

Theorie van Concurrency

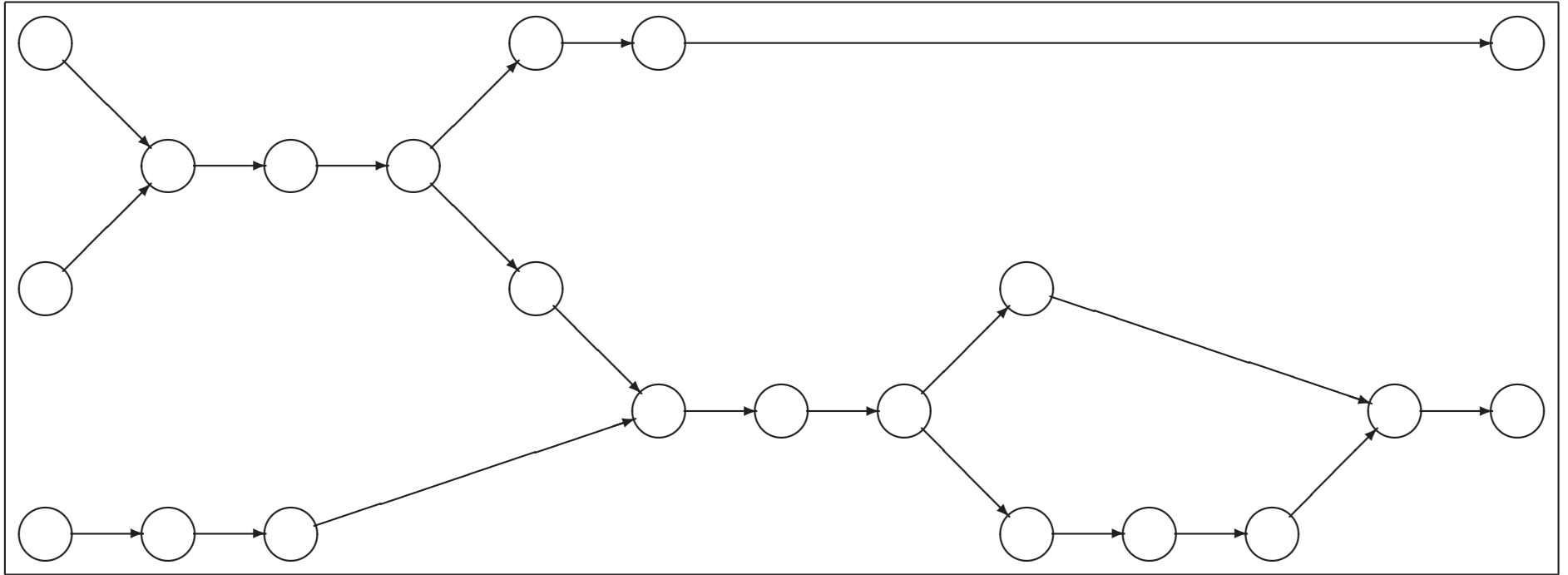
najaar 2011

<http://www.liacs.nl/home/rvvliet/tvc/>

negende college: 13 oktober 2011

6. Processes

6.2 Process Nets



Graphical representation of a partially ordered set (A, ρ)
(except for ...).

li_ρ , co_ρ , cliques, lines, cuts, ${}^\circ A$, A°

Definition 73. A net $N = (P, T, F)$ is a *process net* if:

(1) N is acyclic, and

(2) $\#(\bullet p) \leq 1$ and $\#(p\bullet) \leq 1$ for all $p \in P$.

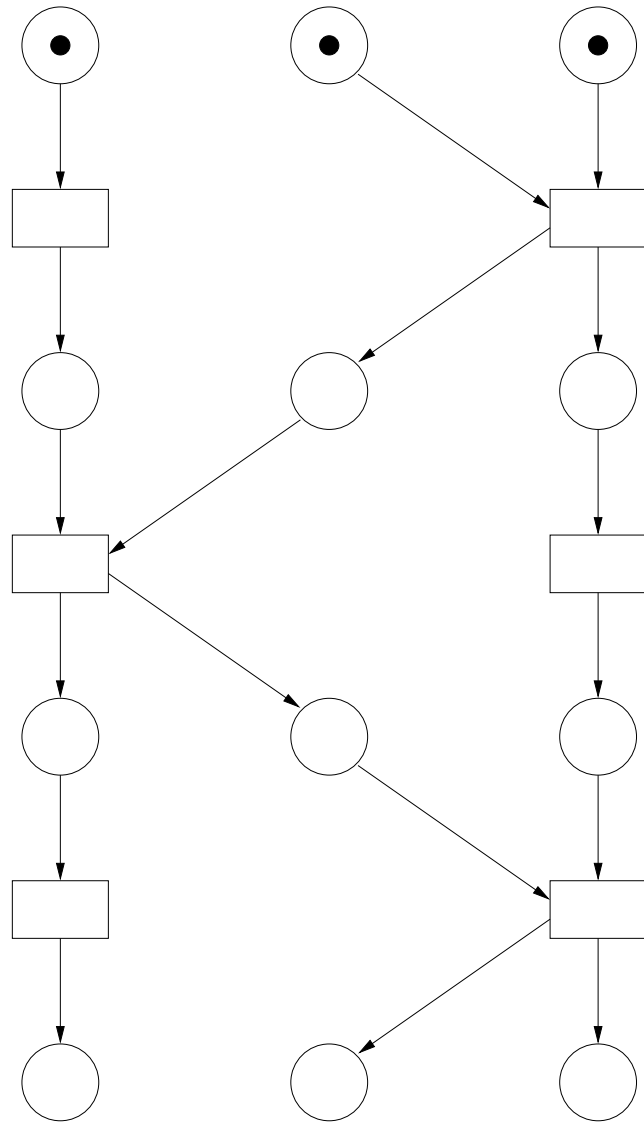


Fig. 52. A process net.

Definition 73. A net $N = (P, T, F)$ is a *process net* if:

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Definition 62. Let A be a finite set.

A binary relation $\rho \subseteq A \times A$ is a *partial order on A* if

ρ is irreflexive and transitive;

(A, ρ) is also called a *partially ordered set*.

Partial order 'is' transitive, directed acyclic graph.

Lemma 74. For every process net N ,
 F_N^+ is a partial order on X_N .

$$\mathbf{li}_N = \mathbf{li}_{F^+}$$

$$\mathbf{co}_N = \mathbf{co}_{F^+}$$

Definition 75. A *slice* of a process net N is a cut C of N such that $C \subseteq P_N$.

Lemma 76. Let $N = (P, T, F)$ be a process net and let $C \subseteq P$.

C is a slice of N iff

(1) for all $p, q \in C$, $p \text{ co}_N q$, and

(2) for every $p \in P - C$ there exists $q \in C$ such that $\neg p \text{ co}_N q$.

Lemma 76. Let $N = (P, T, F)$ be a process net and let $C \subseteq P$.
 C is a slice of N iff

(1) for all $p, q \in C$, $p \text{ co}_N q$, and

(2) for every $p \in P - C$ there exists $q \in C$ such that $\neg p \text{ co}_N q$.

A cut C is a maximal co-clique

Definition 65'.

C is a *co-clique* if

$a \text{ co } b$ for all $a, b \in C$, and

C is a *maximal co-clique* if

C is a co-clique and

for every $a \in X_N - C$ there exists $b \in C$ such that $\neg a \text{ co } b$.

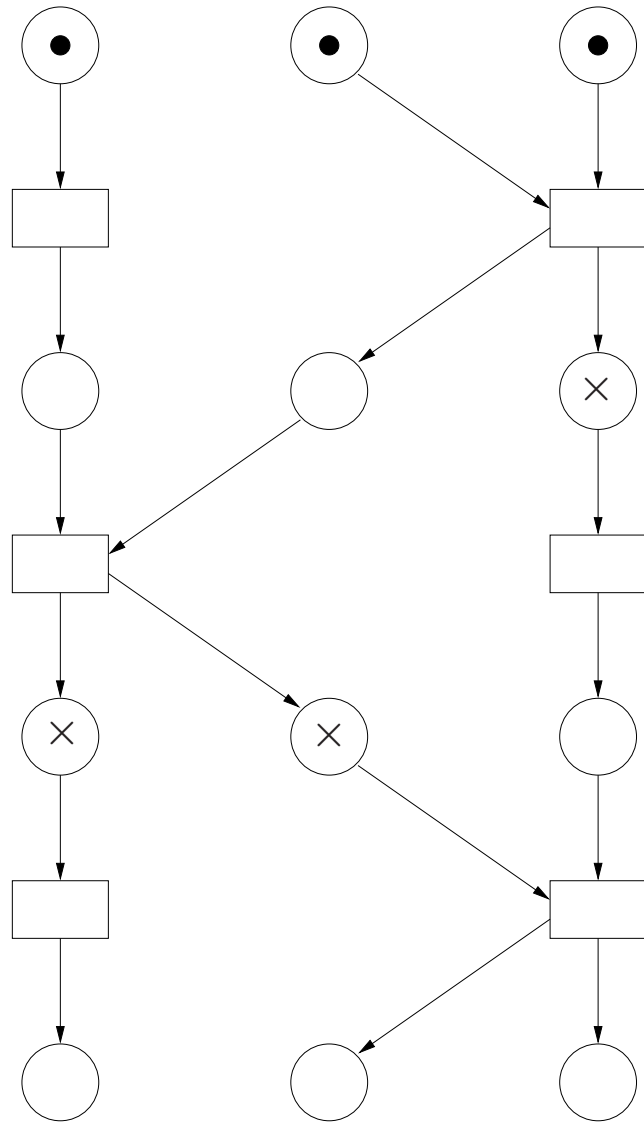


Fig. 52. A process net.

Lemma 66. Let A be a finite set and let $\sigma \subseteq A \times A$ be a reflexive symmetric relation.

For every σ -clique B
there exists a maximal σ -clique C with $B \subseteq C$.

Lemma 77. Let $N = (P, T, F)$ be a process net.
For every co_N -clique $B \subseteq P$
there exists a slice C of N with $B \subseteq C$.

$${}^{\circ}N = {}^{\circ}X_N$$

$$N^{\circ} = X_N^{\circ}$$

Process net $N = (P, T, F)$ corresponds to EN system $(P, T, F, {}^{\circ}N)$.

$(P, T, F, {}^{\circ}N)$ is conflict-free

If $N = (P, T, F)$ is isomorphic to $N' = (P', T', F')$,
with bijection α on places,
then $\alpha({}^{\circ}N) = {}^{\circ}N'$.

Aim:

Theorem 79. Let $N = (P, T, F, \circ N)$ be a process net and let $C \subseteq P$.

$C \in \mathbb{C}_N$ iff C is a slice of N .

Lemma 78. Let $N = (P, T, F)$ be a process net.

(1) For every $U \subseteq T$, if U is a co-clique, then $\bullet U$ and U^\bullet are co-cliques.

In particular, $\bullet t$ and t^\bullet are co-cliques for every $t \in T$.

(2) For every co-clique $U \subseteq T$ there exists a slice C such that $\bullet U \subseteq C$.

(3a) For every slice C and every $t \in T$, if $\bullet t \subseteq C$, then

$t^\bullet \cap C = \emptyset$ and

$D = (C - \bullet t) \cup t^\bullet$ is a slice such that $\rightarrow D = \rightarrow C \cup \{t\} \cup t^\bullet$.

Lemma 78 Ctd. Let $N = (P, T, F)$ be a process net.

(3b) For every slice C and every $t \in T$,

if $t^\bullet \subseteq C$, then

${}^\bullet t \cap C = \emptyset$ and

$D = (C - t^\bullet) \cup {}^\bullet t$ is a slice such that $\rightarrow D = \rightarrow C - t^\bullet - \{t\}$.

(4) For every slice C and every transition t ,

if $t \in \rightarrow C$, then

$\text{nbh}(t) \subseteq \rightarrow C$.

(5) For every slice $C \neq {}^\circ N$

there exists $t \in T$ such that $t^\bullet \subseteq C$.

Theorem 79. Let $N = (P, T, F, \circ N)$ be a process net and let $C \subseteq P$.

$C \in \mathbb{C}_N$ iff C is a slice of N .

Since process nets are conflict-free and contact-free:

Theorem 80. Let N be a process net, let $C \in \mathbb{C}_N$, and let $U \subseteq T_N$ with $U \neq \emptyset$.

If $\bullet U \subseteq C$, then $U \text{ con } C$.

Theorem 81. Let $N = (P, T, F, \circ N)$ be a process net.

(1) N is reduced.

(2) For every $U \subseteq T$,
($\exists C \in \mathbb{C}_N : U \text{ con } C$) iff U is a co-clique.