Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

vijfde college: 27 september 2011

5. Equivalences and Normal Forms

5.2 Reduction
5.3 Sequential EN Systems

Theorem 36. For every EN system M
there exists a reduced EN system M' such that $M \approx M'$.

This theorem can be strengthened as follows.

Theorem 37. For every EN system M
there exists a strongly reduced EN system M' such that $M \approx M'$.

Theorem 38. Let $M = (P, T, F, C_{in})$ be a strongly reduced EN system.

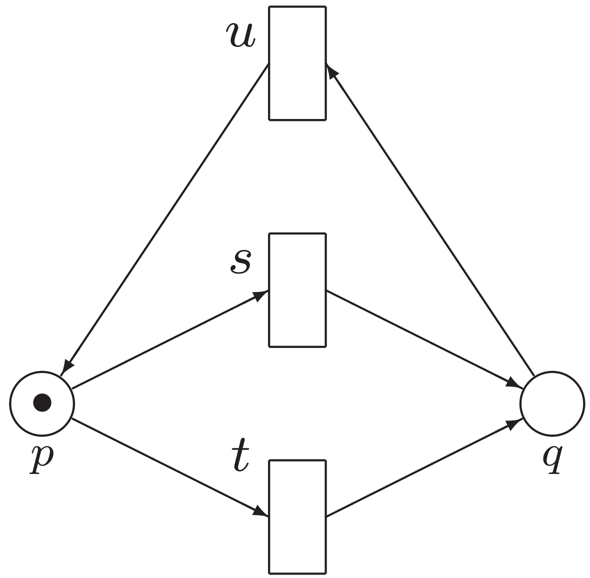
For every $p \in P$
there exist configurations $C, D \in \mathbb{C}_M$ such that

$p \in C$ and $p \notin D$.

Theorem. For every EN system M, there exists a configuration equivalent (strongly reduced) EN system M' that is P-simple.

Theorem 39. There exists an EN system M such that for every EN system M':
if $M' \approx_{fs} M$, then M' is not T-simple.

Proof: Take M as follows:



Use Exercise 4.7: if sus **con** C, then $s^\bullet \subseteq {}^\bullet u$ and ${}^\bullet s \subseteq u^\bullet$.

5.3. Sequential EN Systems.

Definition 44. An EN system M is *concurrency-free* if there do not exist $C \in \mathbb{C}_M$ and *different!* $t_1, t_2 \in T_M$ such that $\{t_1, t_2\}$ $\mathbf{con}$ C.

in other words
M is concurrency-free if it has no concurrent steps: there does not exist $C \in \mathbb{C}_M$ and $U \subseteq T_M$ such that U $\mathbf{con}$ C and $\#U \geq 2$.

Definition 40. An EN system M is *sequential* if $\#C = 1$ for all $C \in \mathbb{C}_M$.

Clearly M sequential implies M concurrency-free.

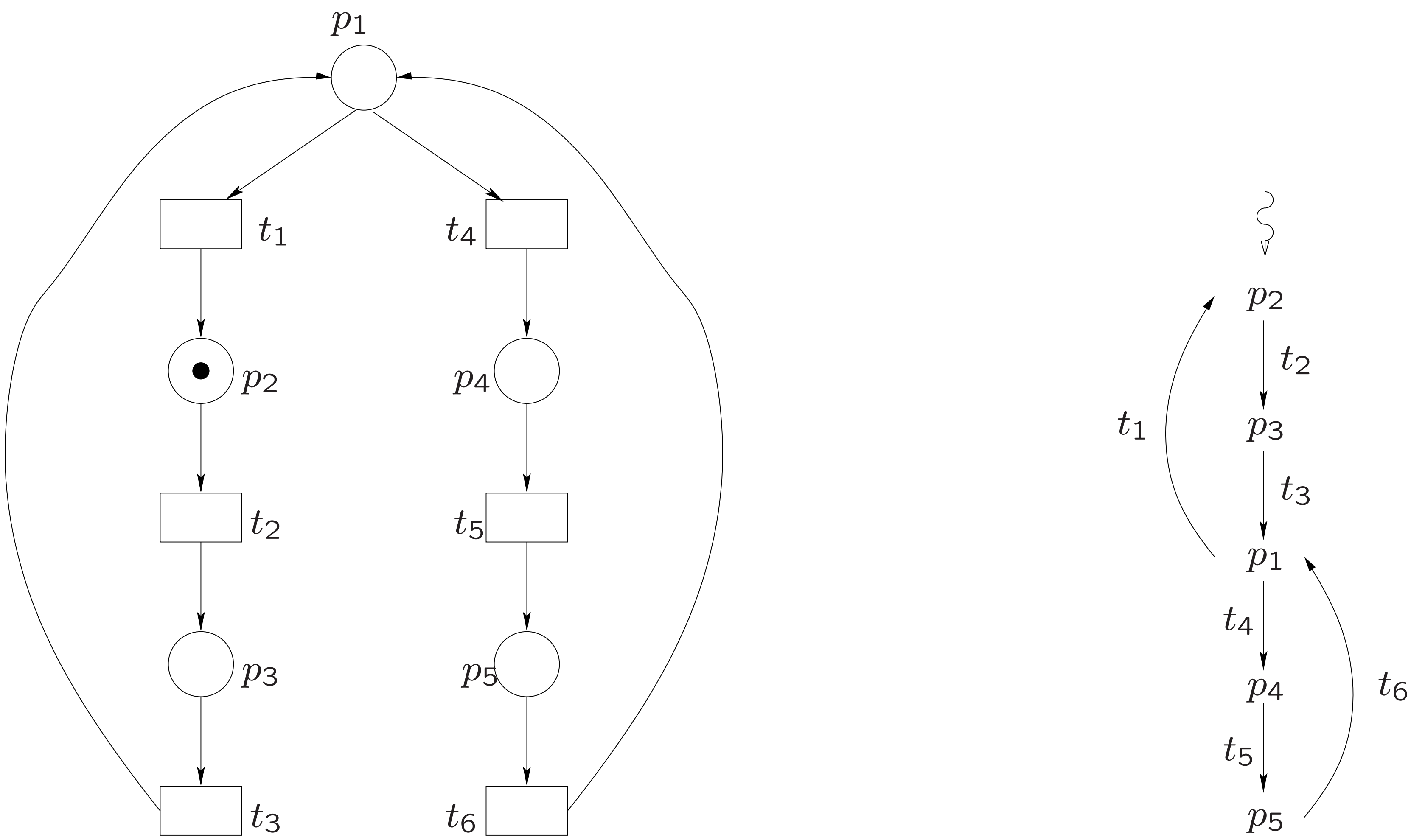


Fig. 30, 32. A sequential EN system M' and its configuration graph.

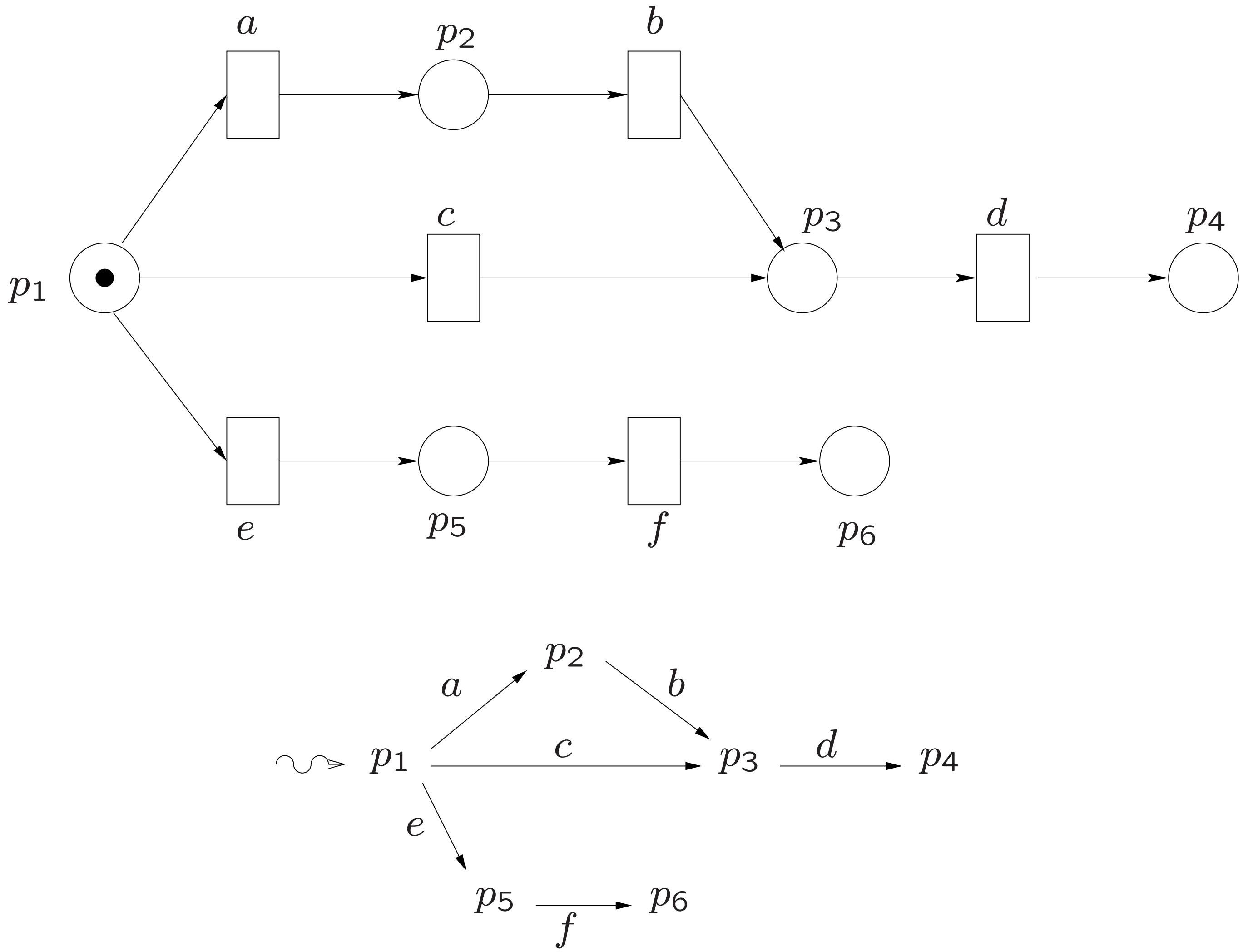


Fig. 33, 34. A seq. EN system M and its config. graph.

Definition 40. An EN system M is *sequential* if $\#C = 1$ for all $C \in \mathbb{C}_M$.

Lemma 41. If $M = (P, T, F, C_{in})$ is an EN system for which
(1) $\#C_{in} = 1$, and
(2) $\#({}^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T$,

then M is sequential.

Lemma 41. If $M = (P, T, F, C_{in})$ is an EN system for which
(1) $\#C_{in} = 1$, and
(2) $\#({}^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T$,
then M is sequential.

Lemma 42. Let $M = (P, T, F, C_{in})$ be a reduced EN system.

(1) M is sequential iff
  (i) $\#C_{in} = 1$, and
  (ii) $\#({}^\bullet t) = \#(t^\bullet) = 1$ for all $t \in T$.

(2) If M is strongly reduced and sequential,
then $\mathbb{C}_M = \{\{p\} \mid p \in P\}$.

Theorem 43. Let M and M' be two strongly reduced sequential EN systems. Then

$M \approx M'$ iff $M \equiv M'$.

Proof. . .

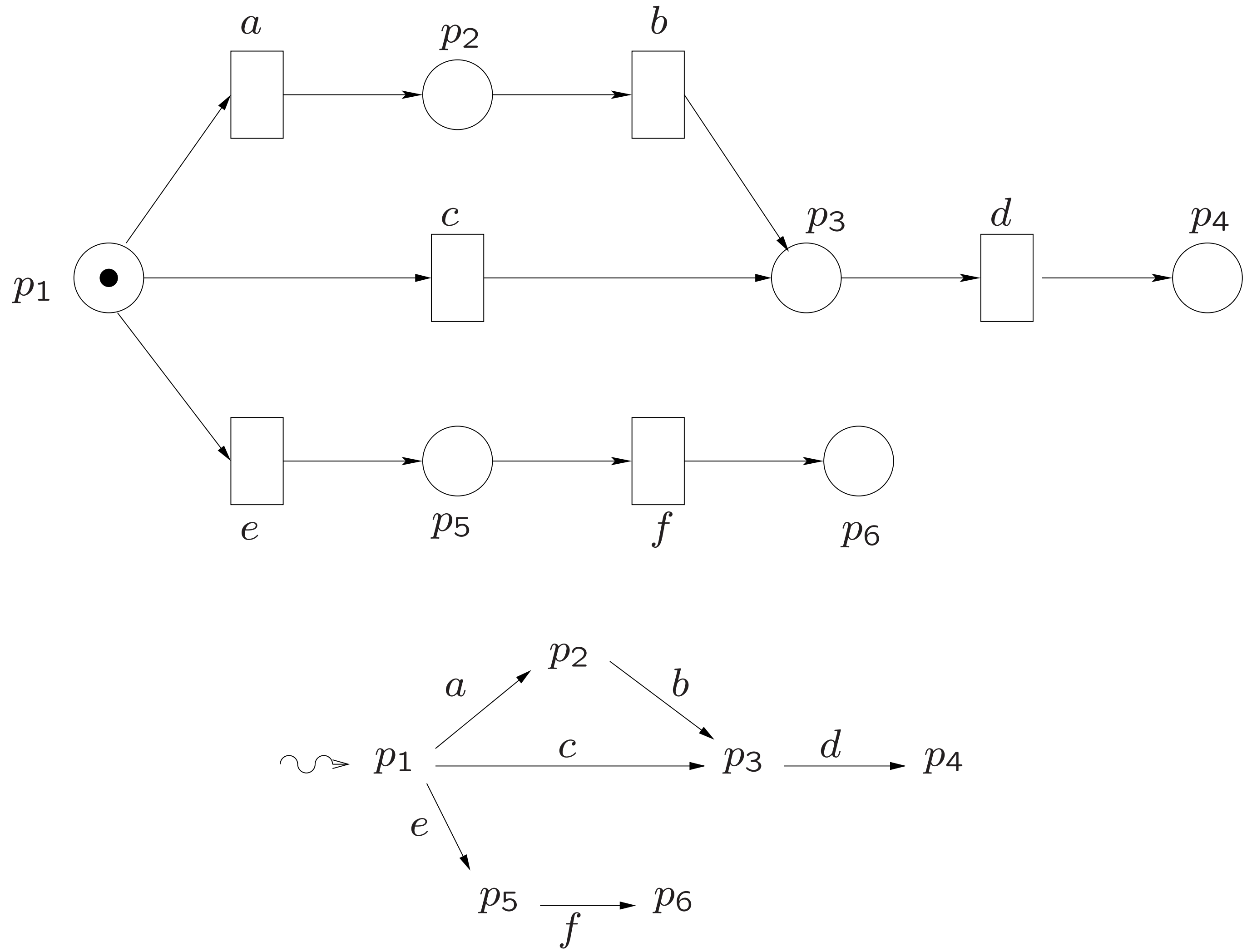


Fig. 33, 34. A seq. EN system M and its config. graph.

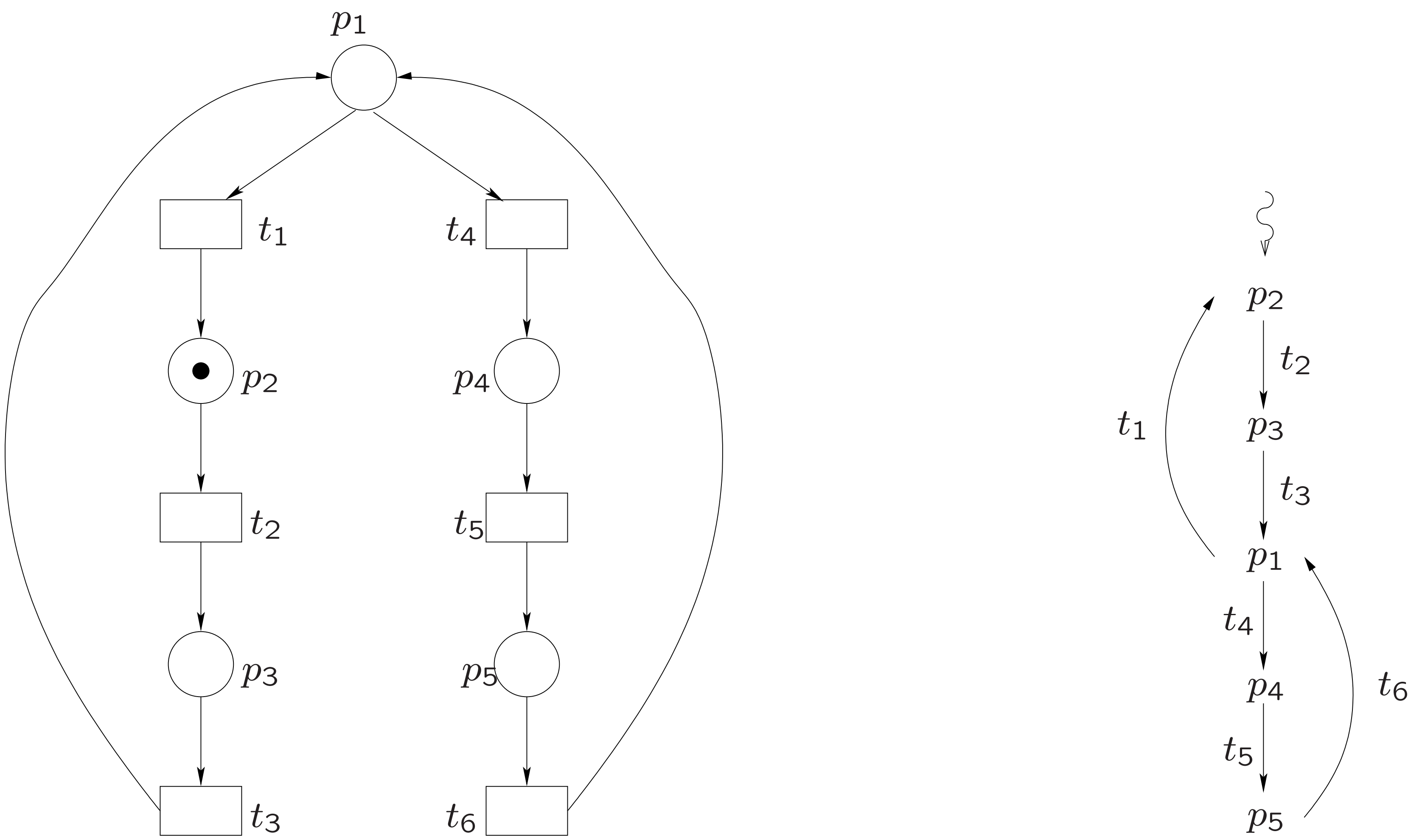


Fig. 30, 32. A sequential EN system M' and its configuration graph.

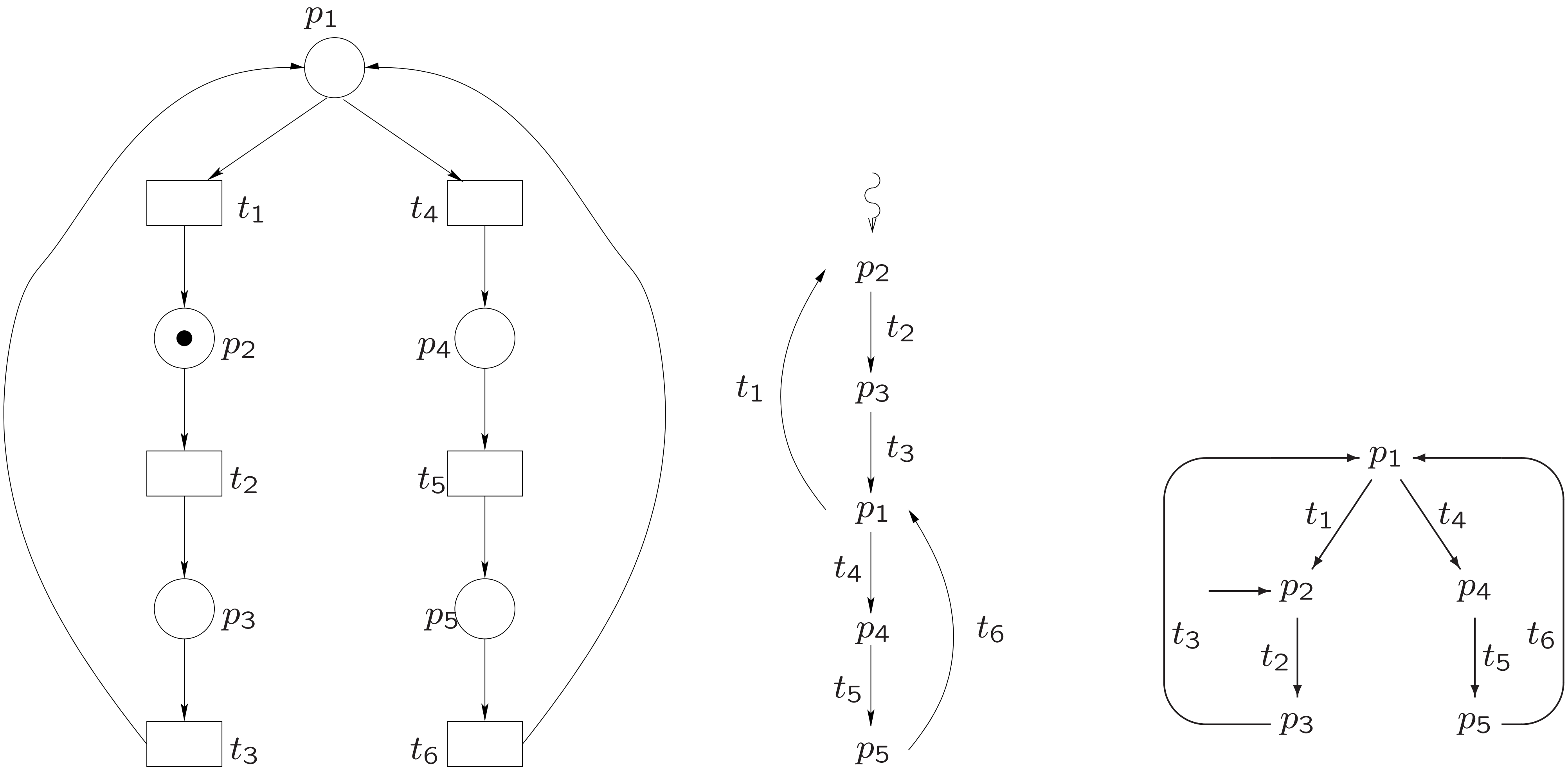


Fig. 30, 32. A sequential EN system M' and its configuration graph (two representations).

Definition 40. An EN system M is *sequential* if $\#C = 1$ for all $C \in \mathbb{C}_M$.

Definition 44. An EN system M is *concurrency-free* if there do not exist $C \in \mathbb{C}_M$ and $t_1, t_2 \in T_M$ such that $\{t_1, t_2\}$ $\mathbf{con}$ C and $t_1 \neq t_2$

Clearly M sequential implies M concurrency-free.
Conversely?

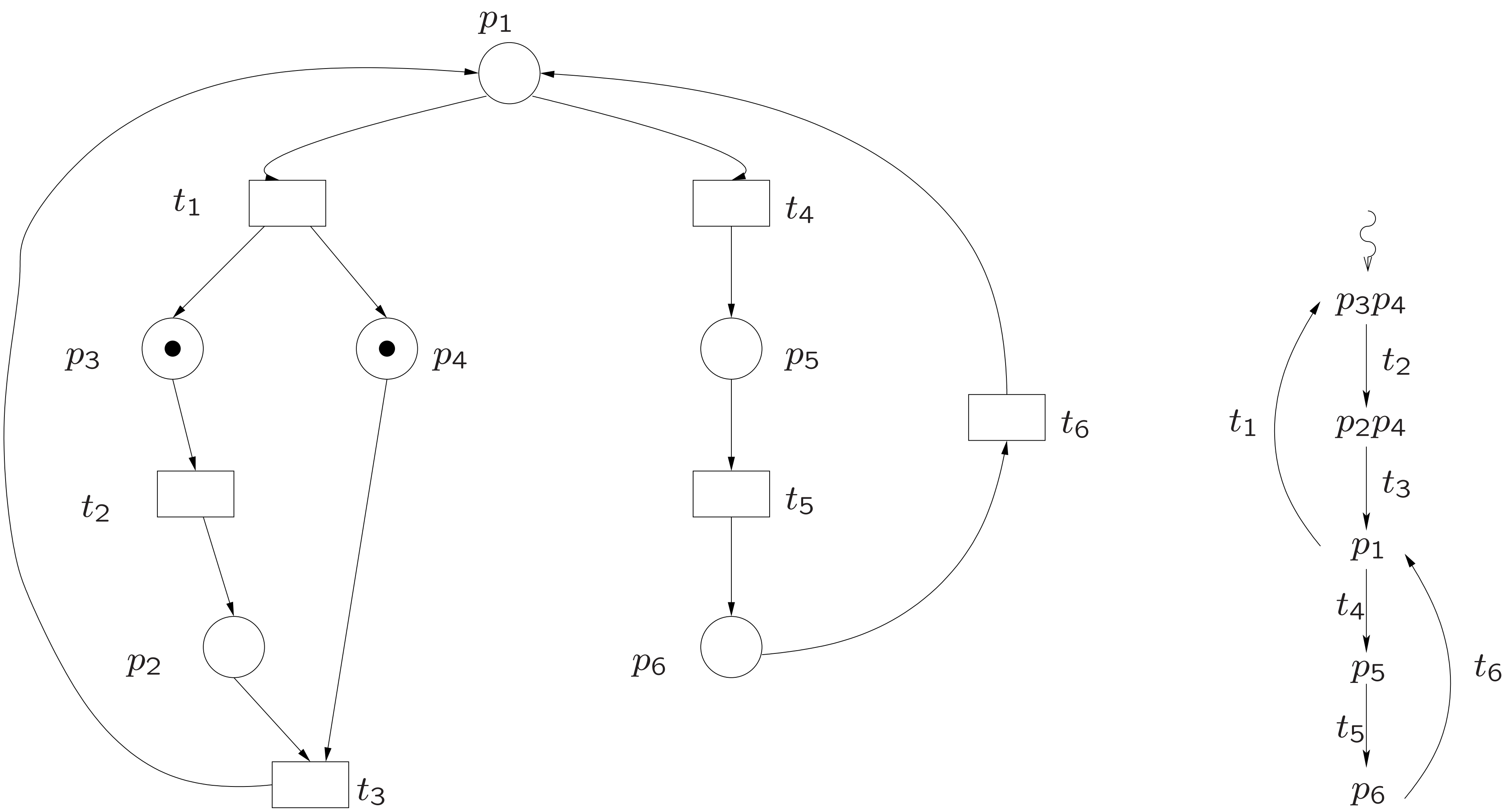


Fig. 29, 31. An EN system M and its configuration graph.

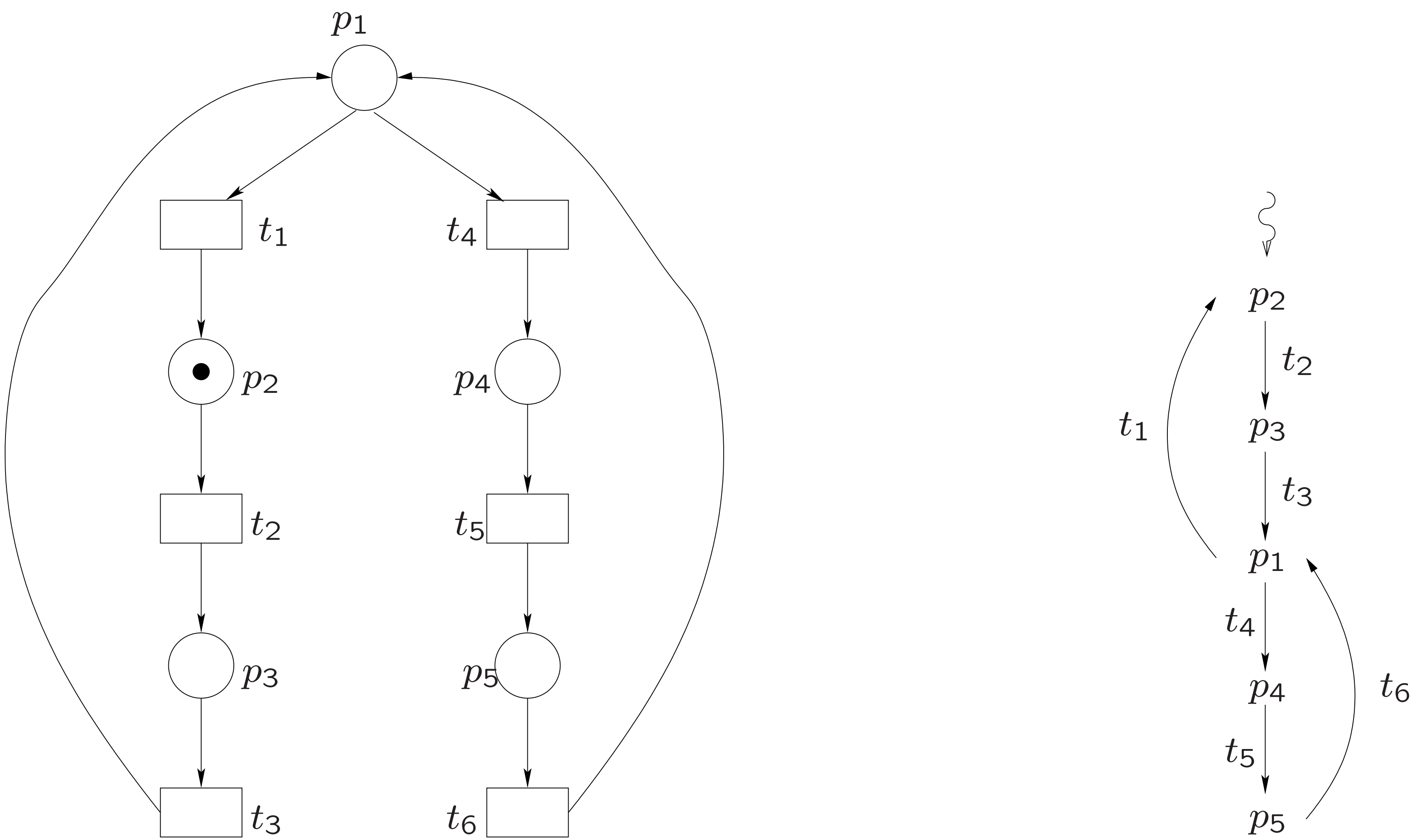
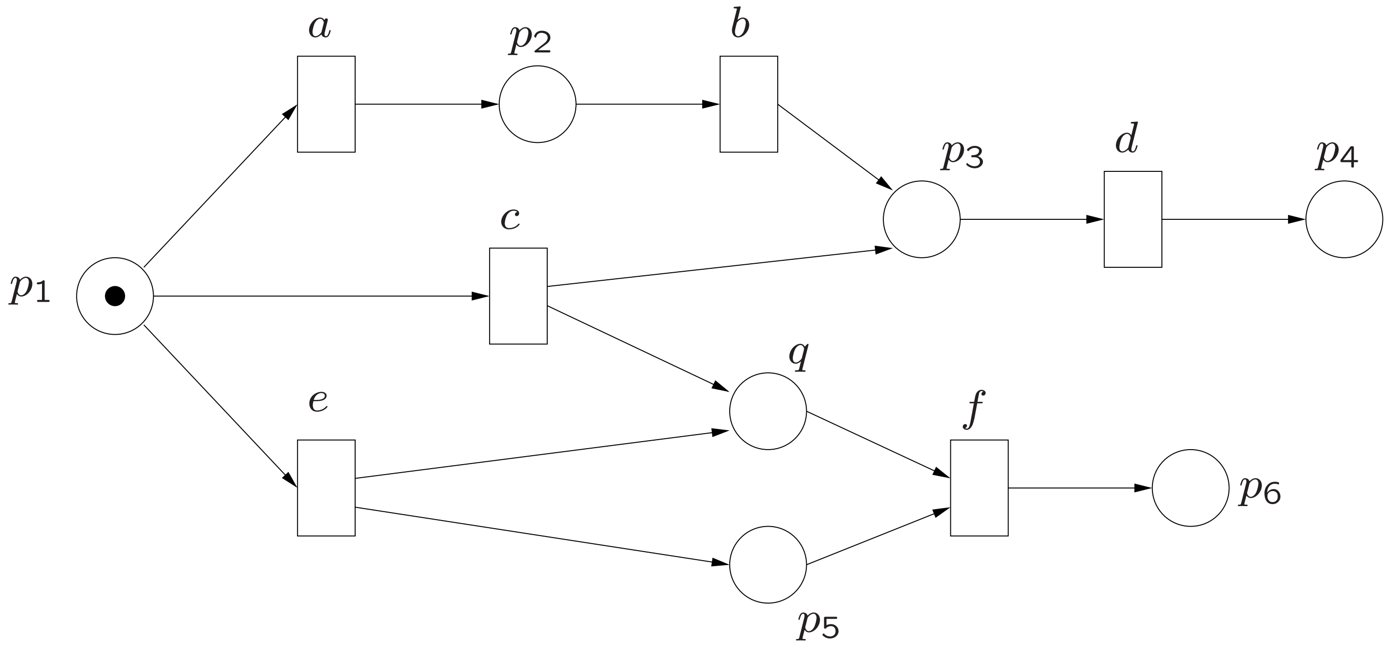


Fig. 30, 32. A sequential EN system M' and its configuration graph.



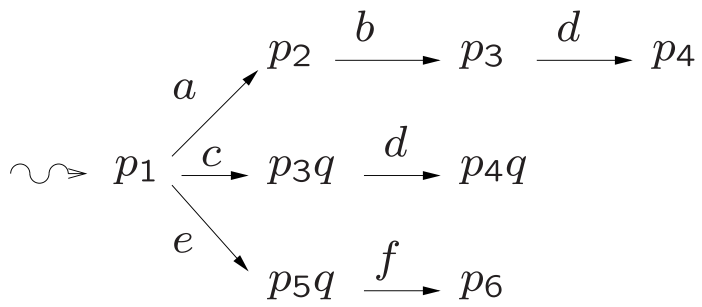
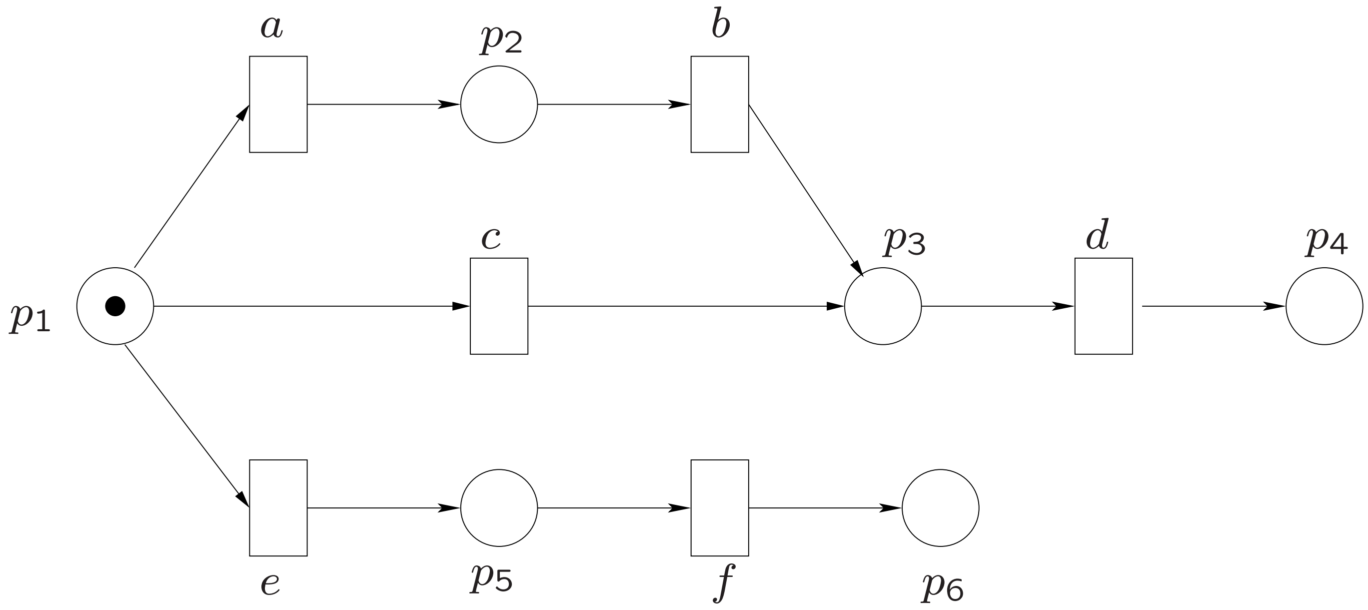


Fig. 35, 36. M', weakly equivalent with M of Fig. 33.



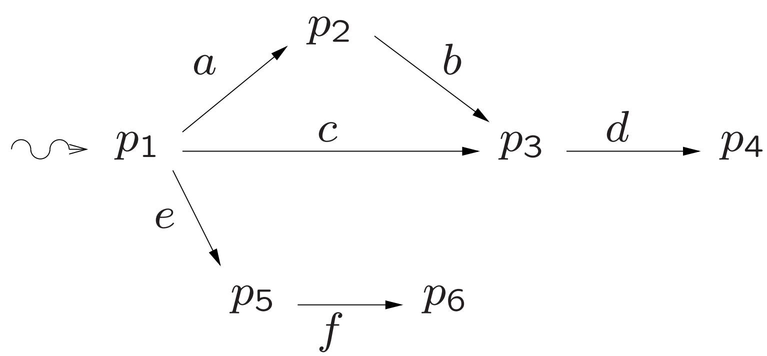


Fig. 33, 34. A seq. EN system M and its config. graph.

Definition 40. An EN system M is *sequential* if $\#C = 1$ for all $C \in \mathbb{C}_M$.

Definition 44. An EN system M is *concurrency-free* if there do not exist $C \in \mathbb{C}_M$ and $t_1, t_2 \in T_M$ such that $\{t_1, t_2\}$ $\mathbf{con}$ C and $t_1 \neq t_2$

Clearly M sequential implies M concurrency-free.
Conversely? Figures 29 vs 30 and 35 vs 33. However:
$\exists$ a concurrency-free EN-system M for which
$\nexists$ sequential EN system M' such that $M \approx_w M'$

Exercises 5.5 and 5.6acd

Exercise 5.5.

Prove that an EN system M is configuration equivalent with a sequential EN system M', if and only if M has the following property:

> For all $C, D \in \mathbb{C}_M$ and $t \in T_M$: if t **con** C and t **con** D, then $C = D$.

In other words:

> Each transition in M occurs at most once in SCG(M).