Theorie van Concurrency

najaar 2011

http://www.liacs.nl/home/rvvliet/tvc/

achtste college: 6 oktober 2011

5. Equivalences and Normal Forms
5.4 Subsystems and Sequential Components
5.5 Contact-freeness
6. Processes
6.1 Partial Orders

derde werkcollege: 11 oktober 2011

Definition 52. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) A set $\{M_1, \ldots, M_n\}$ of subsystems of M, $n \geq 0$,
with $M_i = (S_i, T_i, F_i, (C_{in})_i)$ for $1 \leq i \leq n$,
is a *covering* of M if
$P = \bigcup_{i=1}^{n} S_i$,
$T = \bigcup_{i=1}^{n} T_i$,
$F = \bigcup_{i=1}^{n} F_i$, and
$C_{in} = \bigcup_{i=1}^{n} (C_{in})_i$.

(2) M is *covered by sequential components*
if there exists a covering $\{M_1, \ldots, M_n\}$, $n \geq 0$, of M
such that M_i is a sequential component of M for every $1 \leq i \leq n$.

Lemma 53. Let $M = (P, T, F, C_{in})$ be an EN system, and let, for every $1 \leq i \leq n$ (with $n \geq 0$), $M_i = (S_i, T_i, F_i, (C_{in})_i)$ be a subsystem of M.

Then $\{M_1, \ldots, M_n\}$ is a covering of M
iff $P = \bigcup_{i=1}^{n} S_i$.

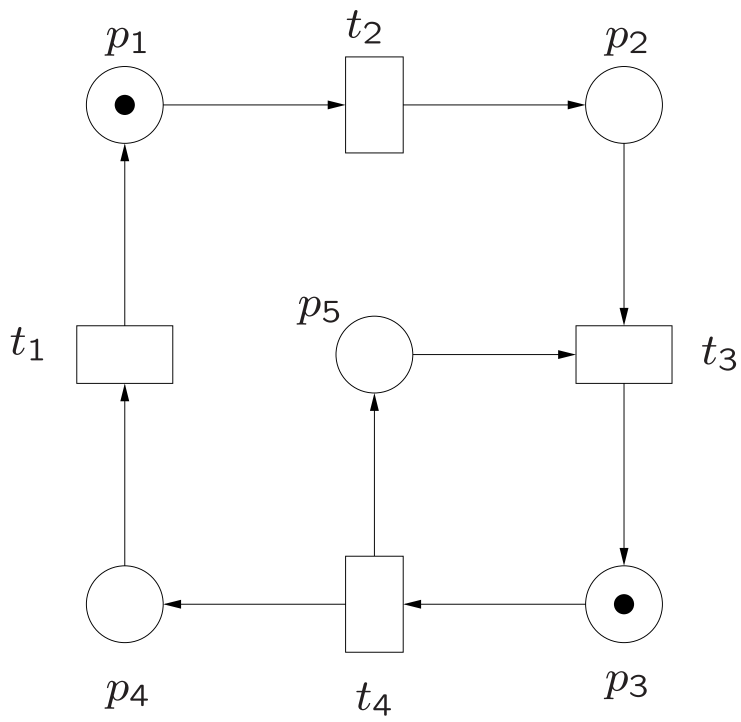

Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

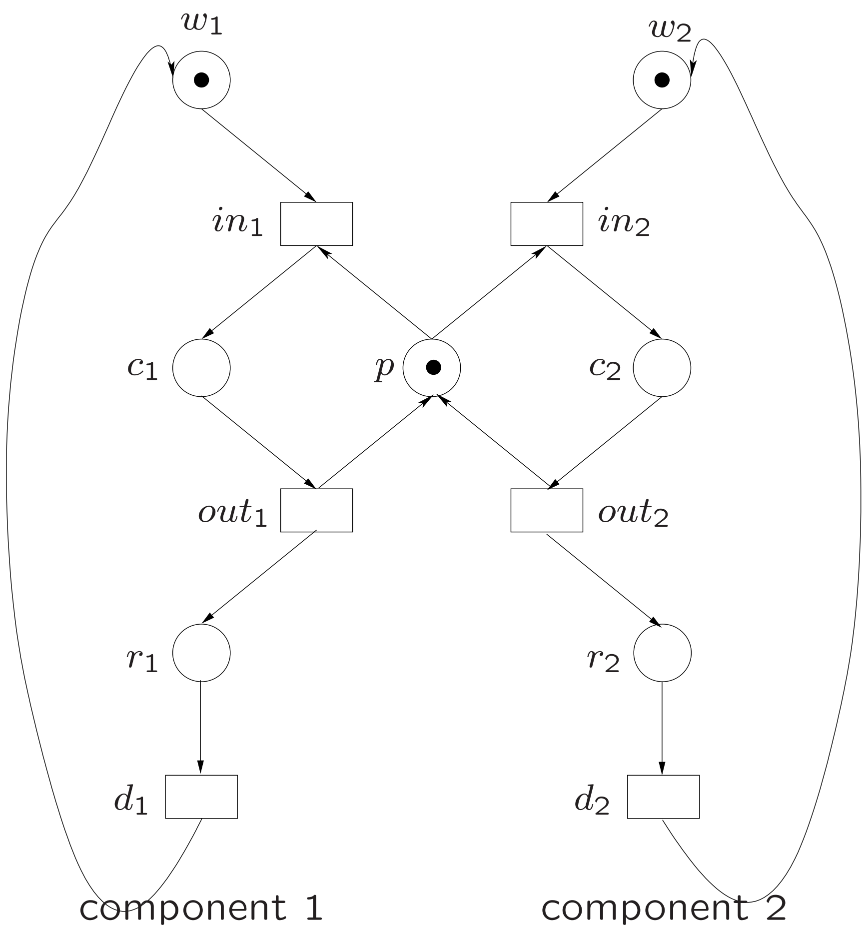

Fig. 5. The mutual exclusion problem

Aim:

Theorem 54. For every EN system M there exists
a reduced EN system M'
that is configuration equivalent with M and
that is covered by at most $\#P_M$ sequential components.

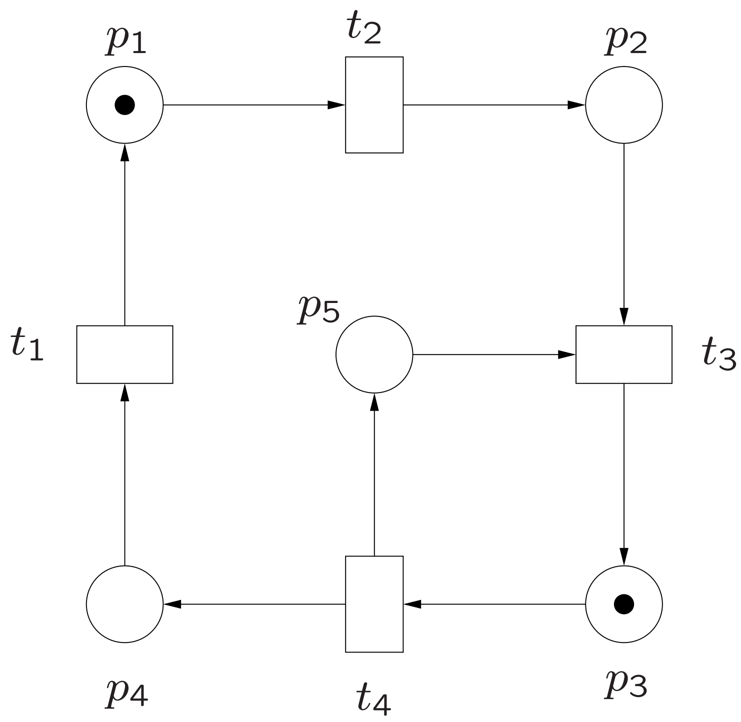

Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

Definition 55. Let M be an EN system and let $p, q \in P_M$.

Then p and q are *complementary*, denoted by $p \ \mathbf{com} \ q$, if

$p^\bullet = {}^\bullet q$ and ${}^\bullet p = q^\bullet$.

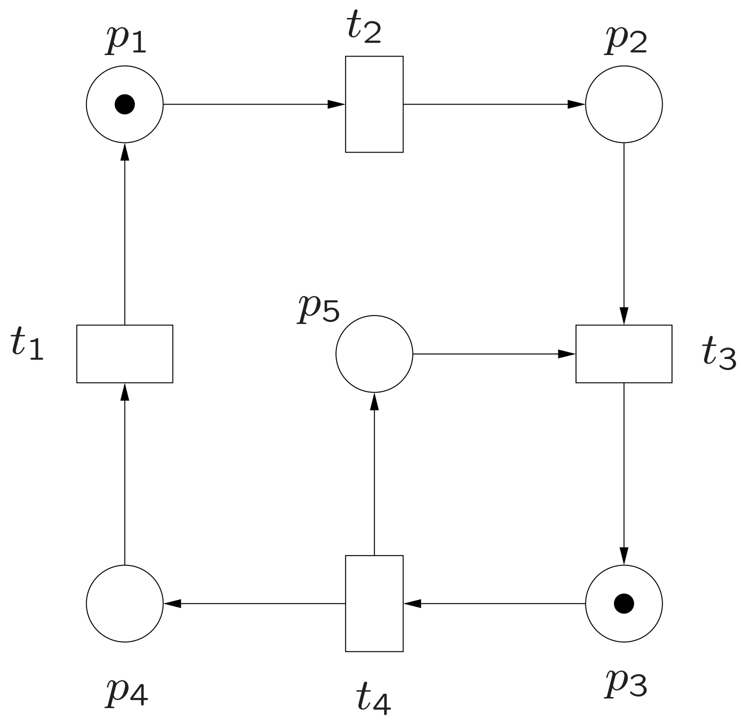

Fig. 39. p_3 and p_5 are complementary.

Lemma 56. Let $M = (P, T, F, C_{in})$ be a reduced EN system. For all $p, q \in P$,

$\{p, q\}$ is a sequential component of M

iff

$\#(C_{in} \cap \{p, q\}) = 1$ and p **com** q.

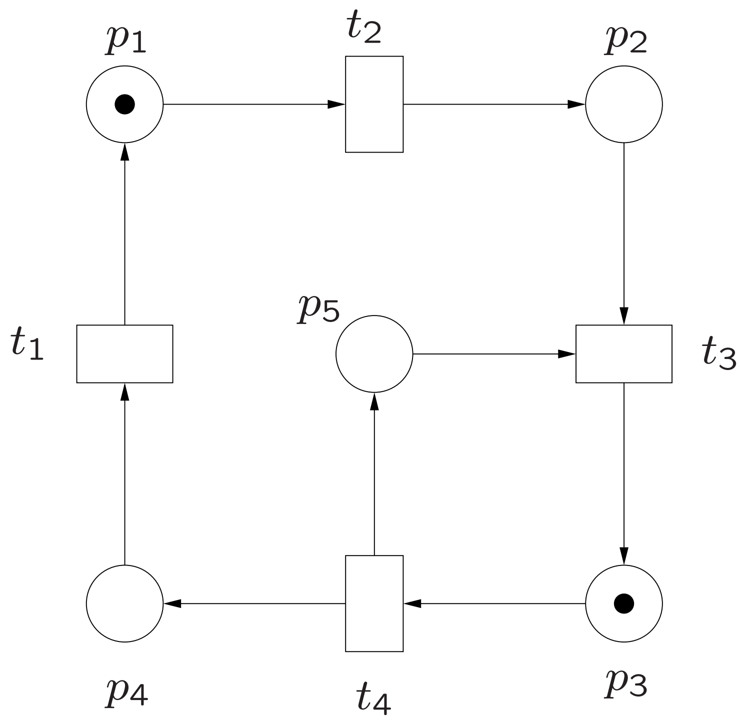

Fig. 39. p_3 and p_5 are complementary.

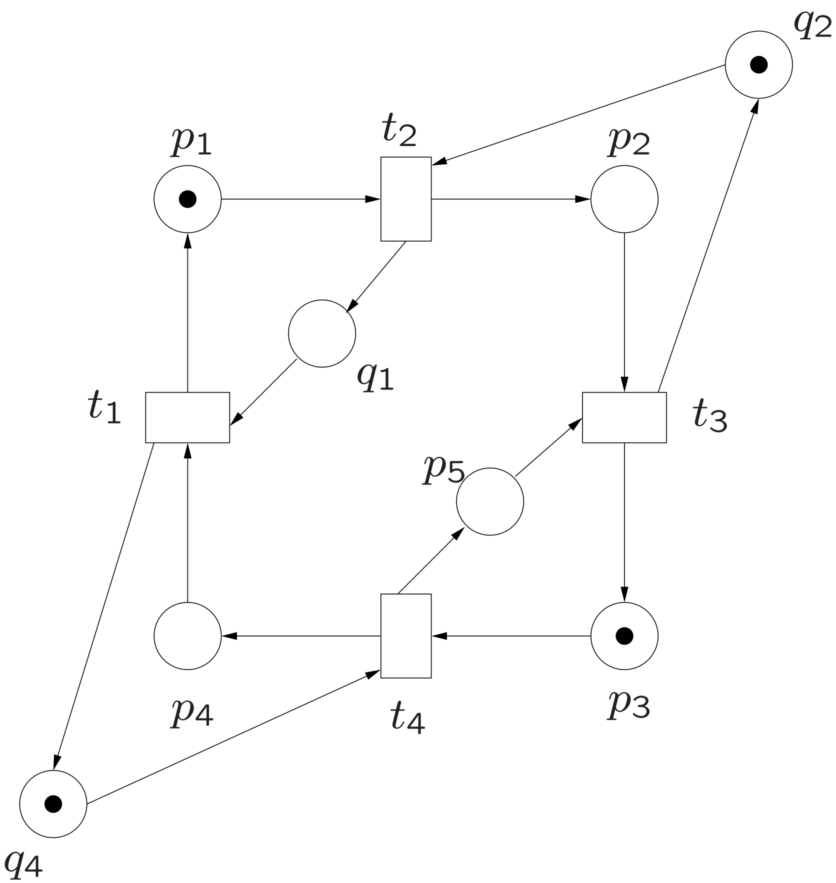

Fig. 48. The result of complementing p_1, p_2, p_4 in Fig. 39.

Theorem 57. Let M be a reduced EN system and let $p_0 \in P_M$.

Then there exists a reduced EN system M' that is configuration equivalent with M, such that:

(1) $P_{M'} = P_M \cup \{q_0\}$ with $q_0 \notin P_M$,
(2) $\{p_0, q_0\}$ is a sequential component of M', and
(3) for every $S \subseteq P_M$,
S is a sequential component of M iff
S is a sequential component of M'.

Lemma 30. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

If α is an injective function, $\alpha : \mathbb{C}_M \to \mathcal{P}(P')$,
and β is a bijective function, $\beta : \mathbf{use}_M(T) \to T'$, such that

(1) $\alpha(C_{in}) = C'_{in}$ and
(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$,
$C[t\rangle_M D$ implies $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$, and
$\beta(t)\ \mathbf{con}_{M'}\ \alpha(C)$ implies $t\ \mathbf{con}_M\ C$,

then $M \approx^{\alpha}_{\beta} M'$.

Theorem 49. Let $M = (P, T, F, C_{in})$ be a reduced EN system and let $S \subseteq P$.
Then the following statements are equivalent.

(1) There is a sequential component M' of M with $P_{M'} = S$,

(2) $\#(C \cap S) = 1$ for all $C \in \mathbb{C}_M$,

(3) (i) $\#(C_{in} \cap S) = 1$, and
(ii) $\forall t \in T$:
$\#(^\bullet t \cap S) = \#(t^\bullet \cap S) = 1$ or $\#(^\bullet t \cap S) = \#(t^\bullet \cap S) = 0$.

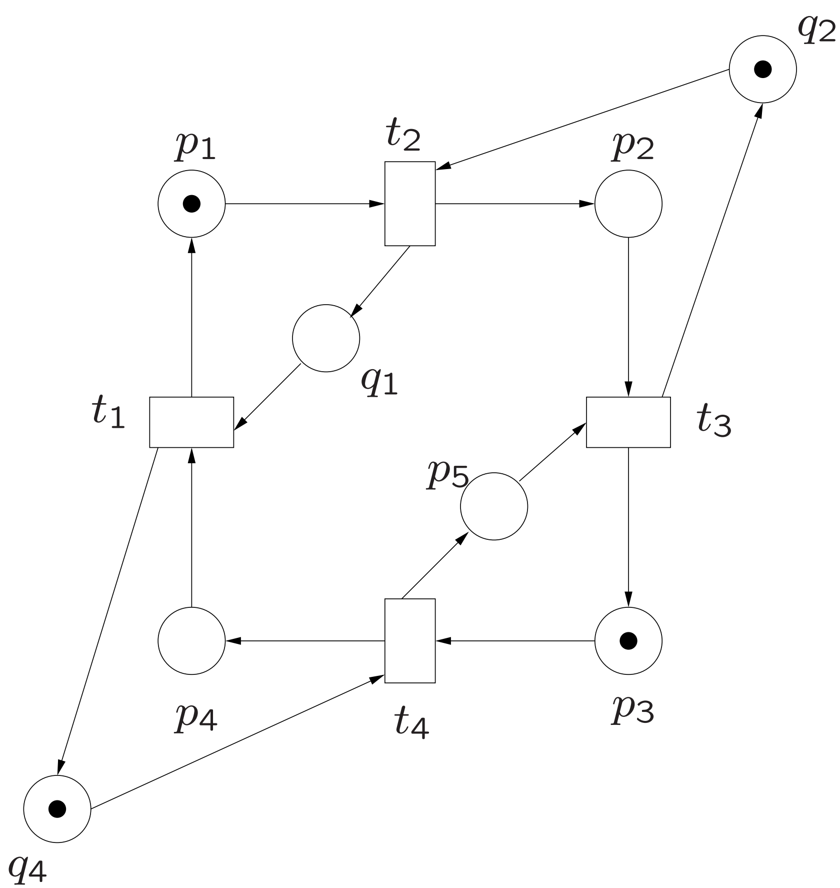

Fig. 48. The result of complementing p_1, p_2, p_4 in Fig. 39.

Our aim was:

Theorem 54. For every EN system M there exists
a reduced EN system M'
that is configuration equivalent with M and
that is covered by at most $\#P_M$ sequential components.

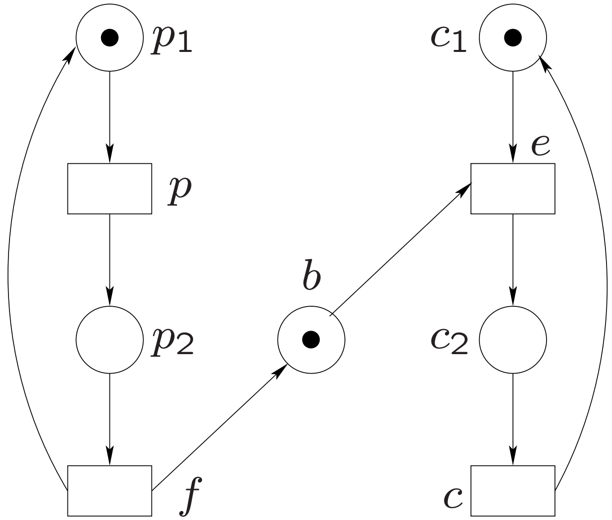

Fig. 12. Subsystems: $\{p_1, p_2\}$, the producer; $\{c_1, c_2\}$, the consumer; and $\{p_1, p_2, c_1, c_2\}$; otherwise trivial. The buffer is NOT a subsystem

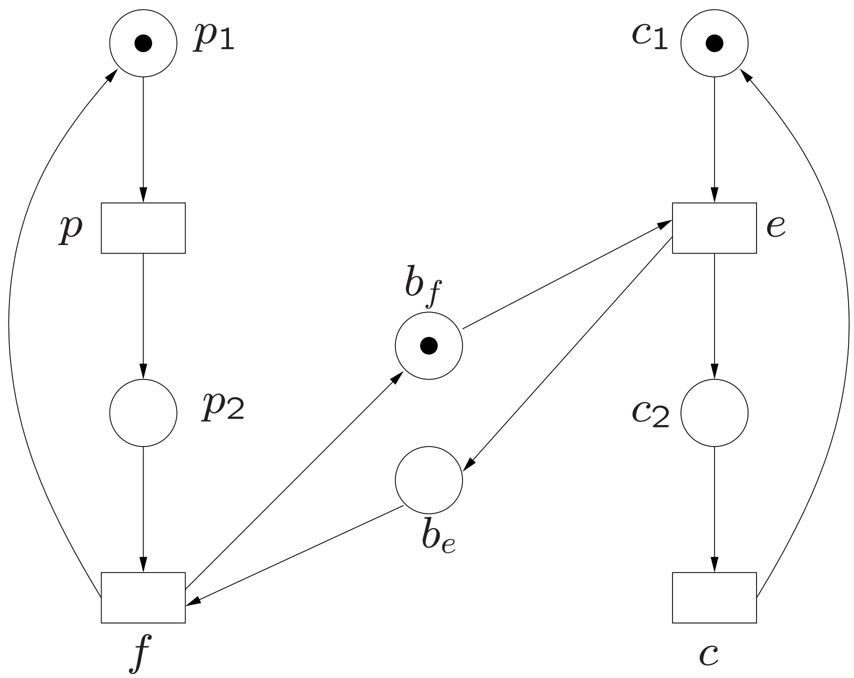

Fig. 49. The producer/consumer system with three sequential components.

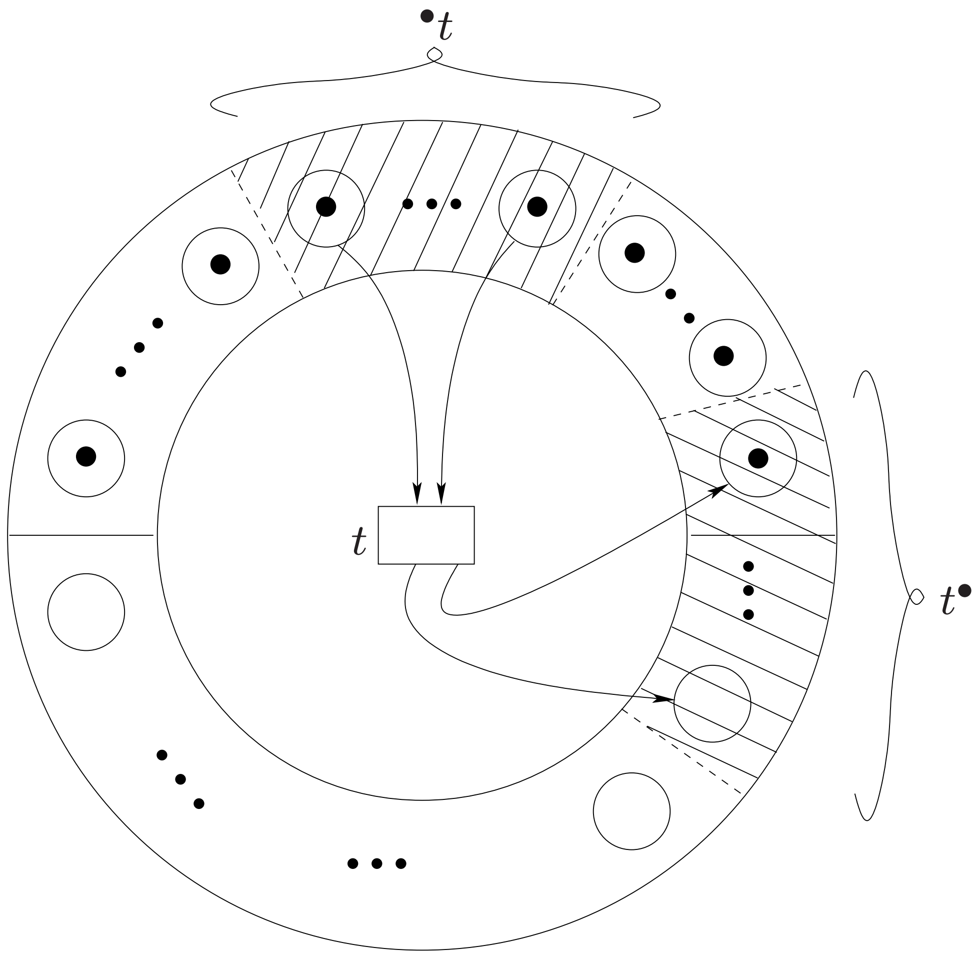

Fig. 50. Contact.

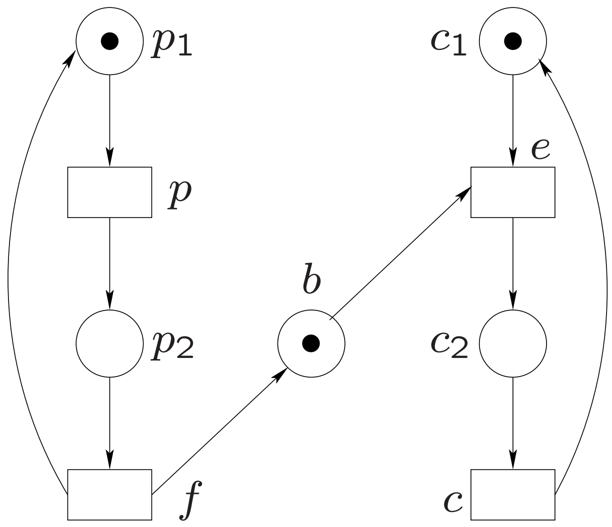

Fig. 12. Contact in $\{p_2, b, c_1\}$.

Definition 58. Let $M = (P, T, F, C_{in})$ be an EN system.

M is *contact-free* if for all $t \in T$ and $C \in \mathbb{C}_M$,

if ${}^\bullet t \subseteq C$ then $t^\bullet \cap C = \varnothing$.

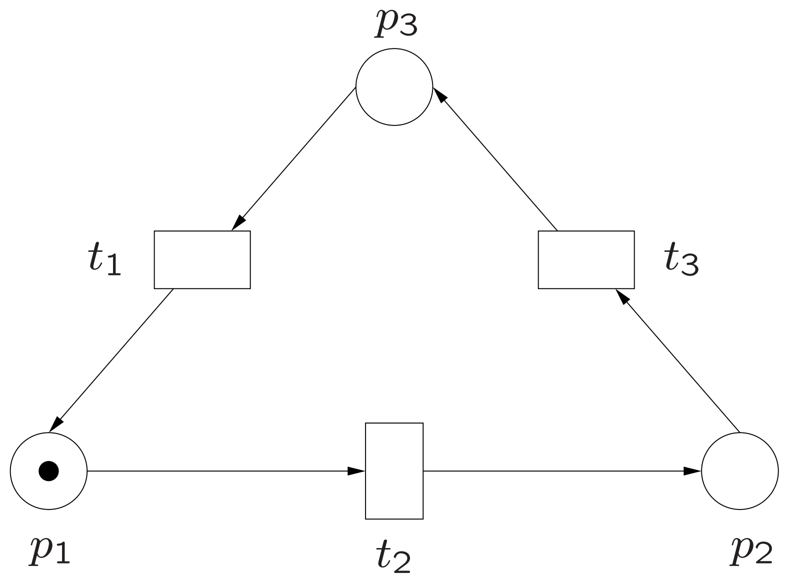

Fig. 47. A (sequential and) contact-free EN system.

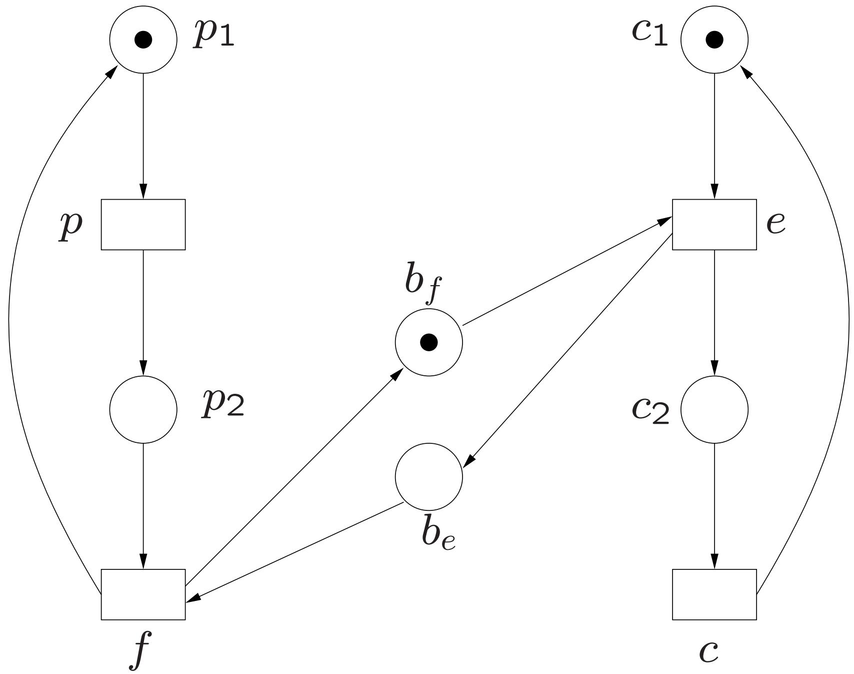

Fig. 49. The (contact-free) producer/consumer system.

Theorem 59. If a reduced EN system M is covered by sequential components, then M is contact-free.

Theorem 54. For every EN system M there exists
a reduced EN system M'
that is configuration equivalent with M and
that is covered by at most $\#P_M$ sequential components.

Theorem 60. For every EN system there exists a configuration equivalent reduced contact-free EN system.

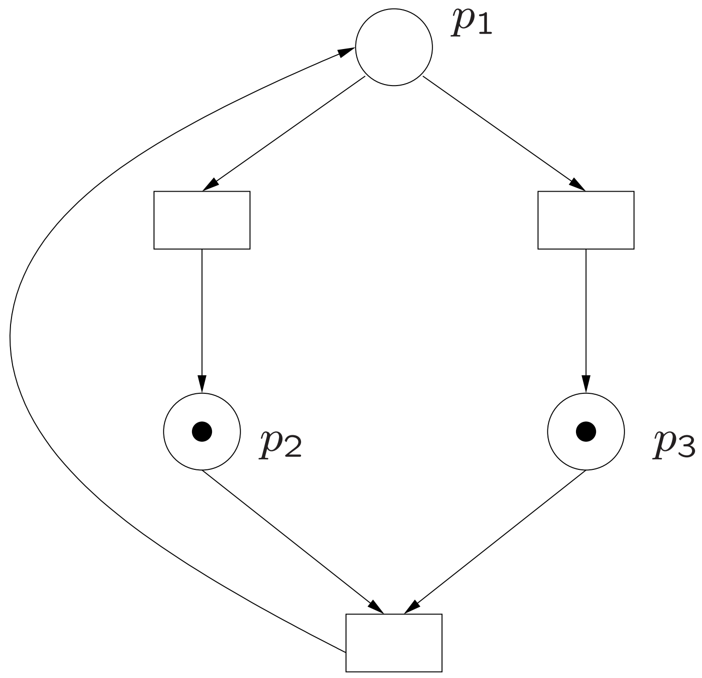

Fig. 51. A contact-free EN system without sequential components.

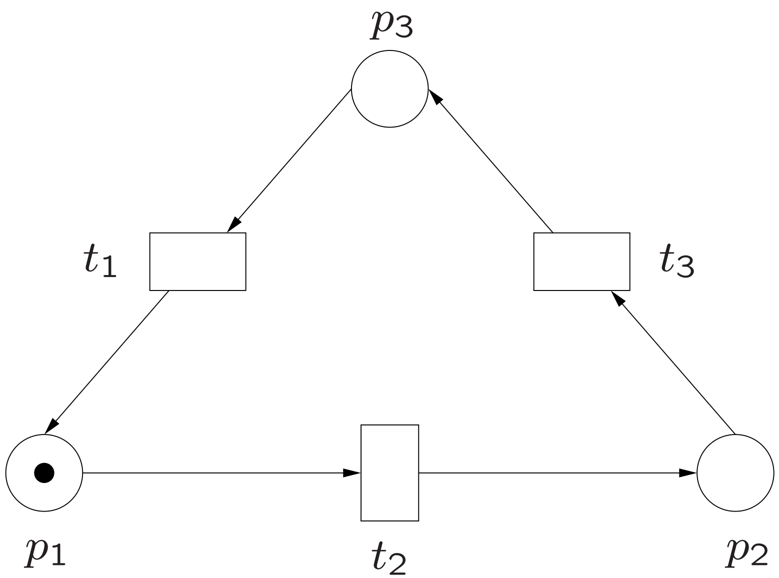

Fig. 47. A sequential EN system.

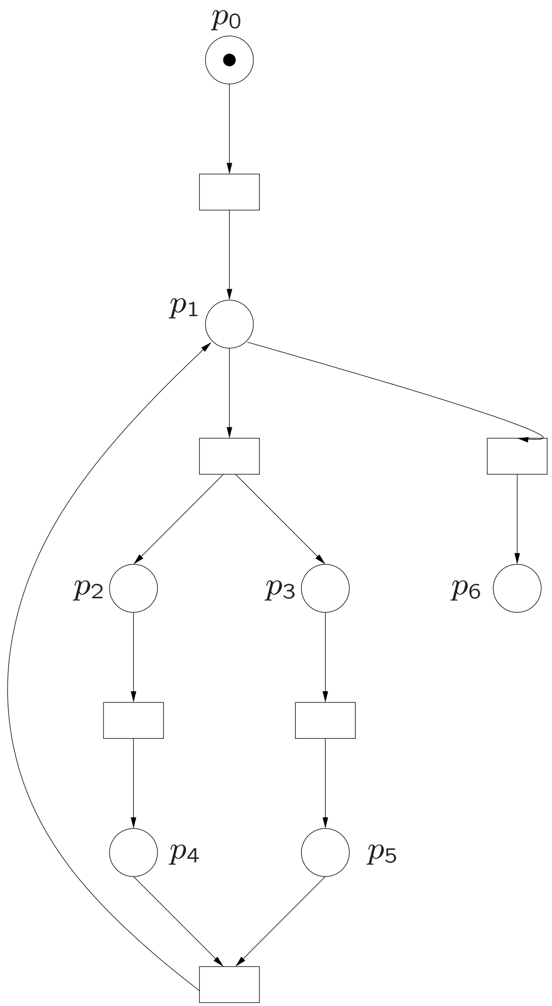

Fig. 15. Covered by (two) sequential components. Hence contact-free.

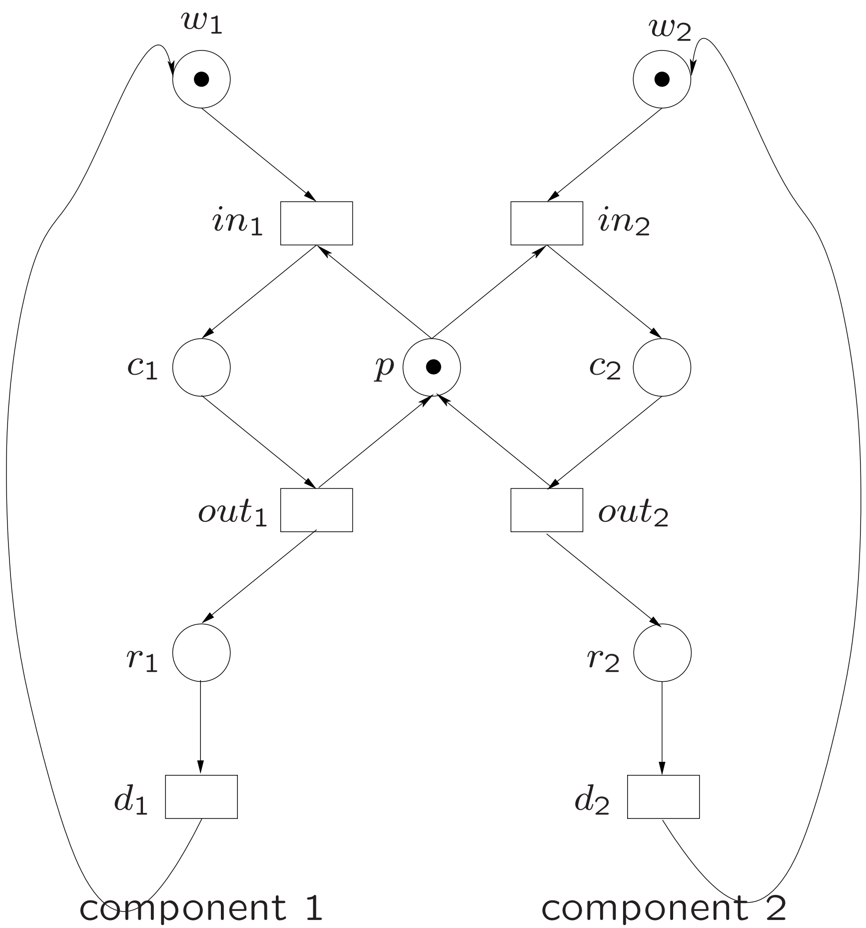

Fig. 5. The mutual exclusion problem.

Theorem 61. Let $M = (P, T, F, C_{in})$ be an EN system.

M is contact-free and conflict-free

iff for all $C \in \mathbb{C}_M$ and all $U \subseteq T$ with $U \neq \varnothing$,

if ${}^{\bullet}U \subseteq C$, then $U \ \mathbf{con} \ C$.

6. Processes

Definition 62. Let A be a finite set.

A binary relation $\rho \subseteq A \times A$ is a *partial order on A* if
ρ is irreflexive and transitive;
(A, ρ) is also called a *partially ordered set*.

A subset B of A is *linearly ordered* if
for all $a, b \in B$: $a \ \rho \ b$ or $b \ \rho \ a$ or $a = b$.

Partial order 'is' transitive, directed acyclic graph.

If (A, ρ) is partially ordered set with $A \neq \emptyset$,
then A has minimal (maximal) elements.

If $B \subseteq A$ is linearly ordered,
then $B = \{a_1, a_2, \ldots, a_k\}$ with $k \geq 1$,
such that $a_1 \ \rho \ a_2 \ \rho \ \ldots \ \rho \ a_k$.

Definition 63. Let (A, ρ) be a partially ordered set.

Then $\mathbf{li}_\rho \subseteq A \times A$ and $\mathbf{co}_\rho \subseteq A \times A$
are the binary relations such that, for every $a, b \in A$,

(1) $a \ \mathbf{li}_\rho \ b$ iff $a \ \rho \ b$ or $b \ \rho \ a$ or $a = b$, and

(2) $a \ \mathbf{co}_\rho \ b$ iff $\neg \ a \ \rho \ b$ and $\neg \ b \ \rho \ a$.

$\mathbf{li}_\rho$ is called the *line relation of* ρ and

$\mathbf{co}_\rho$ is called the *concurrency relation of* ρ.

Lemma 64. Let (A, ρ) be a partially ordered set. Then, for every $a, b \in A$,

(1) $a\ \mathbf{li}_\rho\ b$ or $a\ \mathbf{co}_\rho\ b$, and

(2) ($a\ \mathbf{li}_\rho\ b$ and $a\ \mathbf{co}_\rho\ b$) iff $a = b$.

Definition 65. Let A be a finite set, let $\sigma \subseteq A \times A$ be a reflexive symmetric relation, and let $B \subseteq A$.

B is a σ-*clique* if
$a \ \sigma \ b$ for all $a, b \in B$, and

B is a *maximal* σ-*clique* if
B is a σ-clique and
for every $a \in A - B$ there exists $b \in B$ such that $\neg \ a \ \sigma \ b$.

Lemma 66. Let A be a finite set and
let $\sigma \subseteq A \times A$ be a reflexive symmetric relation.

For every σ-clique B
there exists a maximal σ-clique C with $B \subseteq C$.

Definition 67. Let (A, ρ) be a partially ordered set.

A maximal $\mathbf{li}_\rho$-clique is a *line of* ρ and

a maximal $\mathbf{co}_\rho$-clique is a *cut of* ρ.

Definition 68. Let (A, ρ) be a partially ordered set.

The ordering ρ is *dense* if every line and every cut of ρ have a nonempty intersection.

Definition 69. Let (A, ρ) be a partially ordered set and let $B \subseteq A$. Then

$(^{\rightarrow}B)_\rho = \{a \in A \mid \exists b \in B : a\ \rho\ b$ or $a = b\}$,

$(B^{\rightarrow})_\rho = \{a \in A \mid \exists b \in B : b\ \rho\ a$ or $b = a\}$,

$(^{\circ}B)_\rho = \{b \in B \mid \neg\exists b' \in B : b'\ \rho\ b\}$, and

$(B^{\circ})_\rho = \{b \in B \mid \neg\exists b' \in B : b\ \rho\ b'\}$.

Lemma 70. Let (A, ρ) be a partially ordered set and let $B \subseteq A$.

Then $B \subseteq ({}^{\circ}B)^{\rightarrow}$ and $B \subseteq {}^{\rightarrow}(B^{\circ})$.

Theorem 71. Let (A, ρ) be a partially ordered set and let B be a cut of ρ.

(1) ${}^{\circ}A$ and A° are cuts of ρ,

(2) $({}^{\circ}A)^{\rightarrow} = A$, ${}^{\rightarrow}({}^{\circ}A) = {}^{\circ}A$, $(A^{\circ})^{\rightarrow} = A^{\circ}$, and ${}^{\rightarrow}(A^{\circ}) = A$,

(3) ${}^{\rightarrow}B \cup B^{\rightarrow} = A$ and ${}^{\rightarrow}B \cap B^{\rightarrow} = B$,

(4) ${}^{\circ}({}^{\rightarrow}B) = {}^{\circ}A$, $({}^{\rightarrow}B)^{\circ} = B$, ${}^{\circ}(B^{\rightarrow}) = B$, and $(B^{\rightarrow})^{\circ} = A^{\circ}$.

Lemma 72. Let (A, ρ) be a partially ordered set with $A \neq \varnothing$ and let L be a line of ρ. Then

$L \cap {}^\circ A \neq \varnothing$ and $L \cap A^\circ \neq \varnothing$.