Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

vierde college: 20 september 2011

5. Equivalences and Normal Forms

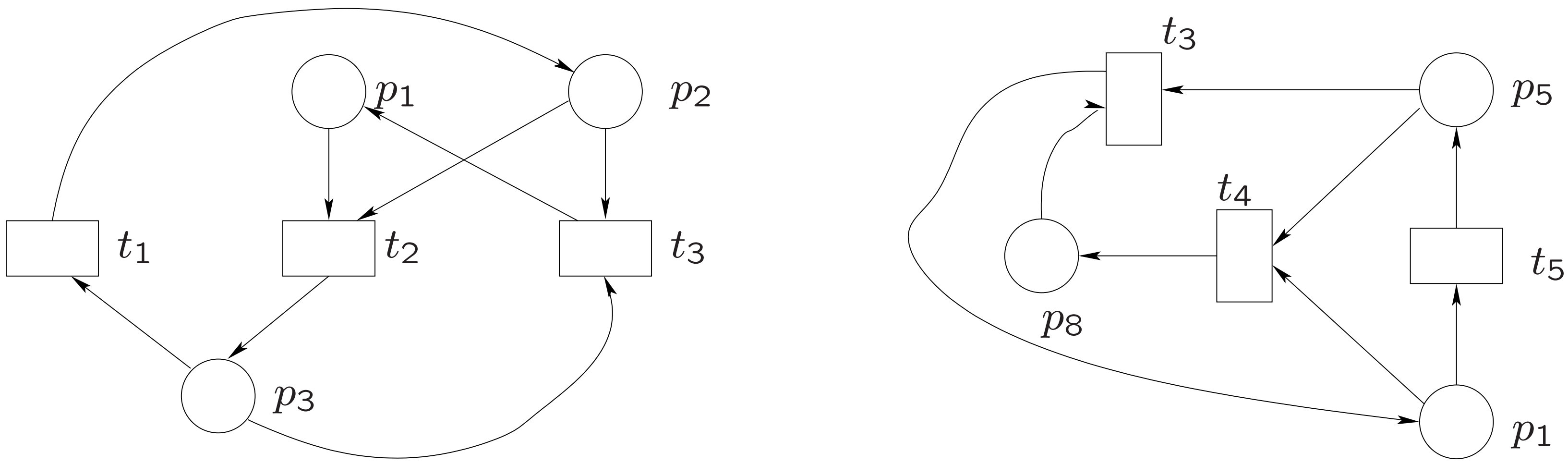
5.1 Equivalence
5.2 Reduction

tweede werkcollege: 22 september 2011
afronden opgaven bij 4. EN Systems
beginnen met opgaven bij 5. Equivalences and Normal Forms

Definition 3. Two nets
$N = (P, T, F)$ and $N' = (P', T', F')$
are *isomorphic*, denoted by $N \equiv N'$,

if there exist two bijections
$\alpha : P \to P'$ and $\beta : T \to T'$,

such that for every $p \in P$ and $t \in T$,
$(p, t) \in F$ iff $(\alpha(p), \beta(t)) \in F'$
and
$(t, p) \in F$ iff $(\beta(t), \alpha(p)) \in F'$.



A net N **Fig. 8** and a net N'' **Fig. 10**, isomorphic.

Definition 27. Two EN systems

$M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$

are *isomorphic*, denoted by $M \equiv M'$,

if there exist two bijections
$\alpha : P \to P'$ and $\beta : T \to T'$

such that $\mathbf{und}(M) \equiv_{\beta}^{\alpha} \mathbf{und}(M')$ and $\alpha(C_{in}) = C'_{in}$.

Definition 28. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

Then M and M' are *configuration equivalent*,

denoted by $M \approx M'$,

if there exist two bijections
$\alpha : \mathbb{C}_M \to \mathbb{C}_{M'}$ and $\beta : \mathbf{use}_M(T) \to \mathbf{use}_{M'}(T')$

such that

(1) $\alpha(C_{in}) = C'_{in}$ and

(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$, $C[t\rangle_M D$ iff $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$.

Theorem 29. Let M and M' be two EN systems.

Then $M \approx M'$ iff

SCG(M) $\equiv$ SCG(M') iff

CG(M) $\equiv$ CG(M').

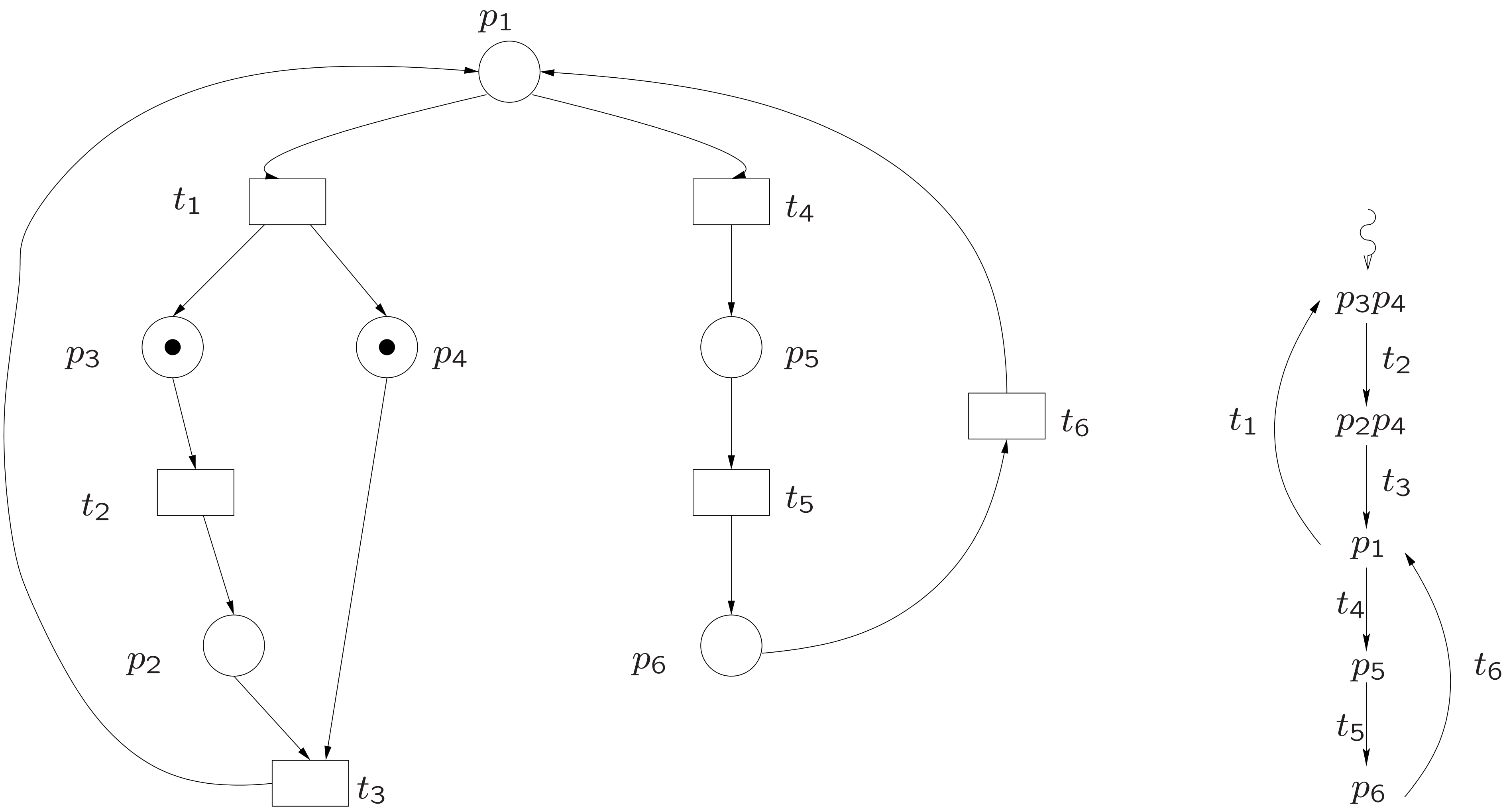


Fig. 29 / 31. An EN system M and its configuration graph.

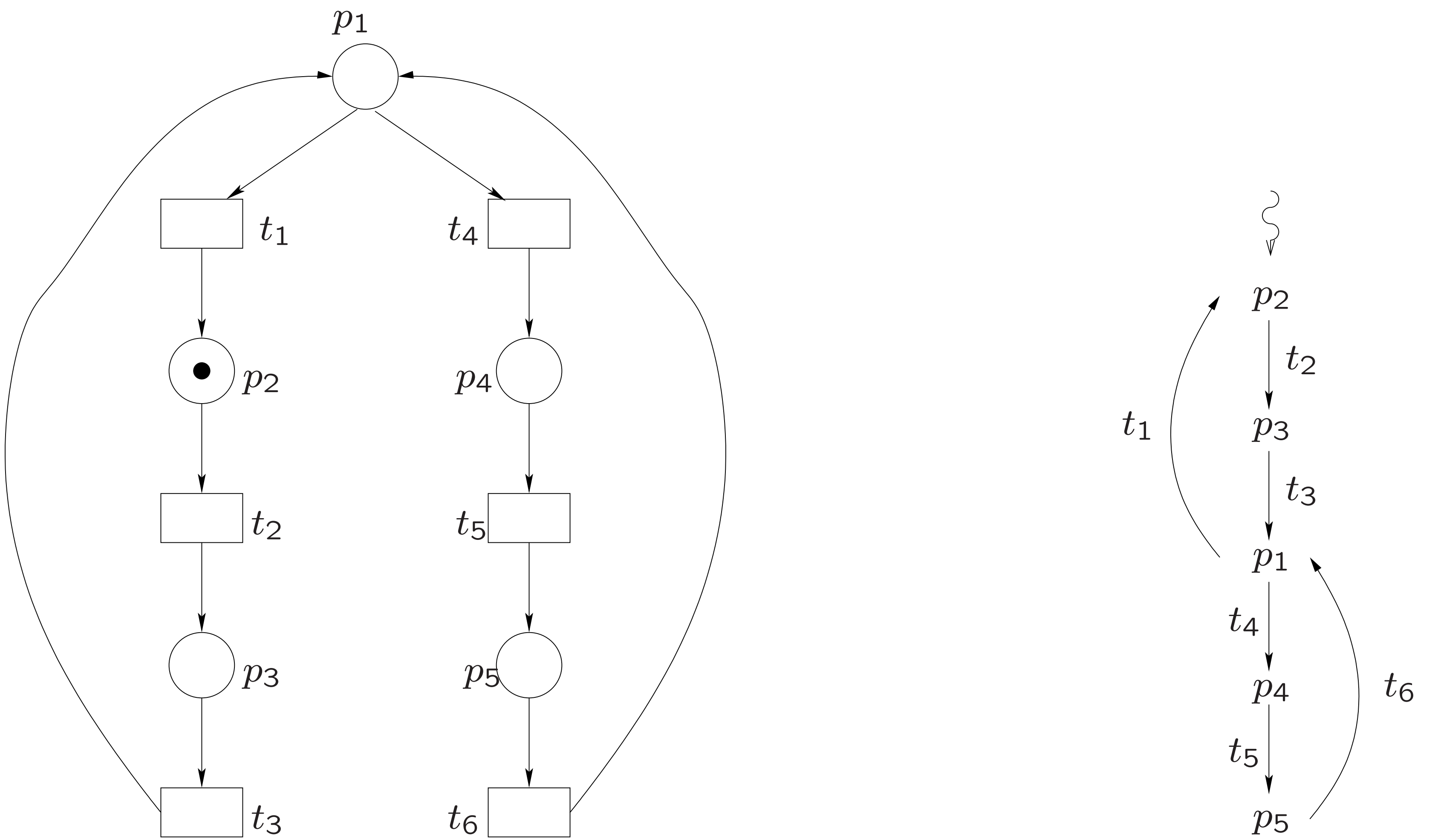


Fig. 30 / 32. An EN system M' and its configuration graph.

Fig. 31 / 32. The configuration graph of M of Fig. 29 and the configuration graph of M' of Fig. 30.

Definition 28. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

Then M and M' are *configuration equivalent*,

denoted by $M \approx M'$,

if there exist two bijections
$\alpha : \mathbb{C}_M \to \mathbb{C}_{M'}$ and $\beta : \mathbf{use}_M(T) \to \mathbf{use}_{M'}(T')$

such that

(1) $\alpha(C_{in}) = C'_{in}$ and

(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$, $C[t\rangle_M D$ iff $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$.

Lemma 30. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

If α is an injective function, $\alpha : \mathbb{C}_M \to \mathcal{P}(P')$,
and β is a bijective function, $\beta : \mathbf{use}_M(T) \to T'$, such that

(1) $\alpha(C_{in}) = C'_{in}$ and
(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$,
$C[t\rangle_M D$ implies $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$, and
$\beta(t)\ \mathbf{con}_{M'}\ \alpha(C)$ implies $t\ \mathbf{con}_M\ C$,

then $M \approx^{\alpha}_{\beta} M'$.

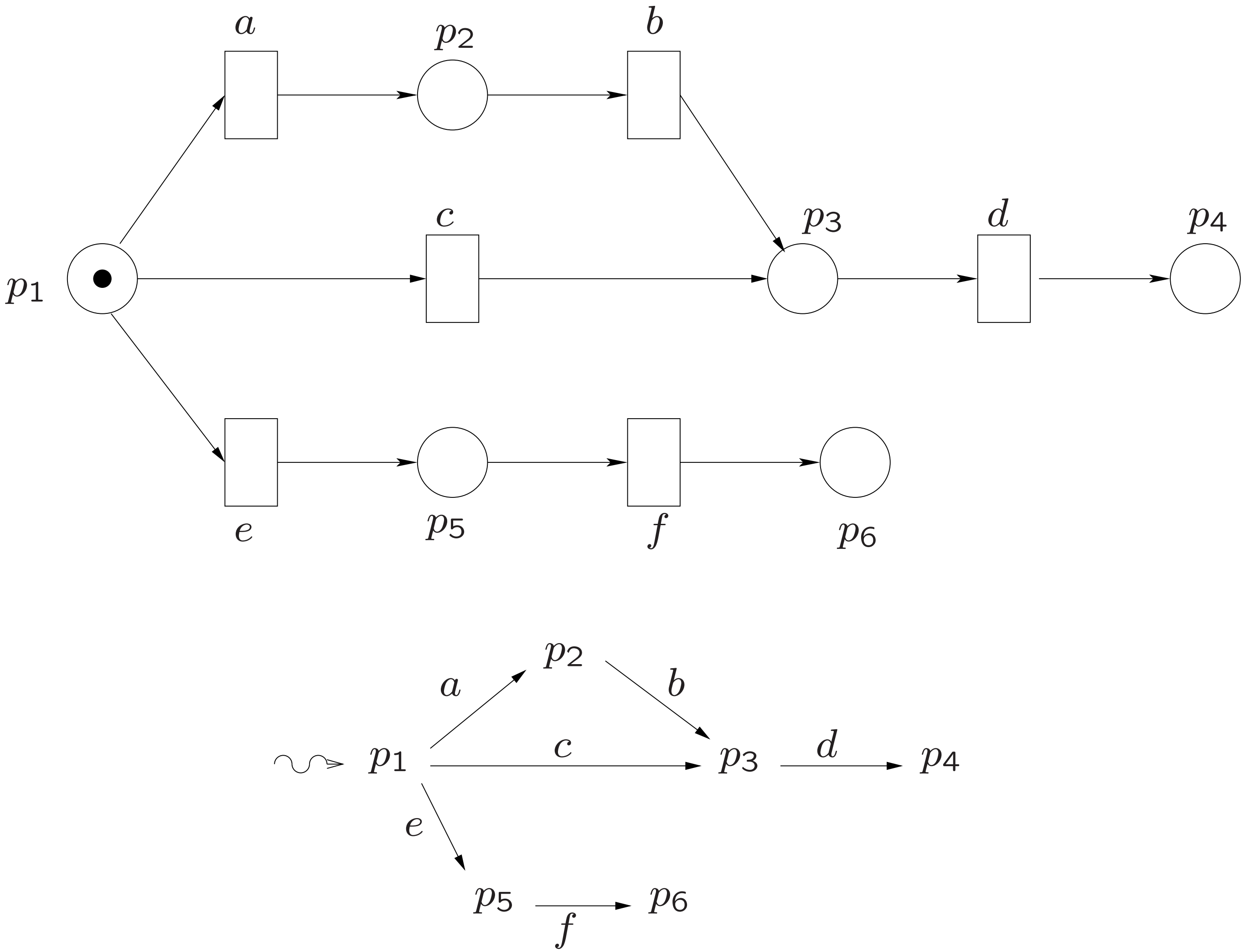


Fig. 33 / 34. An EN system M and its configuration graph.

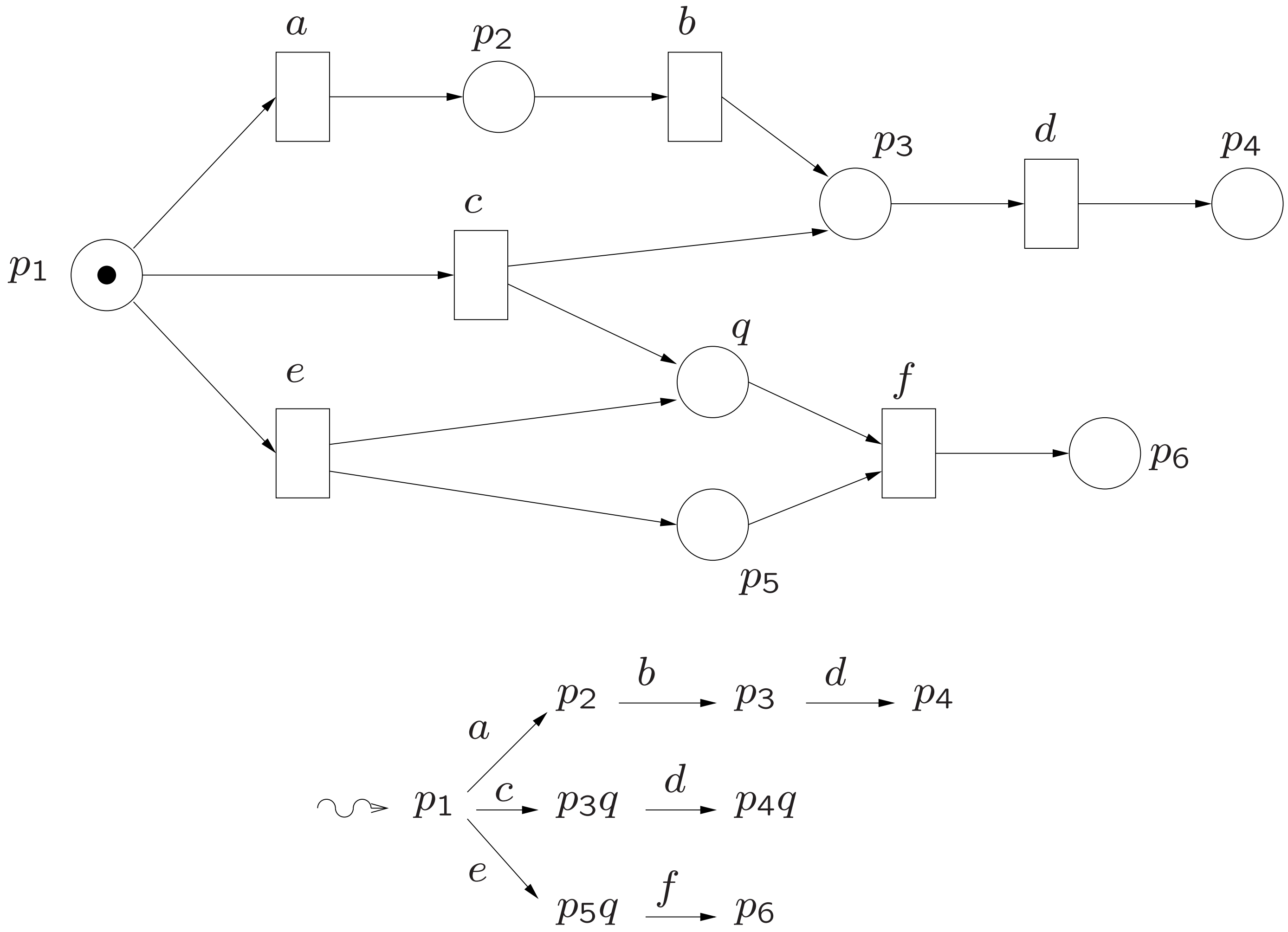
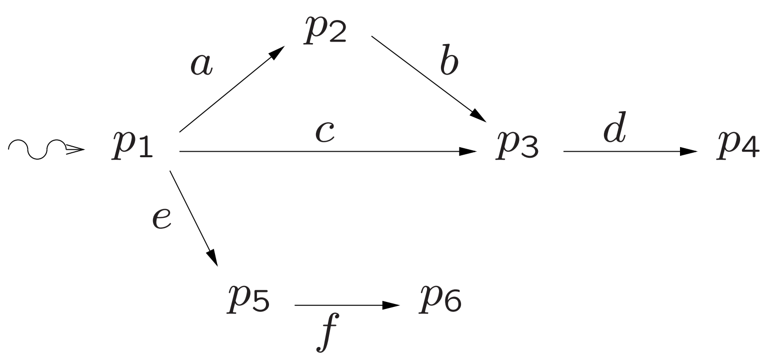


Fig. 35 / 36. M', weakly equivalent with M of Fig. 33.



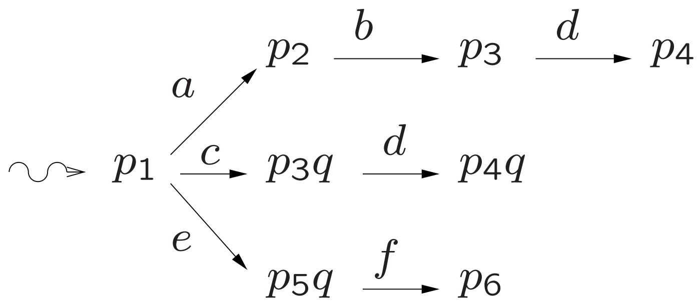


Fig. 34., **Fig. 36.** The configuration graphs of M of Fig. 33 and of M' of Fig. 35.

Definition 31. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.
M and M' are *weakly configuration equivalent*, denoted by $M \approx_w M'$,
if there exists a relation $\alpha \subseteq \mathbb{C}_M \times \mathbb{C}_{M'}$
and a bijection $\beta : \mathbf{use}(T) \to \mathbf{use}(T')$, such that

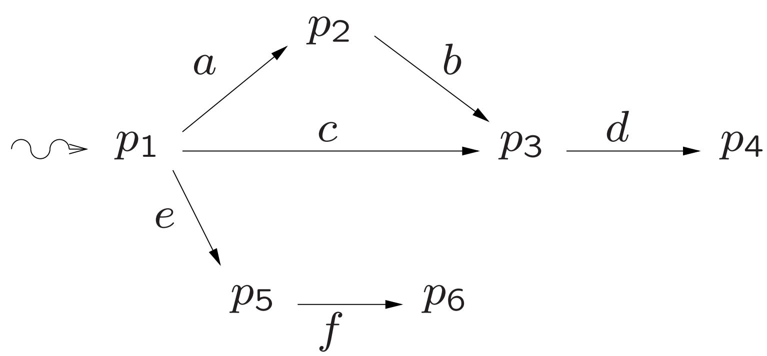
(1) $(C_{in}, C'_{in}) \in \alpha$,

(2) for all $C, D \in \mathbb{C}_M$, $C' \in \mathbb{C}_{M'}$, and $t \in \mathbf{use}(T)$:
if $C[t\rangle_M D$ and $(C, C') \in \alpha$, then
there is a $D' \in \mathbb{C}_{M'}$ such that $C'[\beta(t)\rangle_{M'} D'$ and $(D, D') \in \alpha$, and

(3) for all $C', D' \in \mathbb{C}_{M'}$, $C \in \mathbb{C}_M$, and $t' \in \mathbf{use}(T')$:
if $C'[t'\rangle_{M'} D'$ and $(C, C') \in \alpha$, then
there is a $D \in \mathbb{C}_M$ such that $C[\beta^{-1}(t')\rangle_M D$ and $(D, D') \in \alpha$.

Definition 32. Let M and M' be two EN systems.

M and M' are *firing sequence equivalent*, denoted by $M \approx_{fs} M'$, if there exists a bijection $\beta : \mathbf{use}(T_M) \to \mathbf{use}(T_{M'})$ such that $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$.



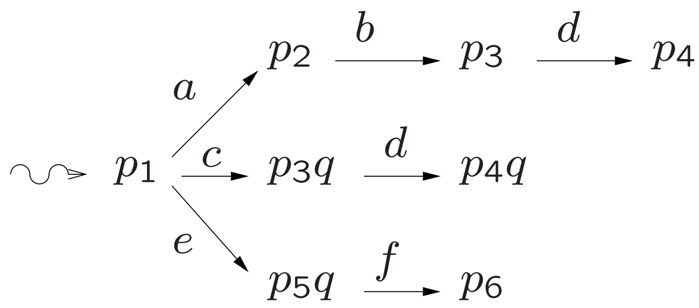


Fig. 34., Fig. 36. The configuration graphs of M of Fig. 33 and of M' of Fig. 35.

Theorem 33. Two EN systems are firing sequence equivalent iff they are weakly configuration equivalent.

Opgave 5.2.

Theorem 33. Two EN systems are firing sequence equivalent iff they are weakly configuration equivalent.

Corollary 34 If two EN systems are configuration equivalent, then they are also firing sequence equivalent.

$M \equiv M'$	$\Longrightarrow$	$M \approx M'$	$\Longrightarrow$	$M \approx_w M'$	$\Longleftrightarrow$	$M \approx_{fs} M'$
isomorphic		config. equivalent		weakly config. equivalent		firing seq. equivalent

$$\mathsf{SCG}(M) \equiv \mathsf{SCG}(M')$$
$$\mathsf{CG}(M) \equiv \mathsf{CG}(M')$$

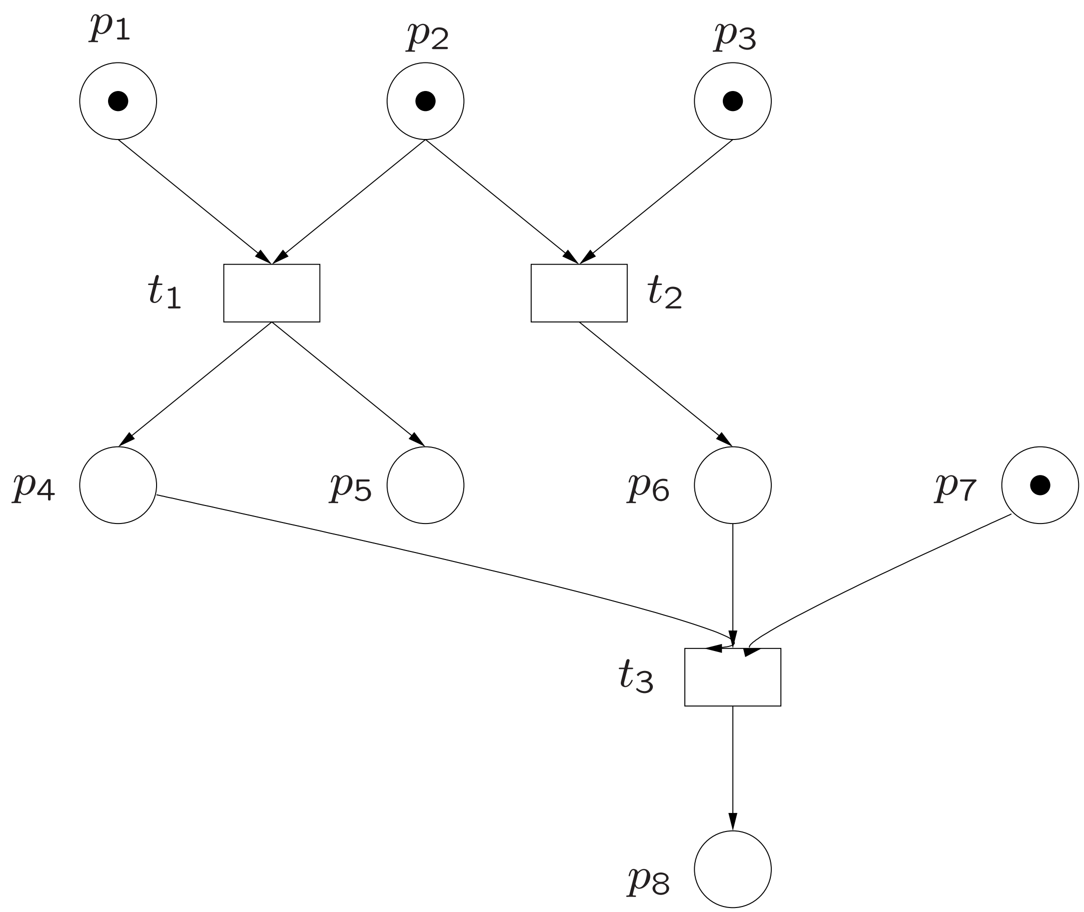


Fig. 37. An EN system with useless transition t_3 and useless places p_7 and p_8.

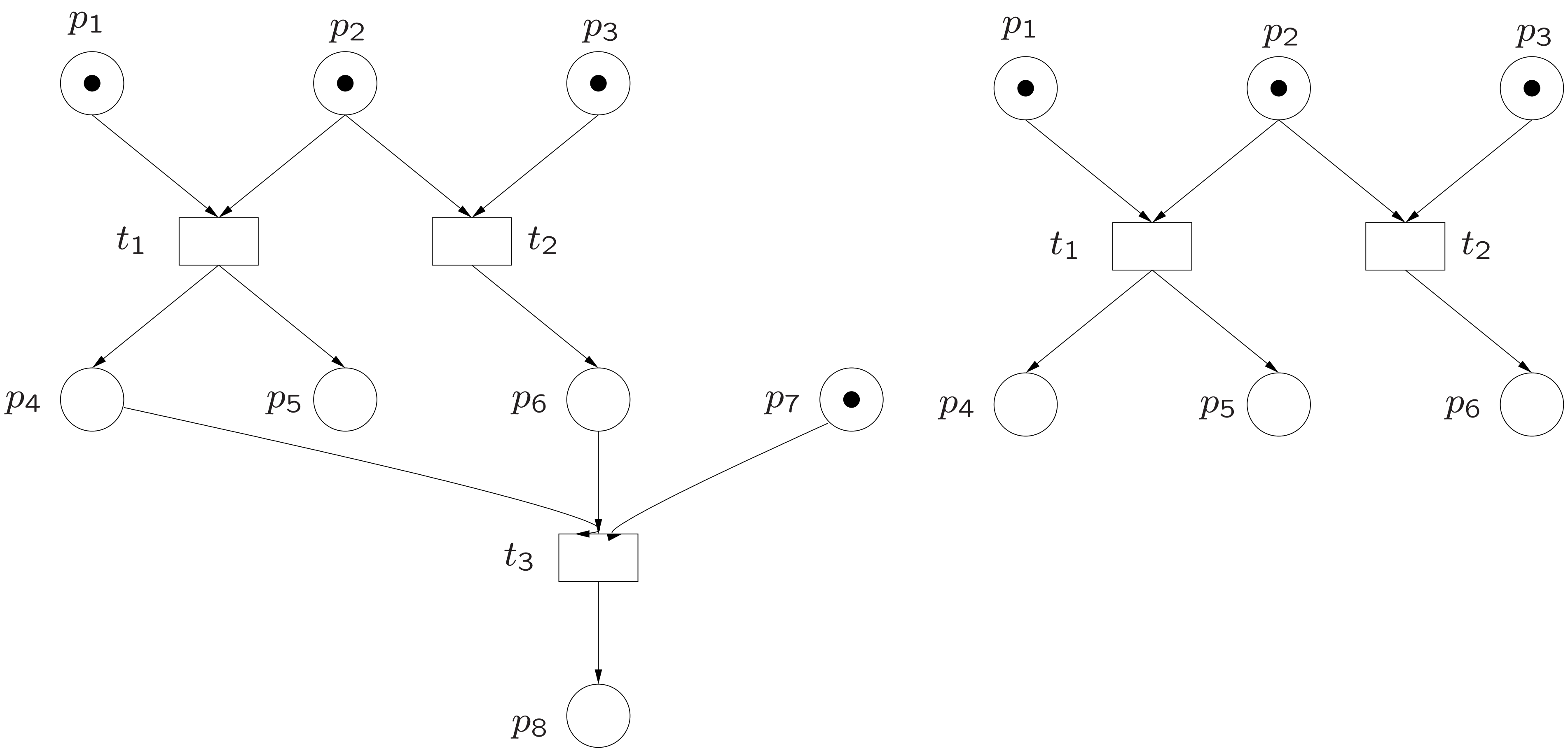


Fig. 37 / 38. An EN system and a strongly reduced, configuration equivalent EN system.

Definition 35 An EN system M
is *reduced* if all transitions of M are useful.
M is *strongly reduced* if M is reduced and has no isolated places.

Theorem 36. For every EN system M
there exists a reduced EN system M' such that $M \approx M'$.

This theorem can be strengthened as follows.

Theorem 37. For every EN system M
there exists a strongly reduced EN system M' such that $M \approx M'$.

Definition 28. Let $M = (P, T, F, C_{in})$ and $M' = (P', T', F', C'_{in})$ be two EN systems.

Then M and M' are *configuration equivalent*,

denoted by $M \approx M'$,

if there exist two bijections
$\alpha : \mathbb{C}_M \rightarrow \mathbb{C}_{M'}$ and $\beta : \mathbf{use}_M(T) \rightarrow \mathbf{use}_{M'}(T')$

such that

(1) $\alpha(C_{in}) = C'_{in}$ and

(2) for all $C, D \in \mathbb{C}_M$ and $t \in \mathbf{use}_M(T)$, $C[t\rangle_M D$ iff $\alpha(C)[\beta(t)\rangle_{M'}\alpha(D)$.

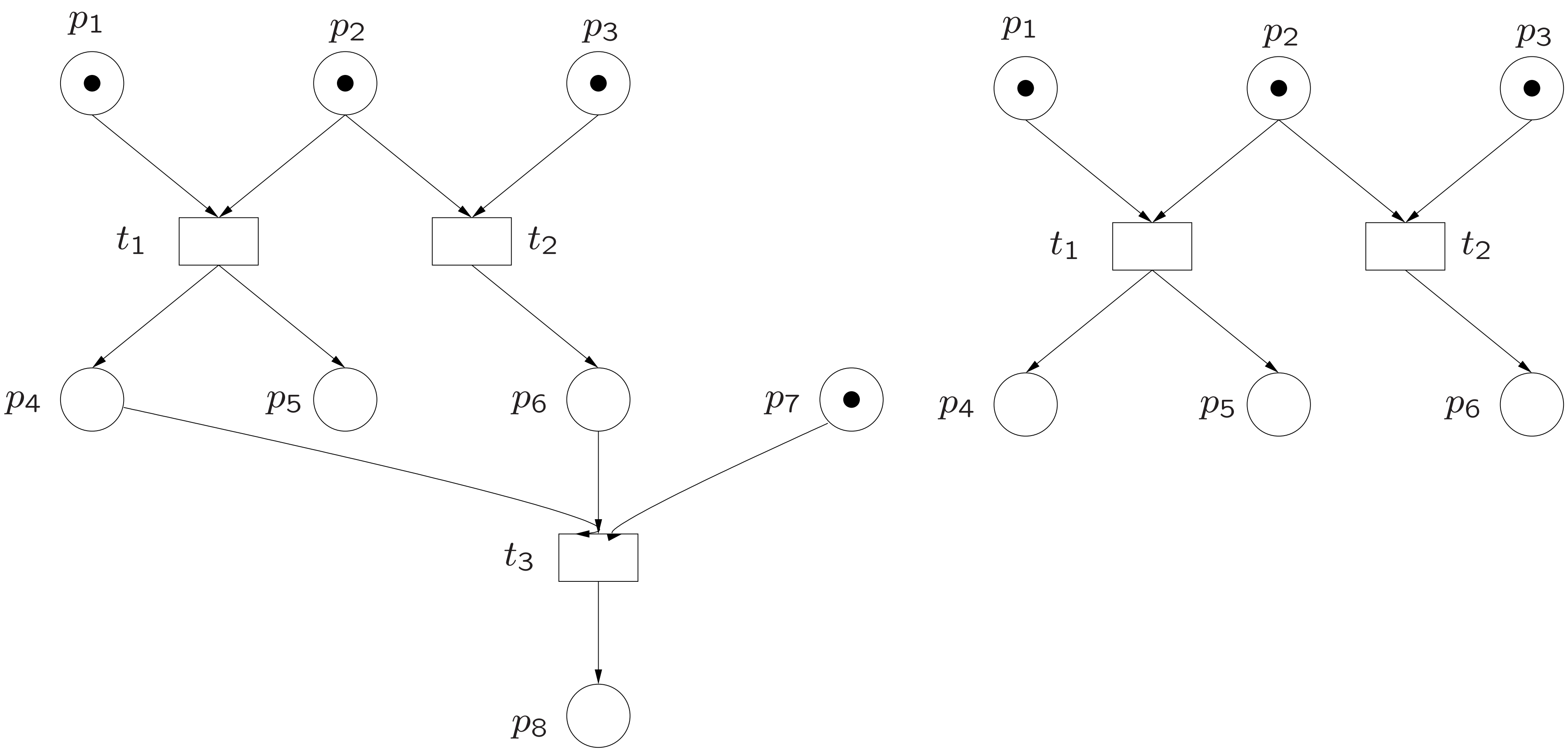


Fig. 37 / 38. An EN system and a strongly reduced, configuration equivalent EN system.