Theorie van Concurrency

najaar 2011

http://www.liacs.nl/home/rvvliet/tvc/

zevende college: 4 oktober 2011

5. Equivalences and Normal Forms

5.4 Subsystems and Sequential Components

Lemma 46. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be EN systems.

(1) M' is a subsystem of M iff
$P' \subseteq P$, $T' = \mathbf{nbh}_M(P')$,
$F' = F \cap ((P' \times T') \cup (T' \times P'))$, and
$C'_{in} = C_{in} \cap P'$.

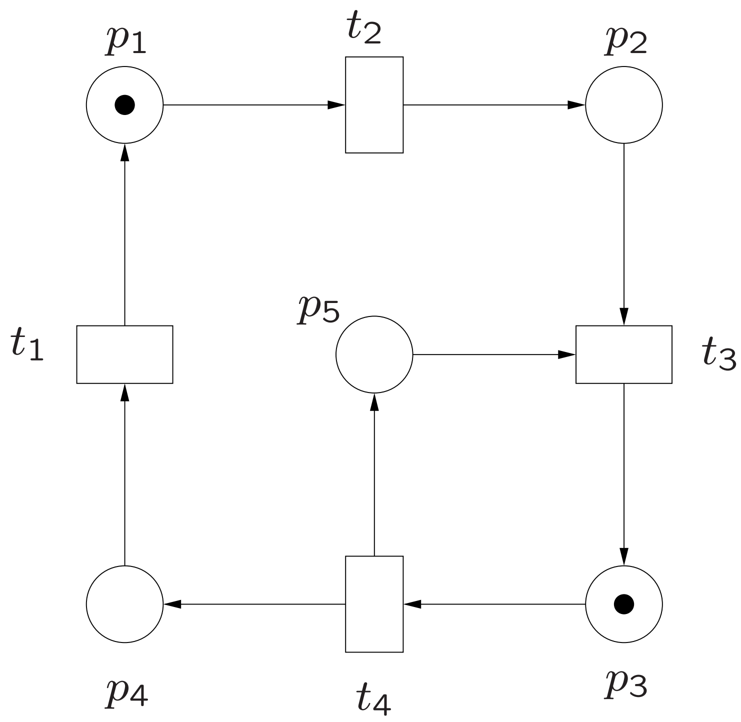

Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

Lemma 47. Let $M = (P, T, F, C_{in})$ be an EN system and let $S \subseteq P$.

There exists a subsystem M' of M with $P_{M'} = S$ iff ${}^\bullet S = S^\bullet$.

Lemma 48. Let $M' = (S, T', F', C'_{in})$ be a subsystem of an EN system $M = (P, T, F, C_{in})$.

(1) For all $C \subseteq P$, if $C \in \mathbb{C}_M$ then $C \cap S \in \mathbb{C}_{M'}$.

(2) For all $t \in T'$, if $t \in \mathbf{use}_M(T)$ then $t \in \mathbf{use}_{M'}(T')$.

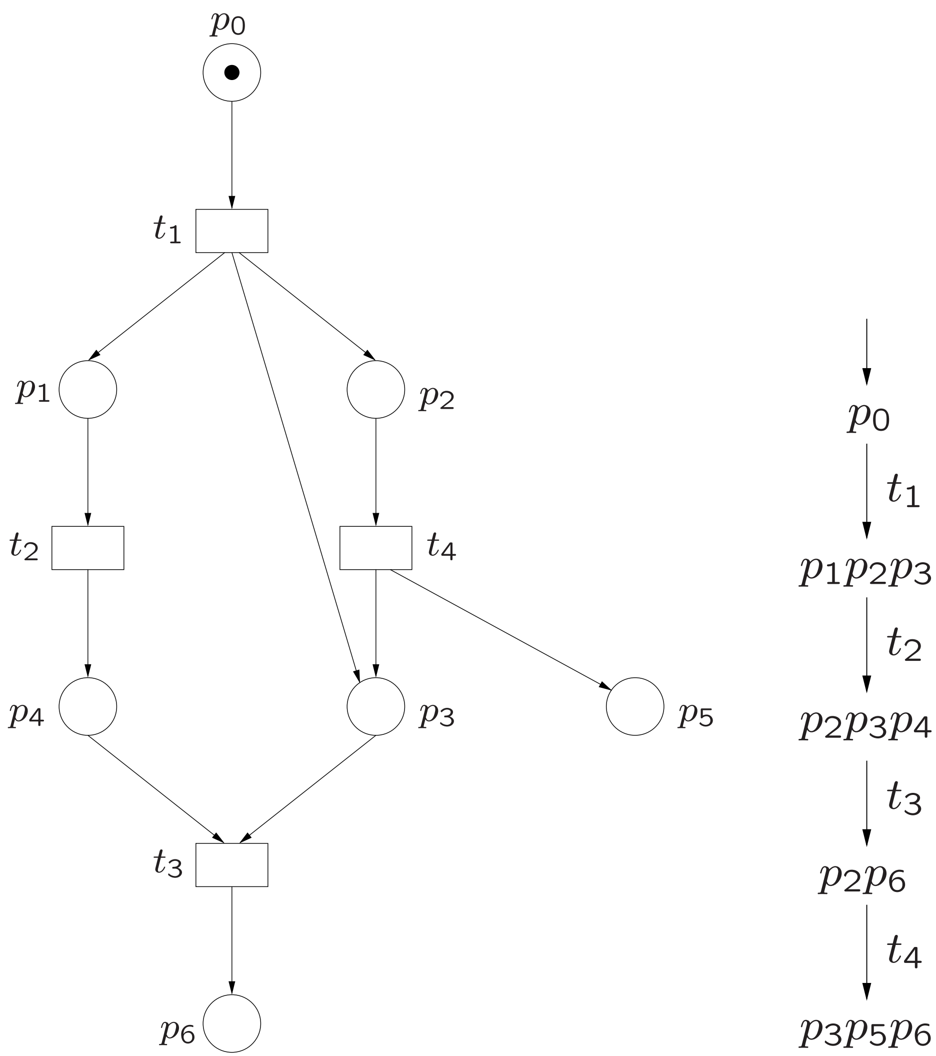

Fig. 40, 41. An EN system M and its configuration graph.

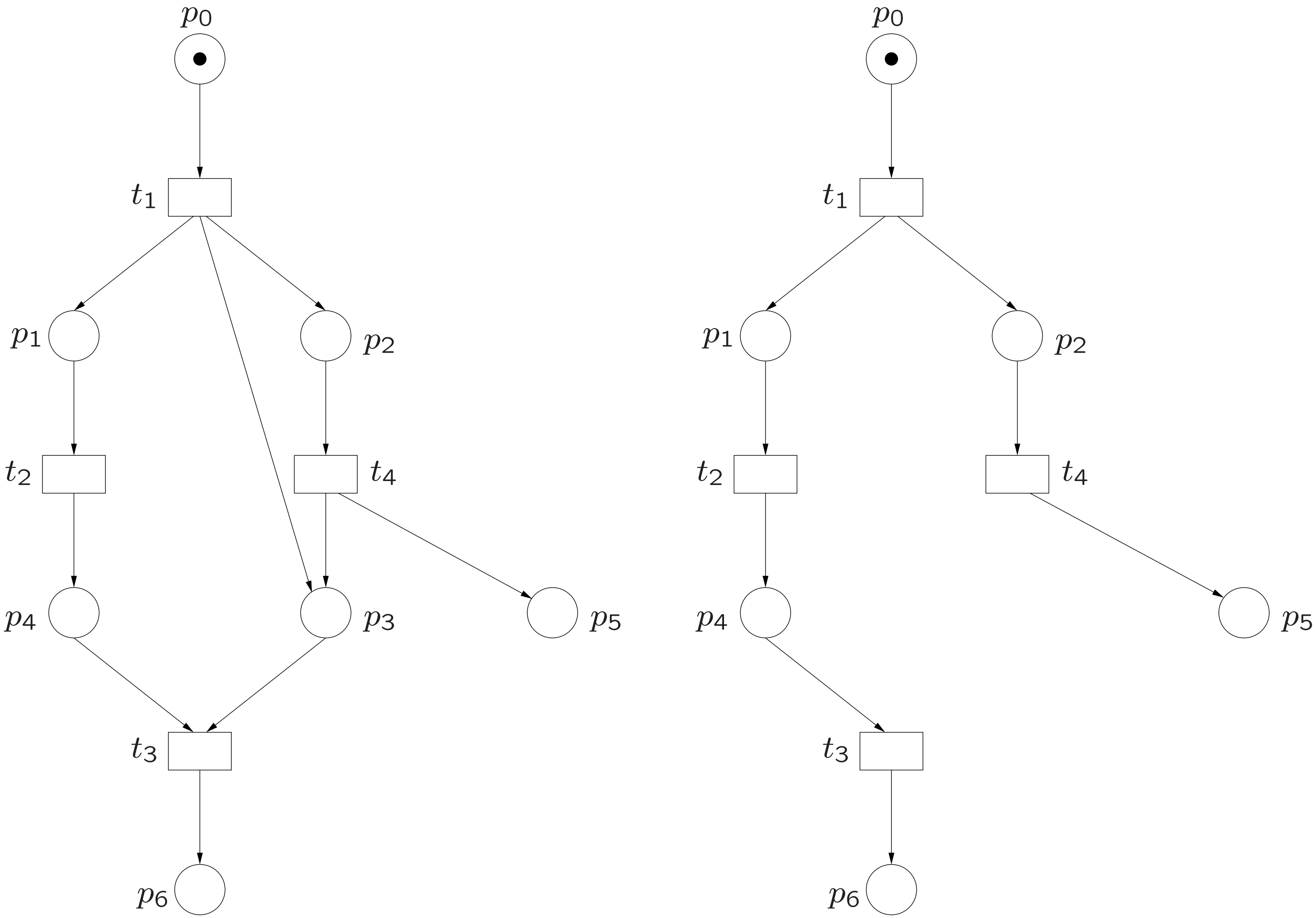

Fig. 40, 42. An EN system M and a subsystem M_1.

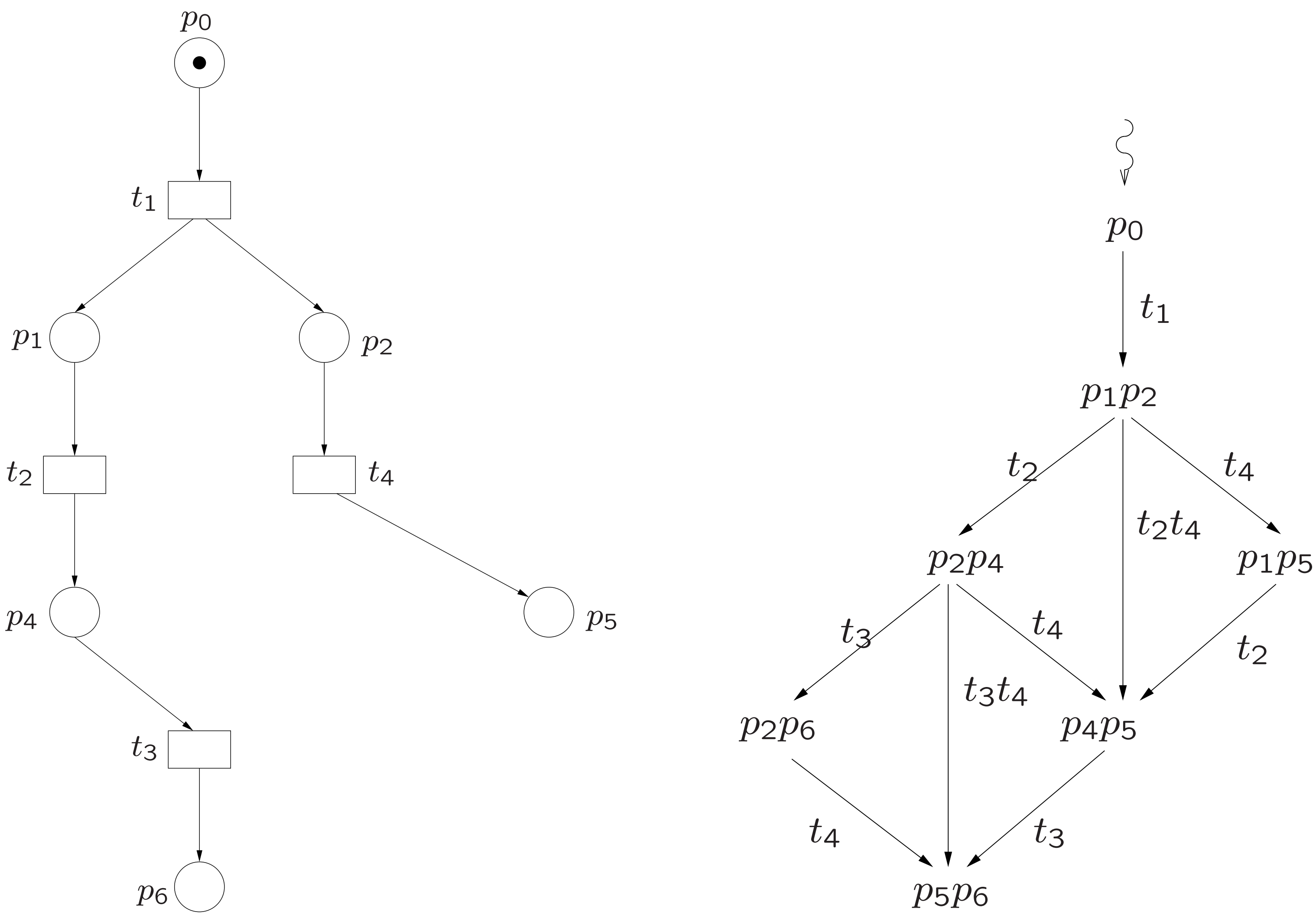

Fig. 42, 43. M_1 and its configuration graph.

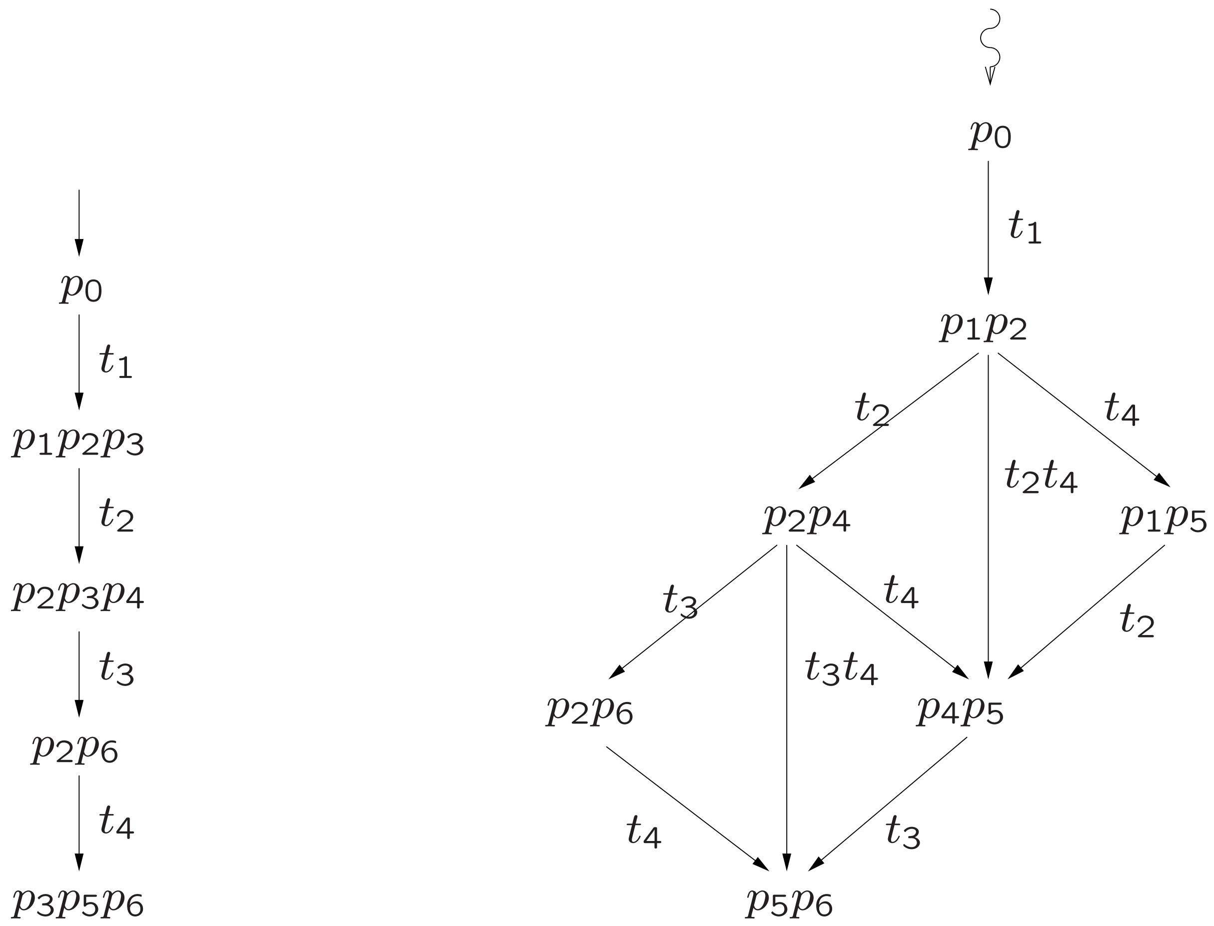

Fig. 41, 43. Configuration graphs of M and its subsystem M_1.

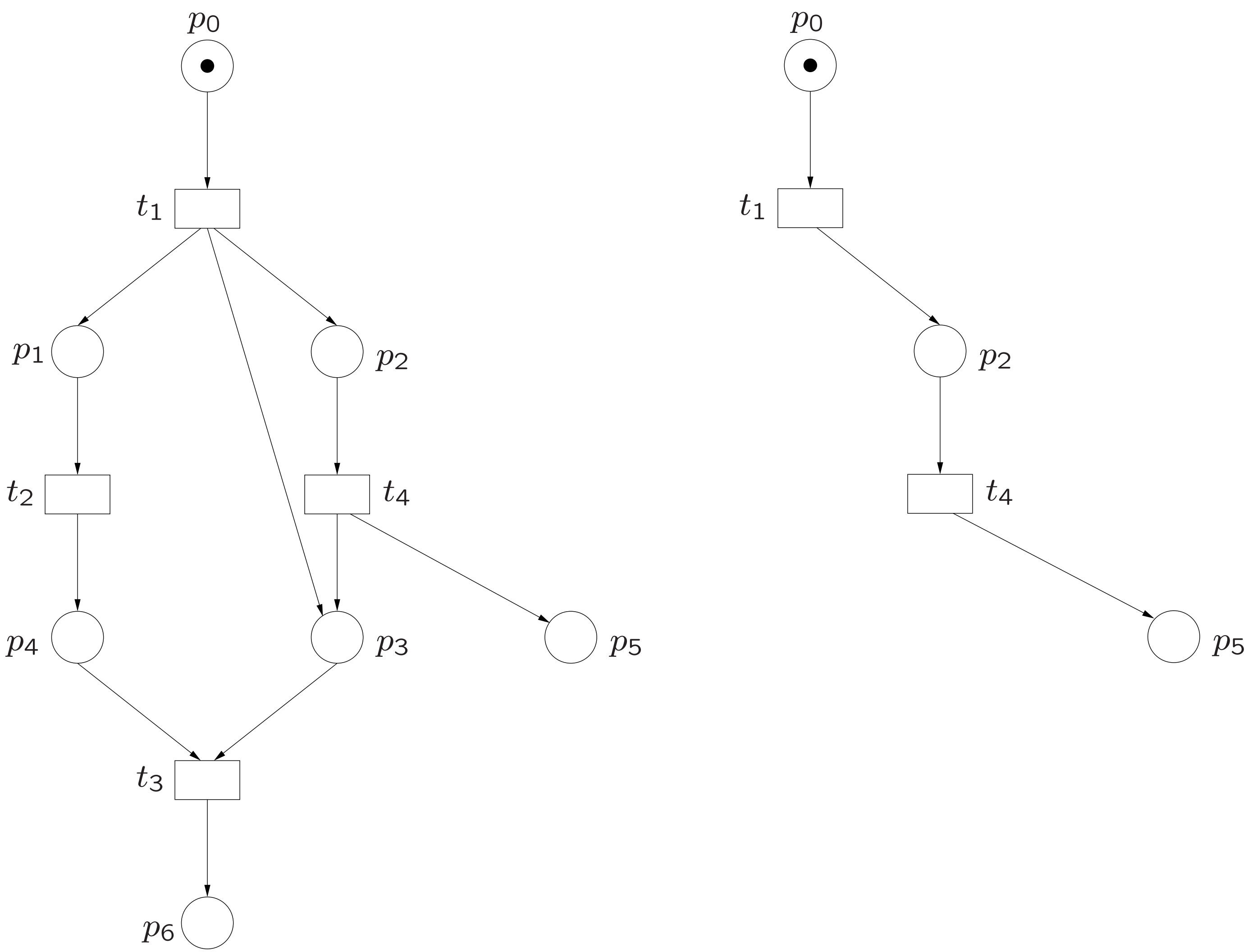

Fig. 40, 44. An EN system M and a subsystem M_2.

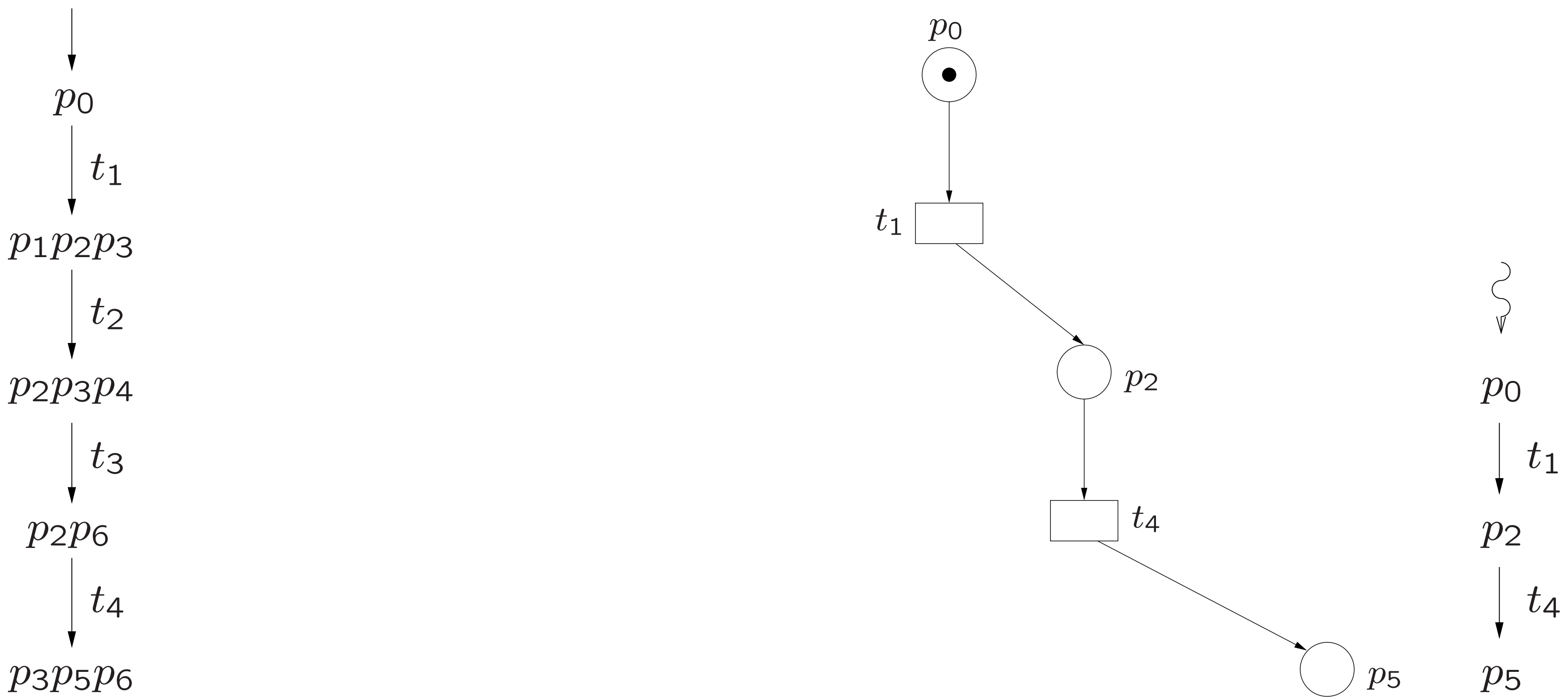

Fig. 41, 44, 45. Configuration graph of M, subsystem M_2 and configuration graph of M_2.

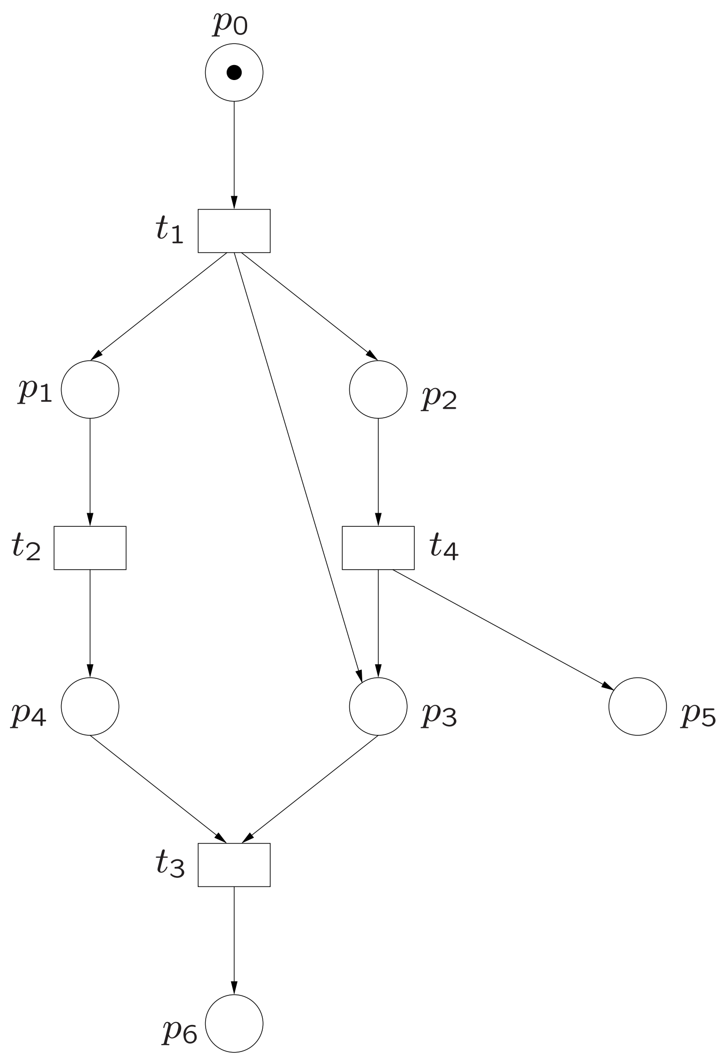

Fig. 40. An EN system M. All subsystems. . .

Lemma 42. Let $M = (P, T, F, C_{in})$ be a reduced EN system.

(1) M is sequential iff

(i) $\#C_{in} = 1$, and

(ii) $\#(^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T$.

Theorem 49. Let $M = (P, T, F, C_{in})$ be a reduced EN system and let $S \subseteq P$.
Then the following statements are equivalent.

(1) There is a sequential component M' of M with $P_{M'} = S$,

(2) $\#(C \cap S) = 1$ for all $C \in \mathbb{C}_M$,

(3) (i) $\#(C_{in} \cap S) = 1$, and
(ii) $\forall t \in T$:
$\#({}^\bullet t \cap S) = \#(t^\bullet \cap S) = 1$ or $\#({}^\bullet t \cap S) = \#(t^\bullet \cap S) = 0$.

Lemma 50. Let M be a strongly reduced sequential EN system and let M' be a subsystem of M.

Then M' is trivial.

Theorem 51. Let M be a strongly reduced EN system, and let M' be a sequential component of M.

Then M' has no nontrivial subsystems.

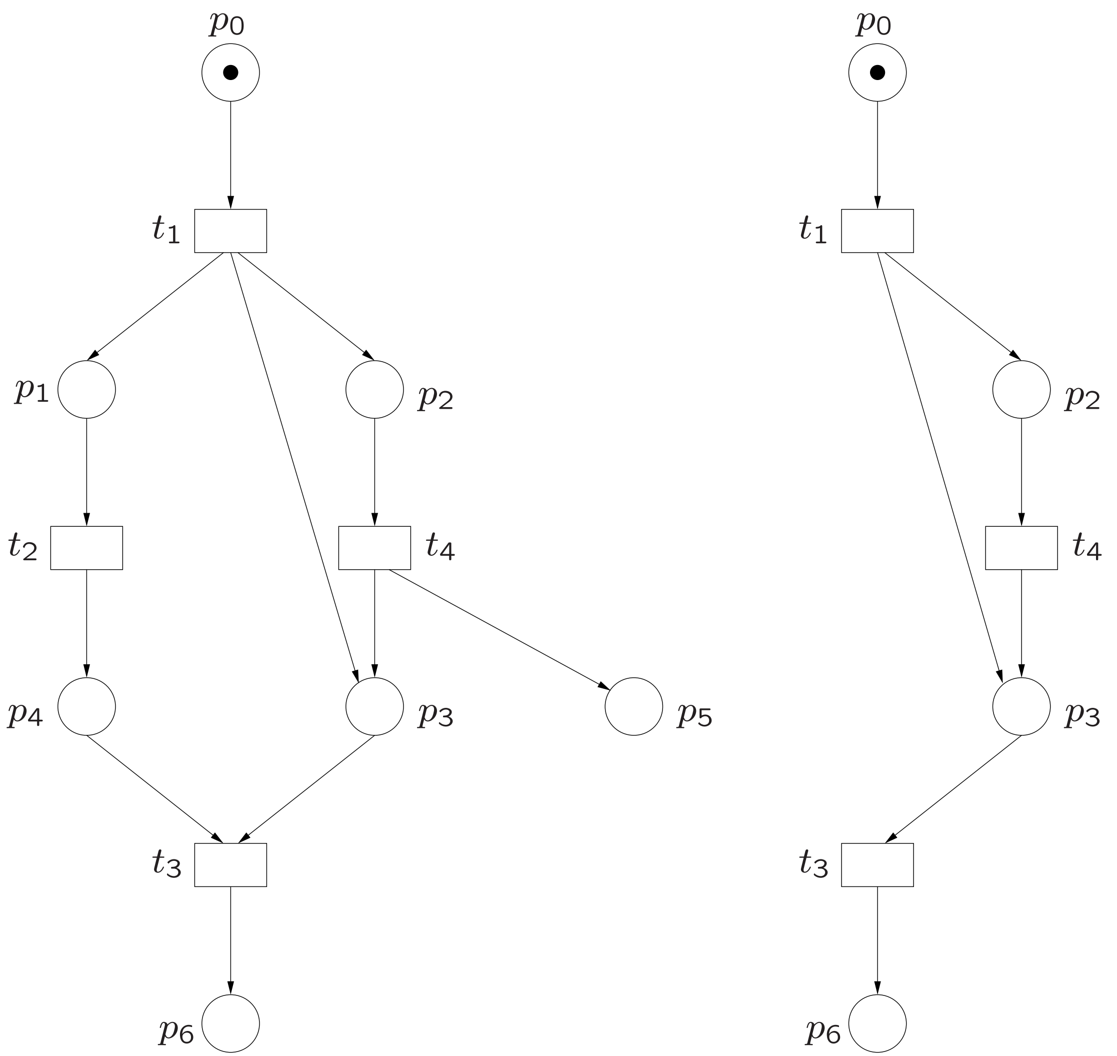

Fig. 40, 46. An EN system M and a subsystem M_3.

Definition 52. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) A set $\{M_1, \ldots, M_n\}$ of subsystems of M, $n \geq 0$,
with $M_i = (S_i, T_i, F_i, (C_{in})_i)$ for $1 \leq i \leq n$,
is a *covering* of M if
$P = \bigcup_{i=1}^{n} S_i$,
$T = \bigcup_{i=1}^{n} T_i$,
$F = \bigcup_{i=1}^{n} F_i$, and
$C_{in} = \bigcup_{i=1}^{n} (C_{in})_i$.

(2) M is *covered by sequential components*
if there exists a covering $\{M_1, \ldots, M_n\}$, $n \geq 0$, of M
such that M_i is a sequential component of M for every $1 \leq i \leq n$.

Lemma 53. Let $M = (P, T, F, C_{in})$ be an EN system, and let, for every $1 \le i \le n$ (with $n \ge 0$), $M_i = (S_i, T_i, F_i, (C_{in})_i)$ be a subsystem of M.

Then $\{M_1, \ldots, M_n\}$ is a covering of M
iff $P = \bigcup_{i=1}^{n} S_i$.

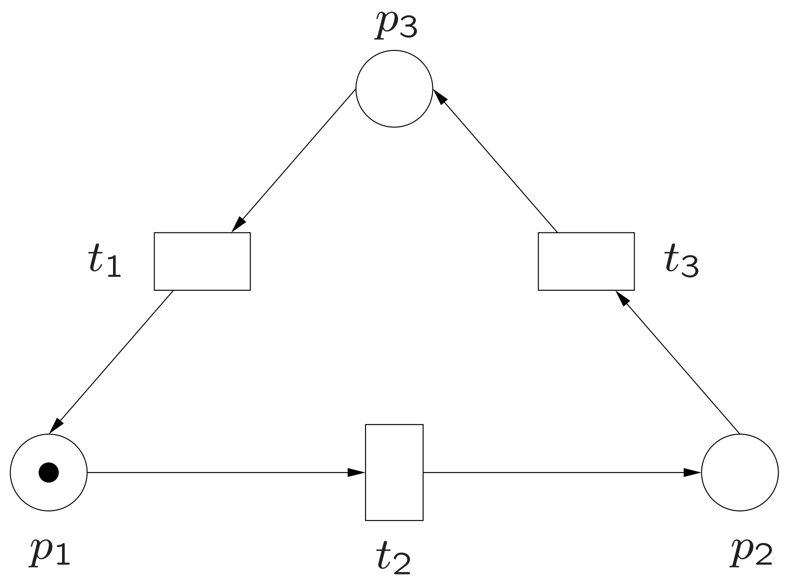

Fig. 47. A sequential EN system.

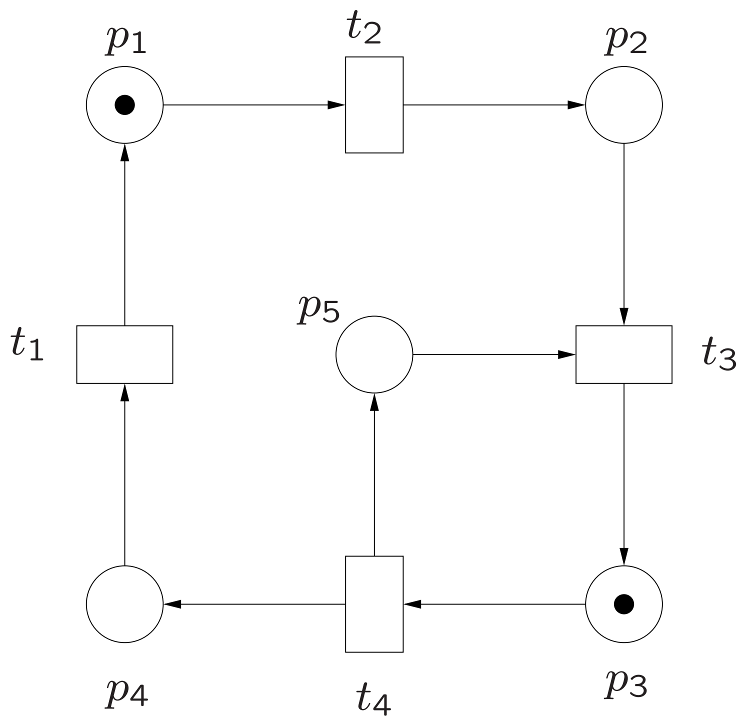

Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

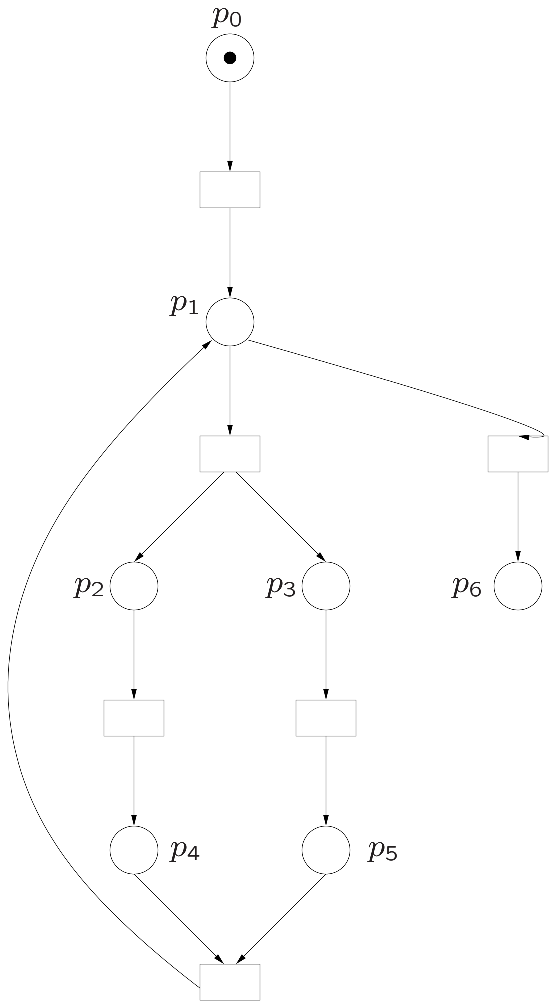

Fig. 15.

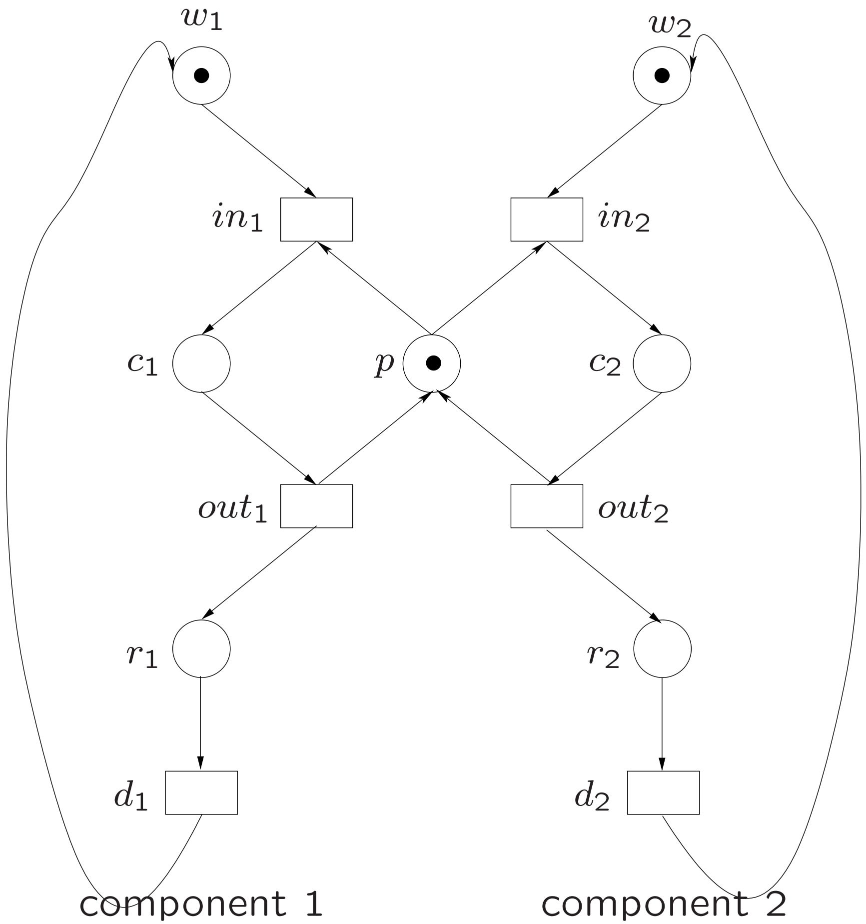

Fig. 5. The mutual exclusion problem

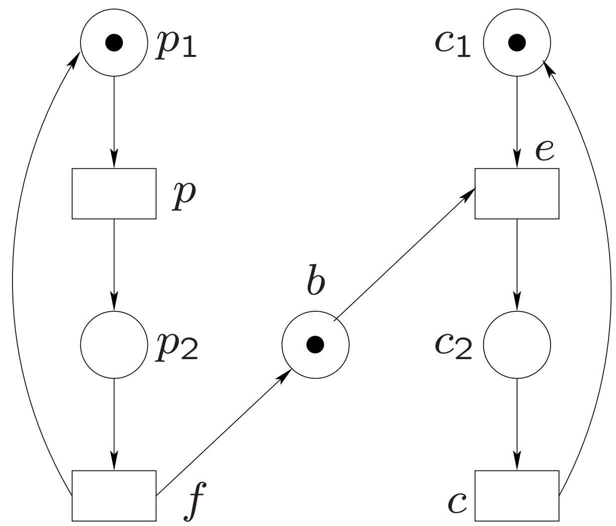

Fig. 12. Subsystems: $\{p_1, p_2\}$, the producer; $\{c_1, c_2\}$, the consumer; and $\{p_1, p_2, c_1, c_2\}$; otherwise trivial.
The buffer is NOT a subsystem

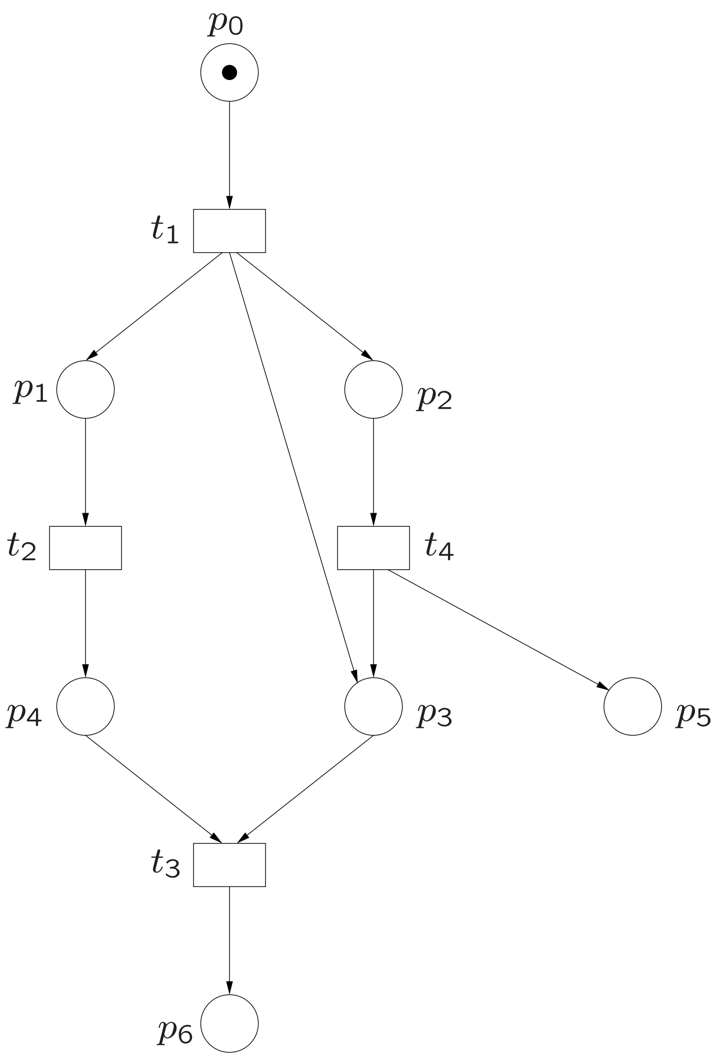

Fig. 40.

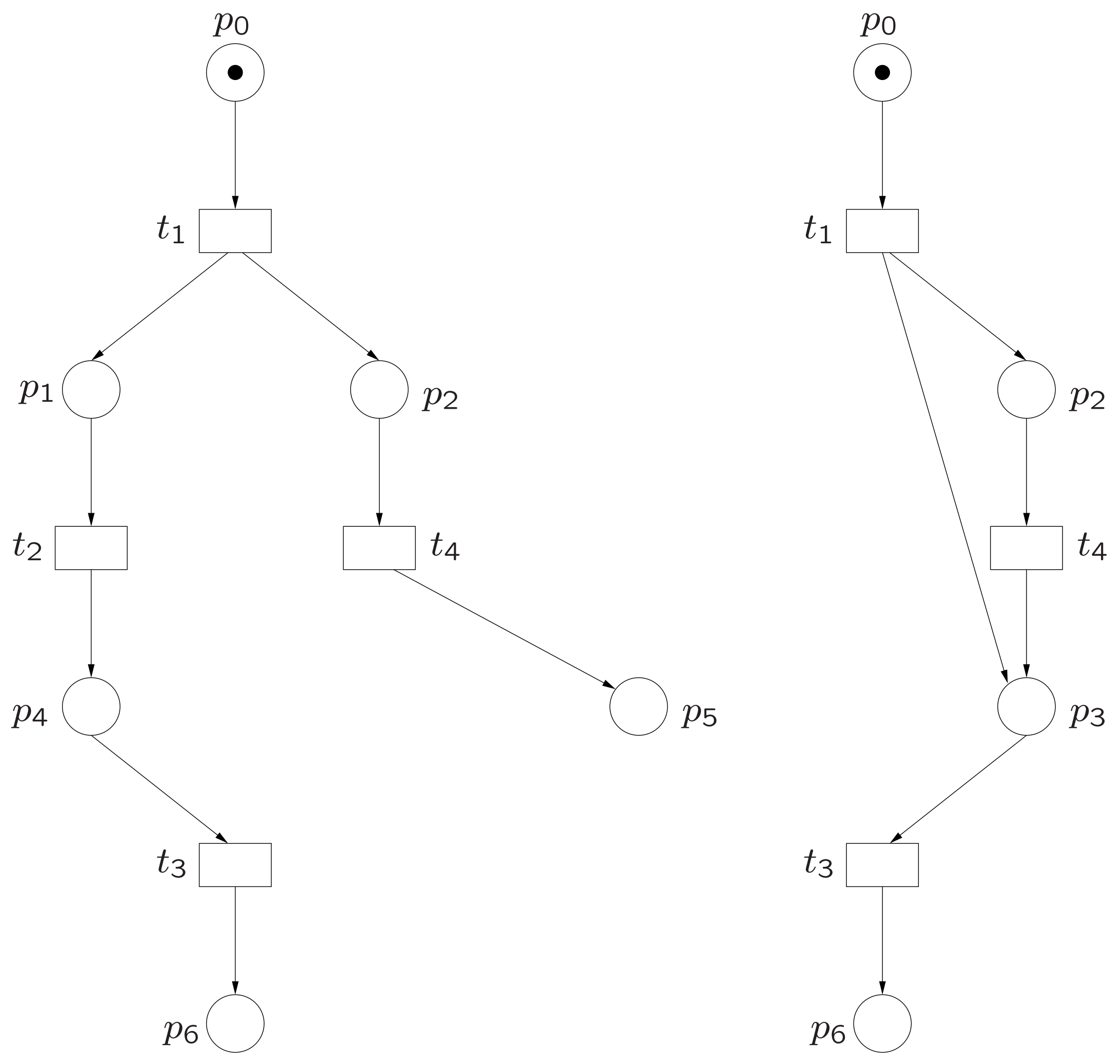

Fig. 42, 46.

Aim:

Theorem 54. For every EN system M there exists
a reduced EN system M'
that is configuration equivalent with M and
that is covered by at most $\#P_M$ sequential components.

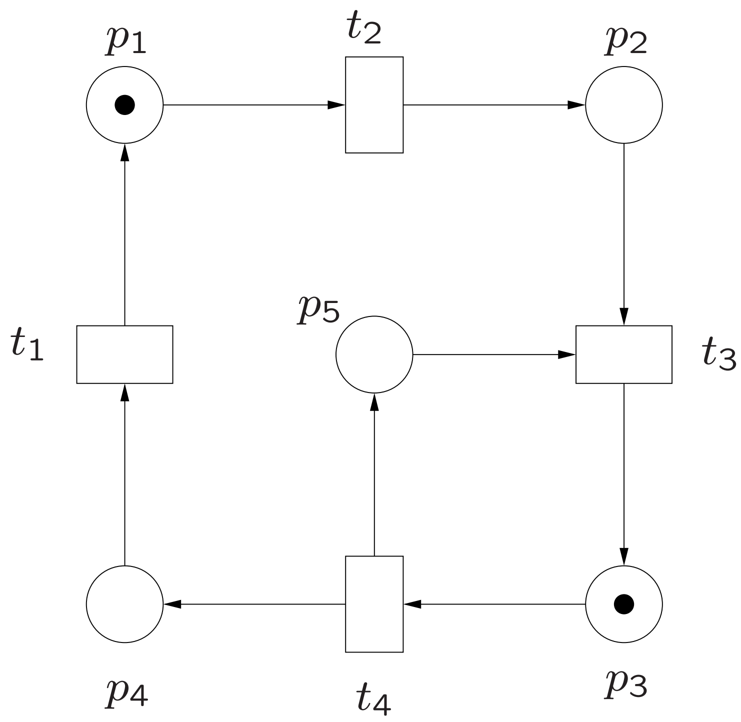

Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

Definition 55. Let M be an EN system and let $p, q \in P_M$.

Then p and q are *complementary*, denoted by $p \mathbf{\ com\ } q$, if

$p^\bullet = {}^\bullet q$ and ${}^\bullet p = q^\bullet$.

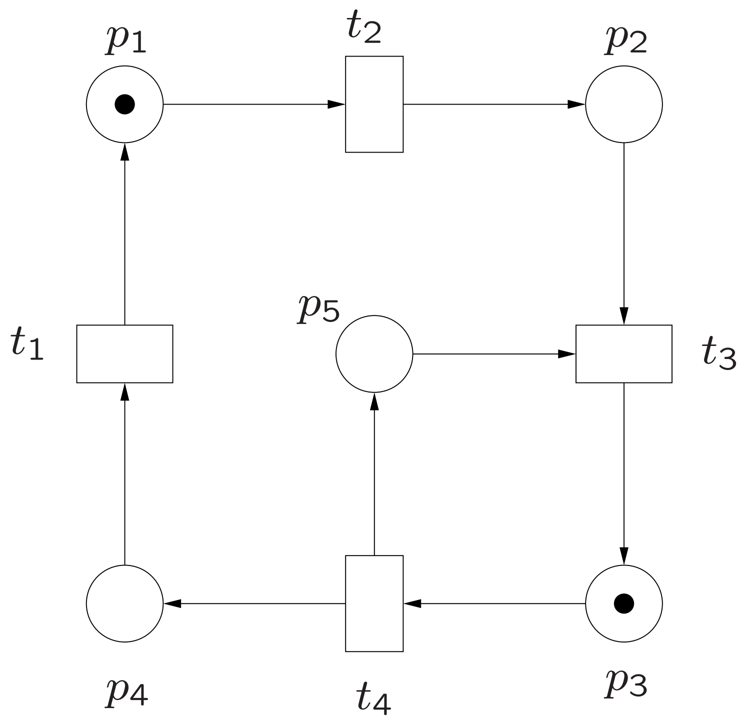

Fig. 39. p_3 and p_5 are complementary.

Lemma 56. Let $M = (P, T, F, C_{in})$ be a reduced EN system. For all $p, q \in P$,

$\{p, q\}$ is a sequential component of M

iff

$\#(C_{in} \cap \{p, q\}) = 1$ and $p \ \mathbf{com} \ q$.