

Theorie van Concurrency

najaar 2011

<http://www.liacs.nl/home/rvvliet/tvc/>

zesde college: 29 september 2011

5. Equivalences and Normal Forms

5.4 Subsystems and Sequential Components

Definition 40. An EN system M is *sequential* if $\#C = 1$ for all $C \in \mathbb{C}_M$.

Definition 44. An EN system M is *concurrency-free* if there do not exist $C \in \mathbb{C}_M$ and $t_1, t_2 \in T_M$ such that $\{t_1, t_2\} \text{ con } C$ and $t_1 \neq t_2$

Clearly M sequential implies M concurrency-free.

Conversely? Figures 29 vs 30 and 35 vs 33. However:

\exists a concurrency-free EN-system M for which

\nexists sequential EN system M' such that $M \approx_w M'$

Exercises 5.5 and 5.6acd

Exercise 5.5.

Prove that an EN system M is configuration equivalent with a sequential EN system M' , if and only if M has the following property:

For all $C, D \in \mathbb{C}_M$ and $t \in T_M$: if t **con** C and t **con** D , then $C = D$.

In other words:

Each transition in M occurs at most once in $\text{SCG}(M)$.

Lemma 42. Let $M = (P, T, F, C_{in})$ be a **reduced** EN system.

(2) If M is strongly reduced and sequential,
then $\mathbb{C}_M = \{\{p\} \mid p \in P\}$.

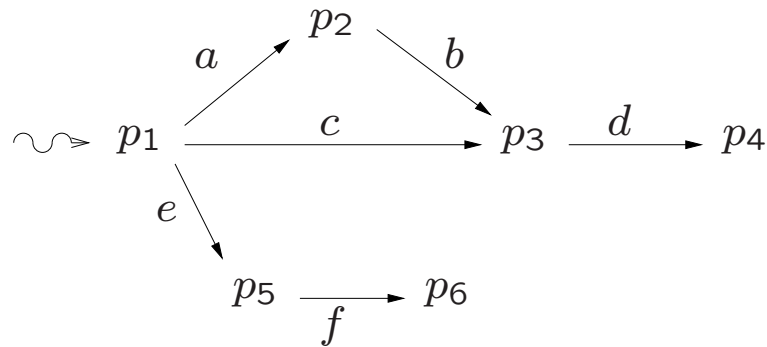
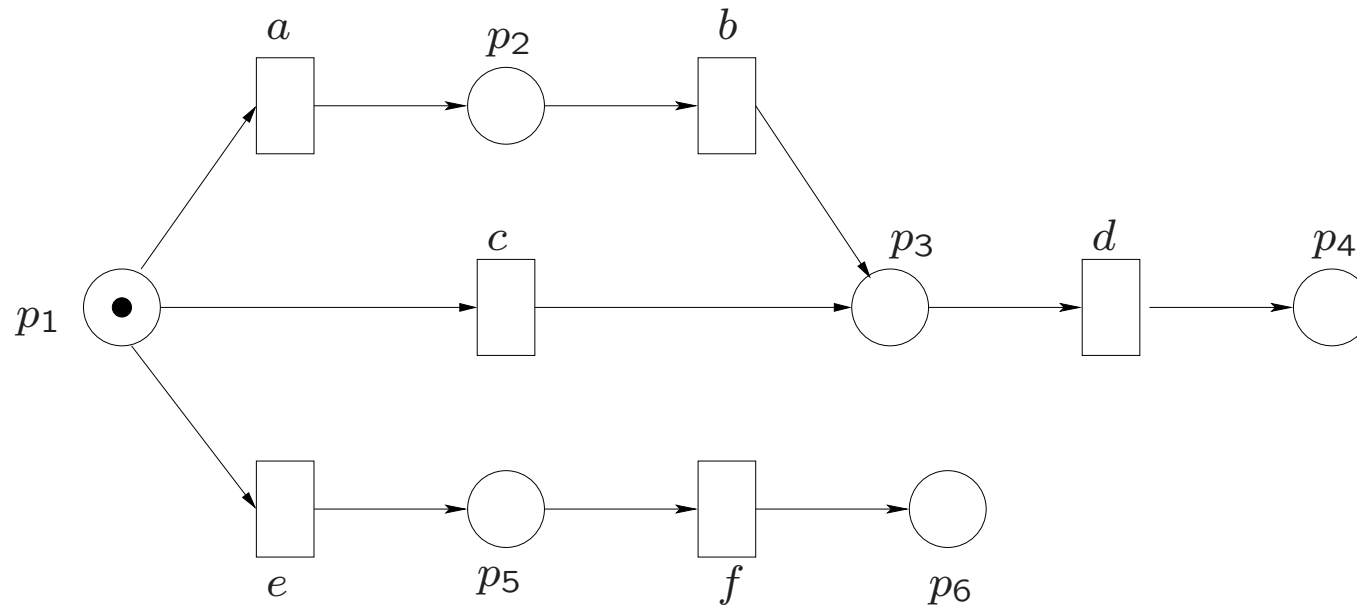


Fig. 33, 34. A seq. EN system M and its config. graph.

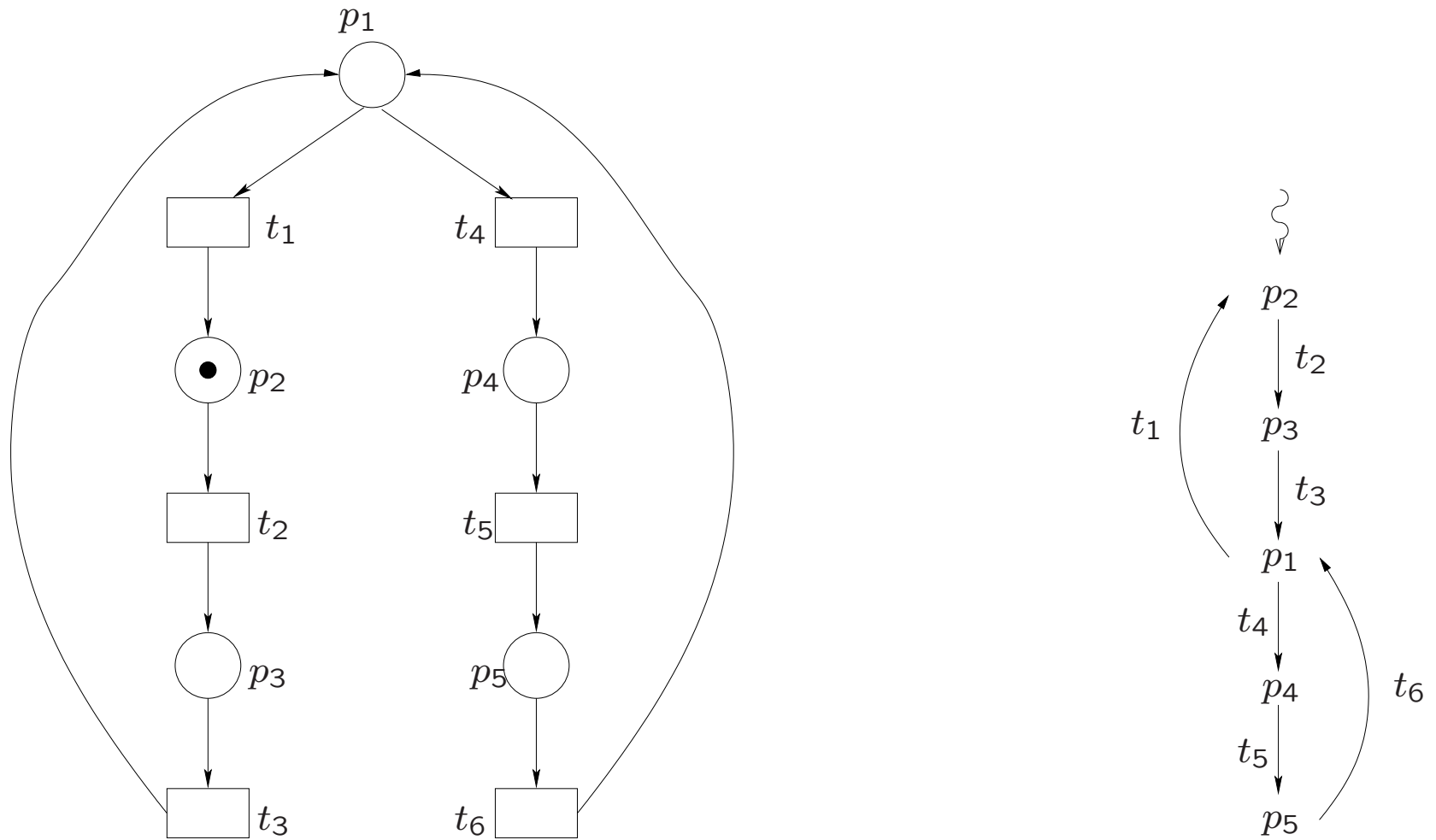


Fig. 30, 32. A sequential EN system M' and its configuration graph.

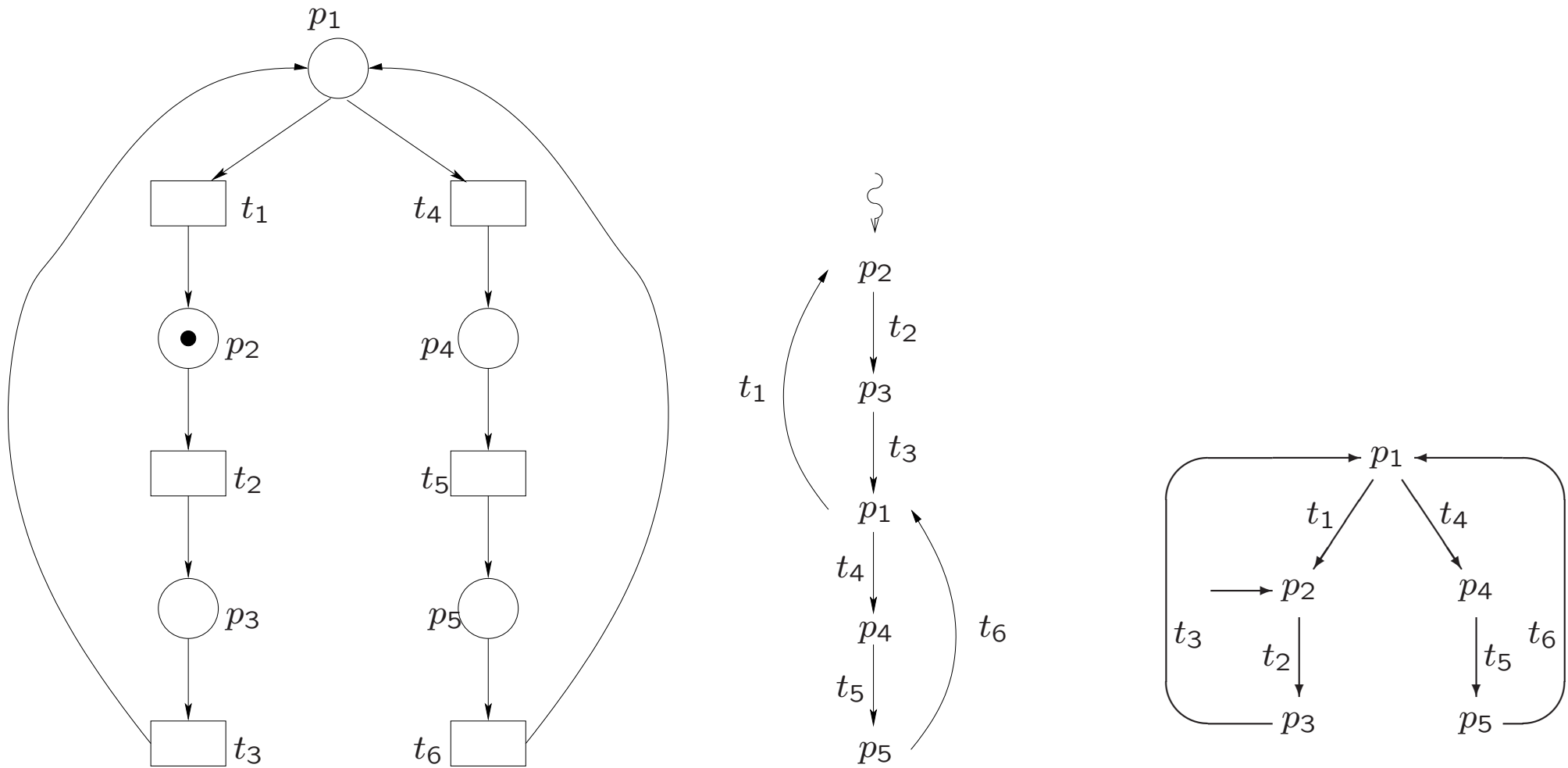


Fig. 30, 32. A sequential EN system M' and its configuration graph (two representations).

Exercise 5.6.

a. Prove that if an EN system M is firing sequence equivalent with a sequential EN system M' , then M has the following property:

For all $C, D \in \mathbb{C}_M$, $t \in T_M$ and $x \in T_M^*$:
if $t \mathbf{con} C$ and $t \mathbf{con} D$, then $(x \mathbf{con} C \iff x \mathbf{con} D)$.

c. Prove that it is decidable whether or not an EN system M is firing sequence equivalent with a sequential system.

d. Determine whether or not the **concurrency-free** EN systems of Figure 9 (from the exercise set) are configuration and/or firing sequence equivalent with a sequential EN system.

b. Construction for converse of a.

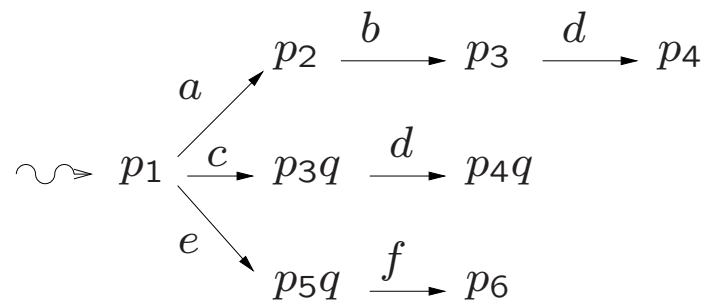
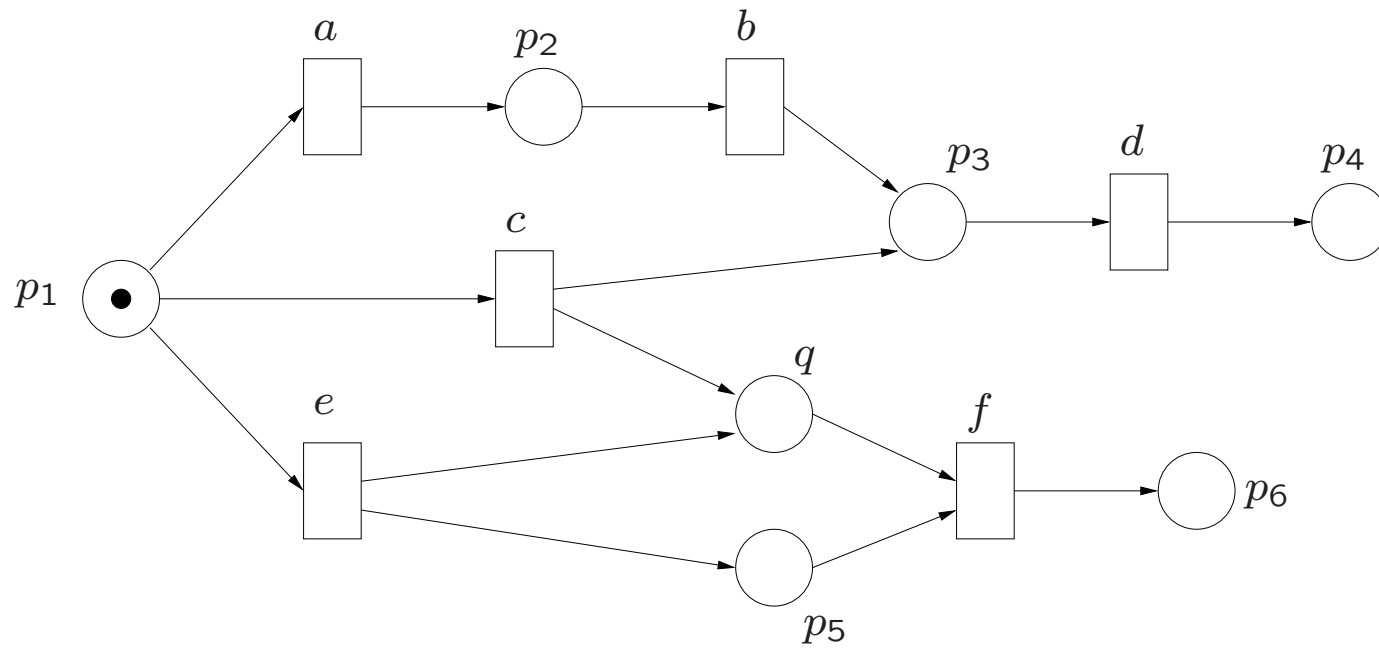


Fig. 35, 36. M' , weakly equivalent with M of Fig. 33.

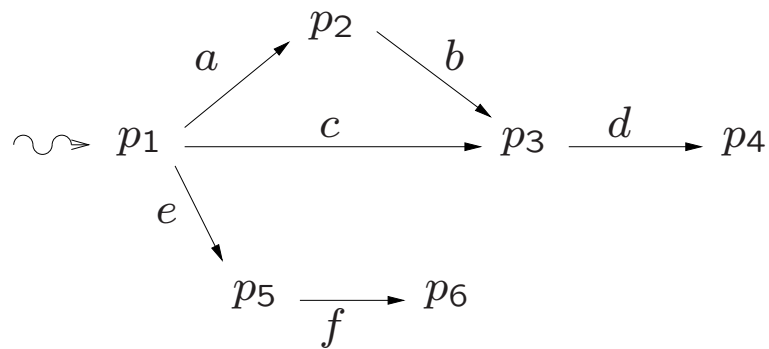
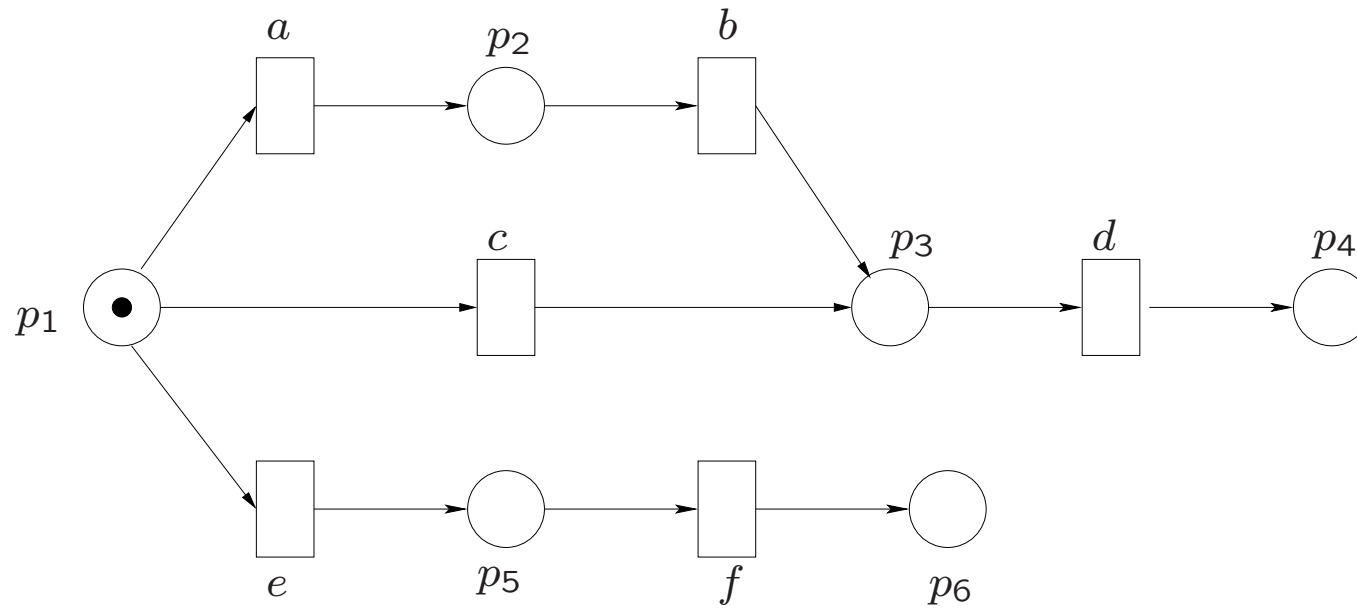


Fig. 33, 34. A seq. EN system M and its config. graph.

5.4 Subsystems and Sequential Components

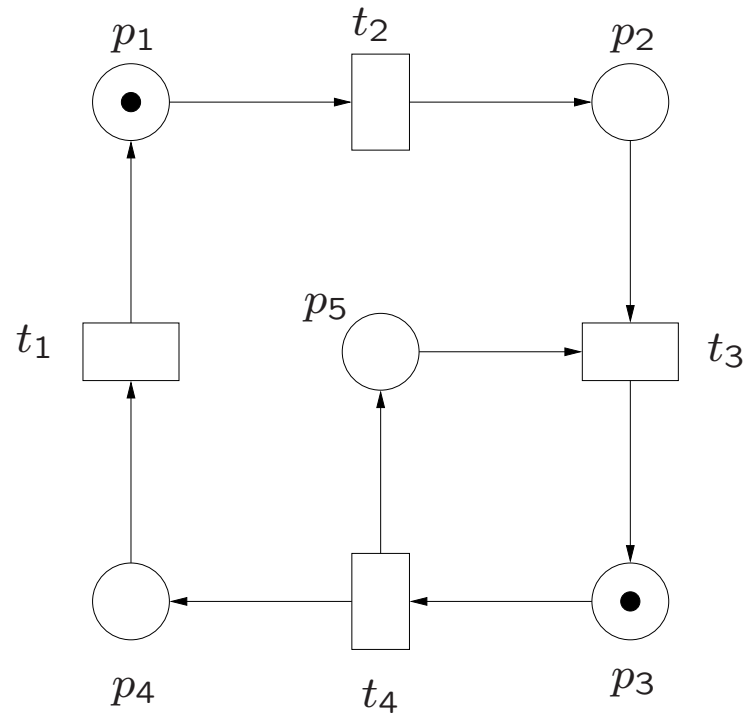


Fig. 39. An EN system with ... nontrivial subsystems ...

Definition 45. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be EN systems.

M' is a *subsystem* of M if:

- (1) $P' \subseteq P, T' \subseteq T, F' \subseteq F, C'_{in} \subseteq C_{in},$
- (2) $\forall p \in P' : \text{nbh}_M(p) \subseteq \text{nbh}_{M'}(p),$ and
- (3) $\forall p \in P' : \text{if } p \in C_{in}, \text{ then } p \in C'_{in}.$

If, moreover, M' is a sequential EN system, then M' is a *sequential component* of M .

Lemma 46. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be EN systems.

(1) M' is a subsystem of M iff

$$P' \subseteq P, T' = \text{nbh}_M(P'),$$

$$F' = F \cap ((P' \times T') \cup (T' \times P')), \text{ and}$$

$$C'_{in} = C_{in} \cap P'.$$

(2) If M' is a subsystem of M then:

$$\text{for every } t \in T', (\bullet t)_{M'} = (\bullet t)_M \cap P' \text{ and } (t\bullet)_{M'} = (t\bullet)_M \cap P',$$

$$\text{for every } t \in T - T', \text{nbh}_M(t) \cap P' = \emptyset,$$

$$\text{for every } p \in P', (\bullet p)_{M'} = (\bullet p)_M \text{ and } (p\bullet)_{M'} = (p\bullet)_M.$$

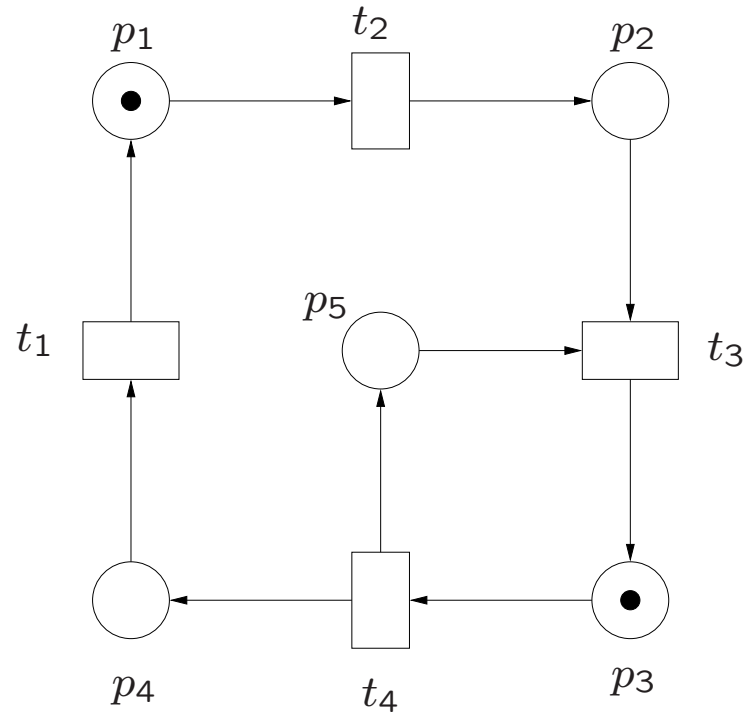


Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

Lemma 47. Let $M = (P, T, F, C_{in})$ be an EN system and let $S \subseteq P$.

There exists a subsystem M' of M with $P_{M'} = S$
iff $\bullet S = S^\bullet$.

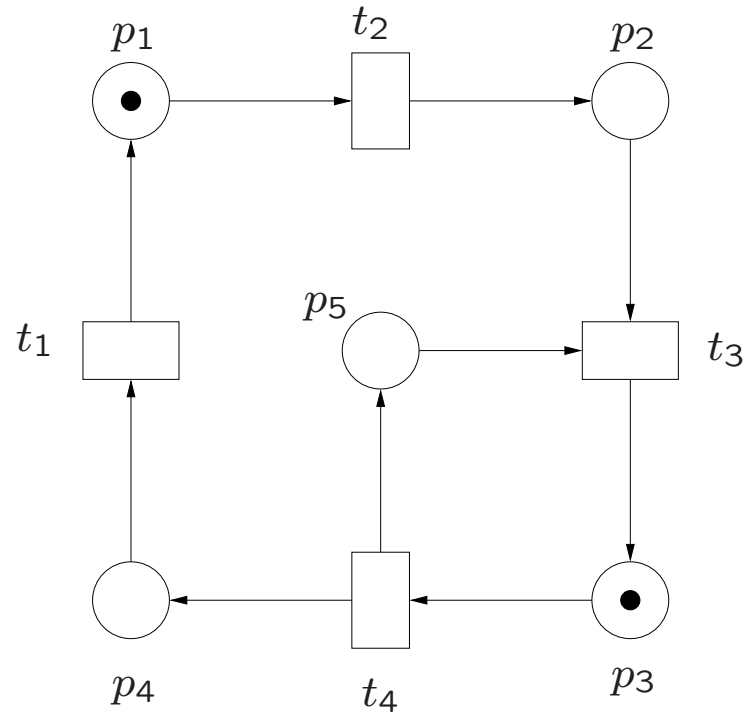


Fig. 39. An EN system with two nontrivial subsystems: $\{p_3, p_5\}$ (a sequential component) and $\{p_1, p_2, p_3, p_4\}$.

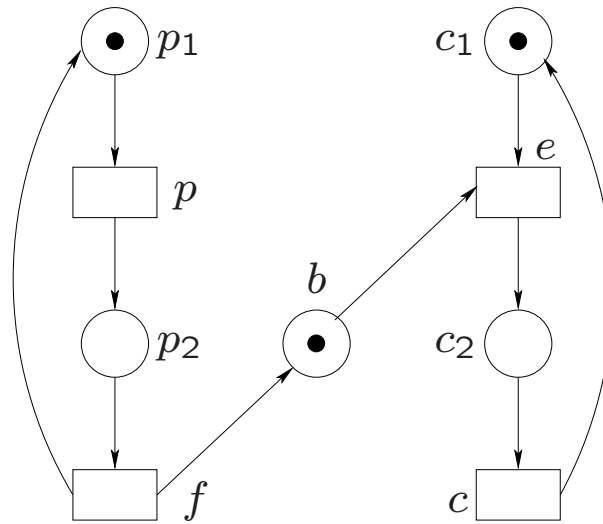


Fig. 12. Subsystems: $\{p_1, p_2\}$, the producer; $\{c_1, c_2\}$, the consumer; and $\{p_1, p_2, c_1, c_2\}$; otherwise trivial.
The buffer is NOT a subsystem