligned}$$

$$(a_1 b_2 - b_1 a_2) x + c_1 b_2 - b_1 c_2 = 0$$

$$x = \frac{b_1 c_2 - c_1 b_2}{a_1 b_2 - b_1 a_2}$$

Intersection Point

(general case)

$$
\begin{aligned}
a_1b_2x + b_1b_2y + c_1b_2 &= 0 \\
b_1a_2x + b_1b_2y + b_1c_2 &= 0
\end{aligned}
$$

$$
(a_1b_2 - b_1a_2)x + c_1b_2 - b_1c_2 = 0
$$

$$
x = \frac{b_1c_2 - c_1b_2}{a_1b_2 - b_1a_2}
$$

$$
y = -\frac{a_1c_2 - c_1a_2}{a_1b_2 - b_1a_2}
$$

unless lines are parallel / equal. . .

Point In Box

```
bool point_in_box (point p, point b1, point b2)
{
  return ( (p[X] >= min (b1[X], b2[X]) - EPSILON) &&
           (p[X] <= max (b1[X], b2[X]) + EPSILON) &&
           (p[Y] >= min (b1[Y], b2[Y]) - EPSILON) &&
           (p[Y] <= max (b1[Y], b2[Y]) + EPSILON) );
}
```

Triangles and Trigonometry

- $\sin(\alpha)$, $\cos(\alpha)$, $\tan(\alpha)$
- $(\sin(\alpha))^2 + (\cos(\alpha))^2 = 1$
- degrees vs. radians: 360 degrees $\approx 2\pi$
- $\cos(\alpha) = \sin(\alpha + (\pi/2))$
- $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$

13.2.3. Solving Triangles

- Pythagoras: $a^2 + b^2 = c^2$

- In general:

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

- In general:

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

13.2.3. Solving Triangles

- In general:

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

- In general:

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

- given two angles and one side...

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13.2.3. Solving Triangles

area $A(T)$ of triangle $T\ldots$

13.2.3. Solving Triangles

area $A(T)$ of triangle T

- $A(T) = (1/2)ab$, for altitude and base
-

$$2A(T) = \begin{vmatrix} a_x & a_y & 1 \\ b_x & b_y & 1 \\ c_x & c_y & 1 \end{vmatrix} = a_x b_y - a_y b_x + a_y c_x - a_x c_y + b_x c_y - c_x b_y$$

(in absolute value)...

Here, $a = (a_x, a_y)$, $b = (b_x, b_y)$, $c = (c_x, c_y)$ are vertices (not lengths of edges)

13.3. Circles

- line tangent to circle

- intersection points of two circles

To Which Side of a Line

point c is to right of $a \to b$, if

$$a_x b_y - a_y b_x + a_y c_x - a_x c_y + b_x c_y - c_x b_y < 0$$

Area of Convex Polygon

Area of Convex Polygon

triangulation from arbitrary vertex

13.6.7. Is This Integration?

13.6.3. The Knights of the Round Table?