Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

derde college: 13 september 2011

4.4 Concurrency

4.5 Fundamental Situations

eerste werkcollege: 15 september 2011

opgaven bij 4. EN Systems

installatie pipe2

Theorie van Concurrency — najaar 2011

http://www.liacs.nl/home/rvvliet/tvc/

- hoorcollege/werkgroep $\sim$ 2/1

Gecorrigeerde data:
dinsdag 6 september - 25 oktober, zaal 403, 11.15–13.00
donderdag 8 september - 27 oktober, zaal 403, 11.15–13.00
donderdag 3 november - 8 december, zaal 403, 10.00–13.00

- dictaat + survey paper
- opgavenbundel + oplossingen + oude tentamens

Samen voor EUR 10,50

- modelleertoets, donderdag 17 november 2011, 10:00–13:00

Definition 13. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $U \subseteq T$. U is a *disjoint set of transitions*, notation $\mathbf{disj}(U)$, if **1.** $U \neq \varnothing$
and **2.** for all transitions $t_1 \neq t_2 \in U$: $\mathbf{nbh}(t_1) \cap \mathbf{nbh}(t_2) = \varnothing$.

(2) Let $U \subseteq T$ and let $C \subseteq P$. Then U *has concession in* C (or U *can be fired in* C, or U *is enabled in* C) if
1. $\mathbf{disj}(U)$, **2.** ${}^\bullet U \subseteq C$, and **3.** $U^\bullet \cap C = \varnothing$.
Notation: U $\mathbf{con}$ C.

(3) Let $U \subseteq T$ and let $C, D \subseteq P$.
Then U *fires from* C *to* D, written as $C[U\rangle D$, if
1. U $\mathbf{con}$ C and **2.** $D = (C - {}^\bullet U) \cup U^\bullet$.
If $\#U \geq 2$, then U is a *concurrent step from* C *to* D.

Lemma 15. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $U \subseteq T$ with $U \neq \varnothing$. Then U $\mathbf{con}$ C iff

(1) t $\mathbf{con}$ C for all $t \in U$, and

(2) for all $t_1 \neq t_2 \in U$, ${}^\bullet t_1 \cap {}^\bullet t_2 = \varnothing$ and $t_1{}^\bullet \cap t_2{}^\bullet = \varnothing$.

Lemma 16. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$, and let $U \subseteq T$. Let $\{U_1, U_2\}$ be a partition of U.*

If $C[U\rangle D$, then there is $E_1 \subseteq P$ such that $C[U_1\rangle E_1$ and $E_1[U_2\rangle D$.

* $U = U_1 \cup U_2$, $U_1 \cap U_2 = \varnothing$ and $U_1, U_2 \neq \varnothing$

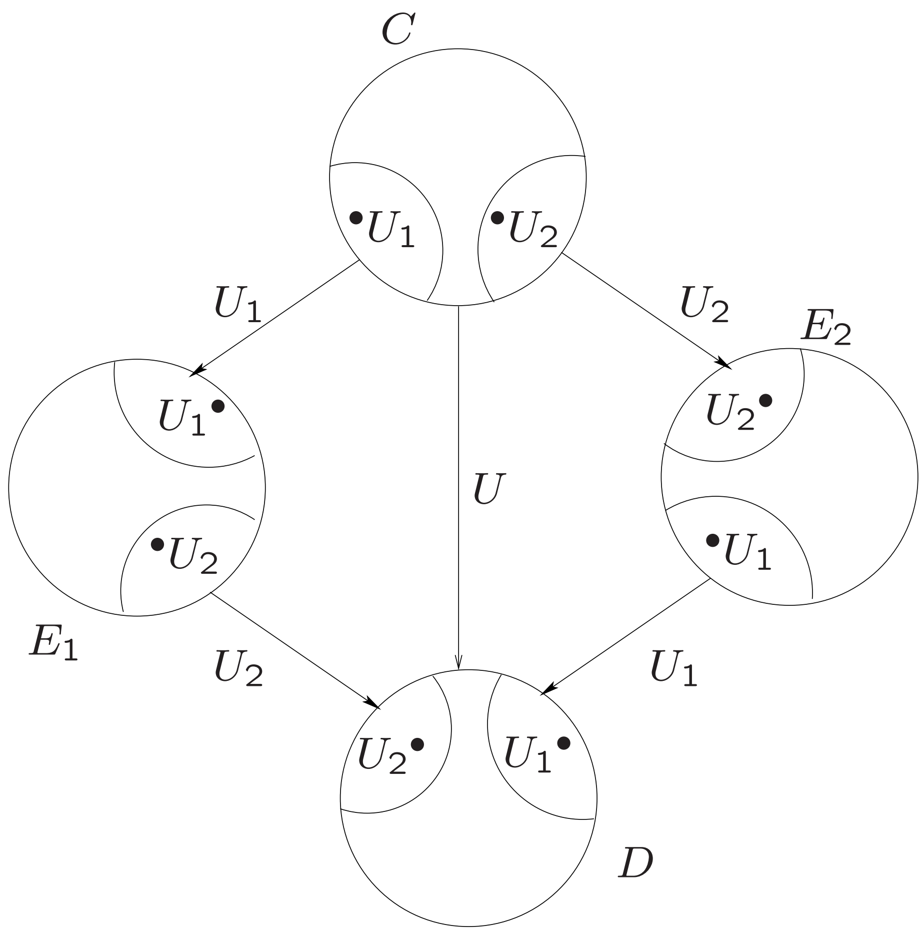

Fig. 17. A diamond.

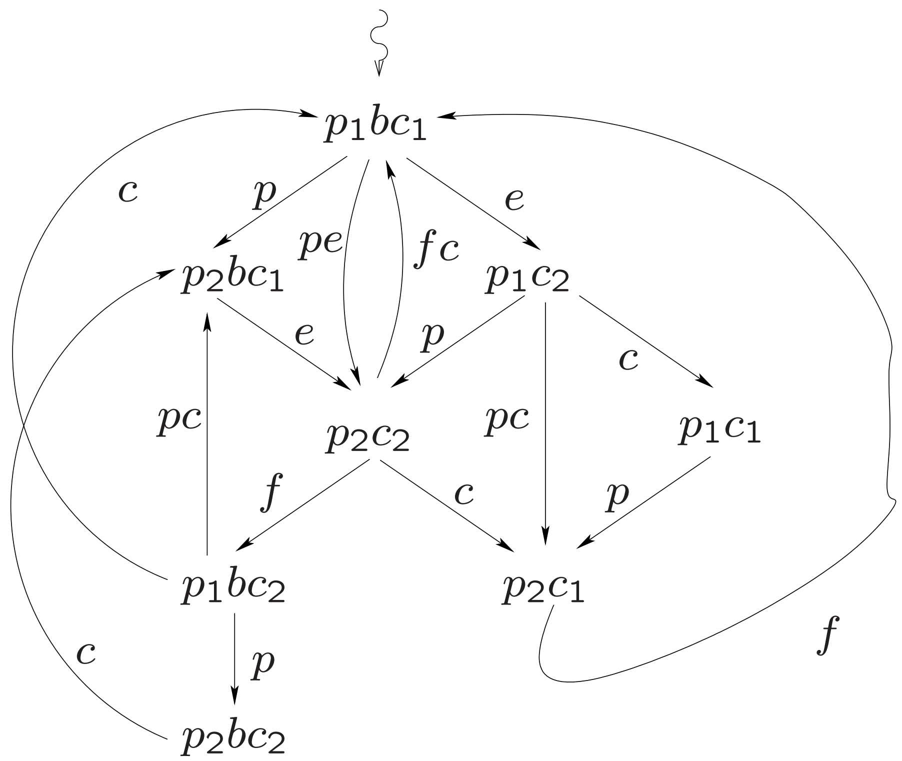

Fig. 18. A configuration graph.

Lemma 17. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$ and let $U \subseteq T$.
If $C[U\rangle D$, then $C[t_1 \cdots t_n\rangle D$ for each ordering $(t_1, \ldots, t_n)$ of the elements of U.

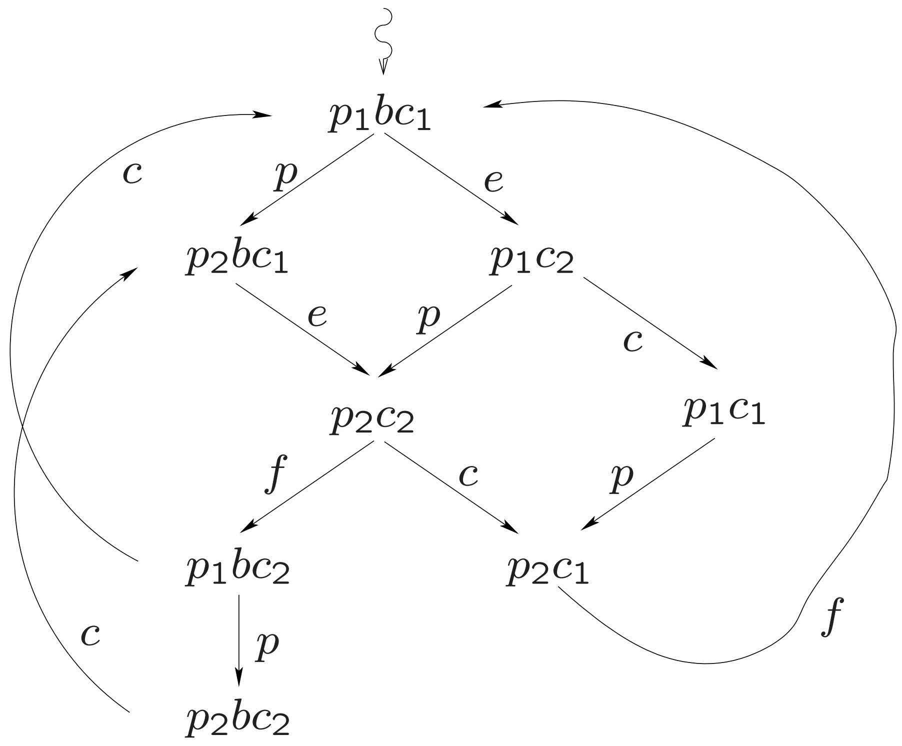

Fig. 16. A sequential configuration graph.

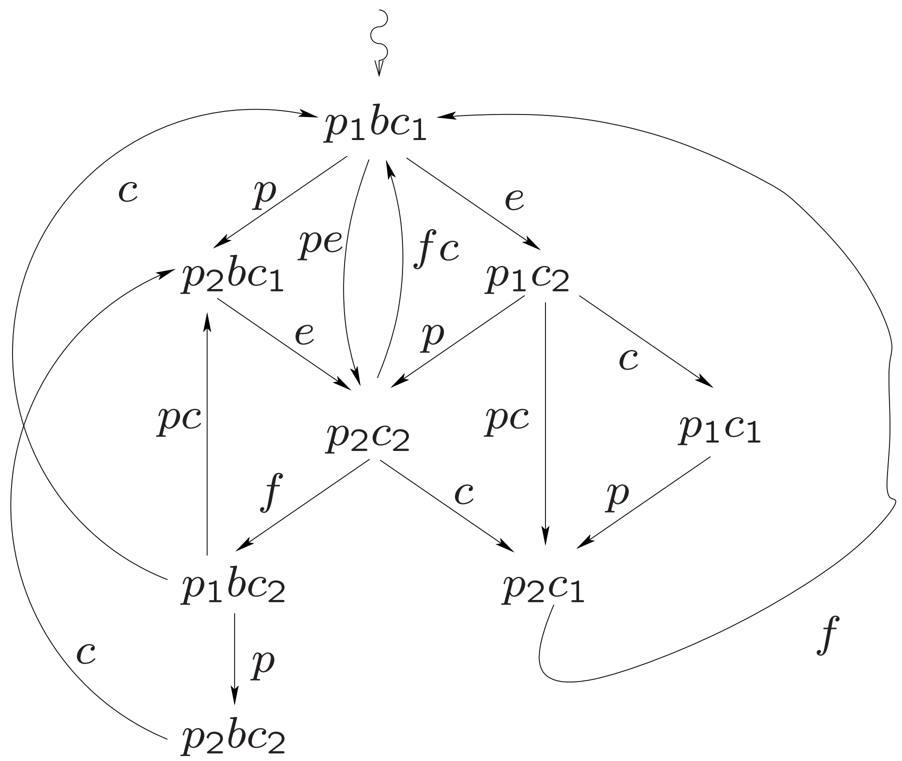

Fig. 18. A configuration graph.

Lemma 19. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $s, t \in T$.
If st $\mathbf{con}$ C and t $\mathbf{con}$ C, then $\{s, t\}$ $\mathbf{con}$ C.

Lemma 15. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $U \subseteq T$ with $U \neq \varnothing$. Then U $\mathbf{con}$ C iff
(1) t $\mathbf{con}$ C for all $t \in U$, and
(2) for all $t_1 \neq t_2 \in U$, ${}^\bullet t_1 \cap {}^\bullet t_2 = \varnothing$ and $t_1{}^\bullet \cap t_2{}^\bullet = \varnothing$.

Lemma 19.5 Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $U \subseteq T$.
If t_i **con** C for every $t_i \in U$ and $t_1 t_2 \ldots t_n$ **con** C for some order of the elements of $U = \{t_1, t_2, \ldots, t_n\}$, then U **con** C.

Theorem 20. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$ and let $U \subseteq T$ with $U \neq \varnothing$. Then

(1) U **con** C iff $t_1 \cdots t_n$ **con** C for every ordering $(t_1, \ldots, t_n)$ of the elements of U, and

(2) $C[U\rangle D$ iff $C[t_1 \cdots t_n\rangle D$ for every ordering $(t_1, \ldots, t_n)$ of the elements of U.

Lemma 17. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C, D \subseteq P$ and let $U \subseteq T$.
If $C[U\rangle D$, then $C[t_1 \cdots t_n\rangle D$ for each ordering $(t_1, \ldots, t_n)$ of the elements of U.

Definition 10. Let $G_1 = (V_1, \Gamma_1, \Sigma_1, v_1)$ and $G_2 = (V_2, \Gamma_2, \Sigma_2, v_2)$ be edge-labelled graphs.
Then G_1 and G_2 are *isomorphic*,
denoted by $G_1 \equiv G_2$, if there exist

two bijections $\alpha : V_1 \to V_2$ and $\beta : \Sigma_1 \to \Sigma_2$

such that $\alpha(v_1) = v_2$ and,

for all $v, w \in V_1$ and all $U \subseteq \Sigma_1$,
$(v, U, w) \in \Gamma_1$ iff $(\alpha(v), \beta(U), \alpha(w)) \in \Gamma_2$.

Theorem 21. For EN systems M and M', $\mathsf{SCG}(M) \equiv \mathsf{SCG}(M')$ iff $\mathsf{CG}(M) \equiv \mathsf{CG}(M')$.

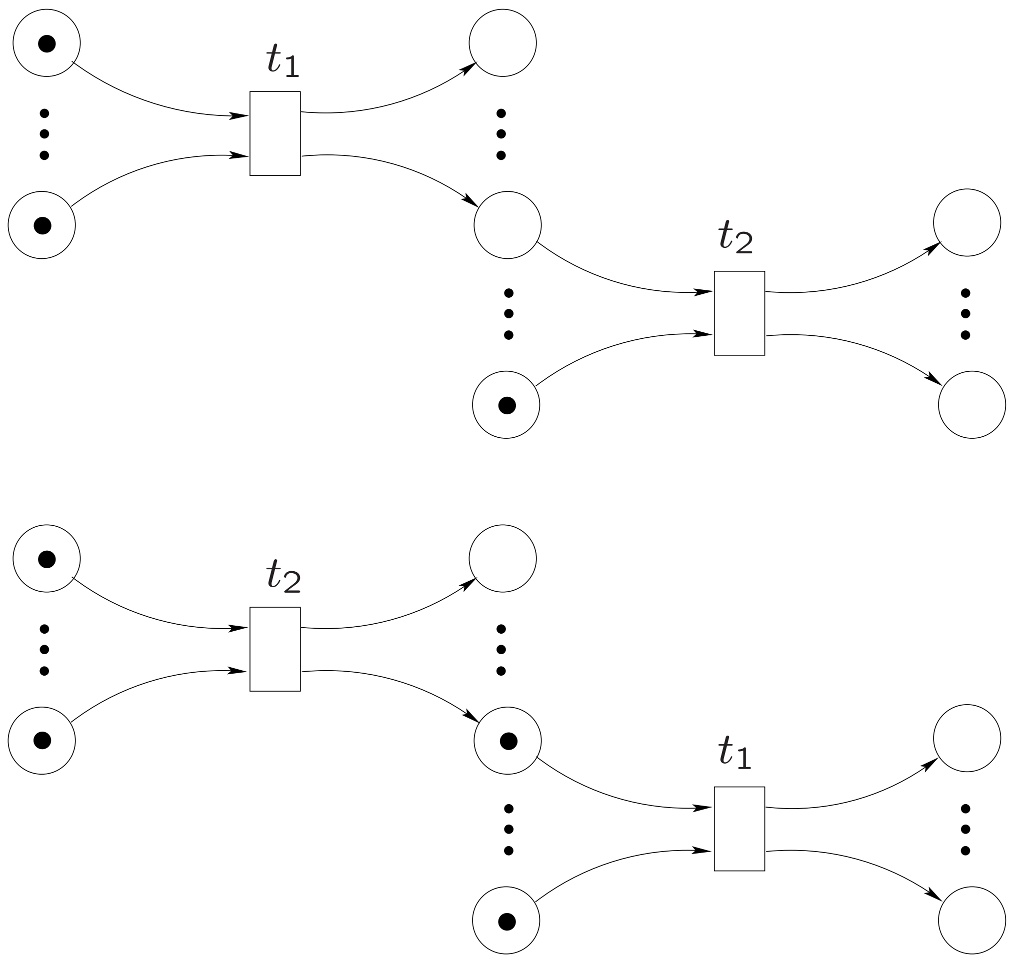

Fig. 19, 20. Causality.

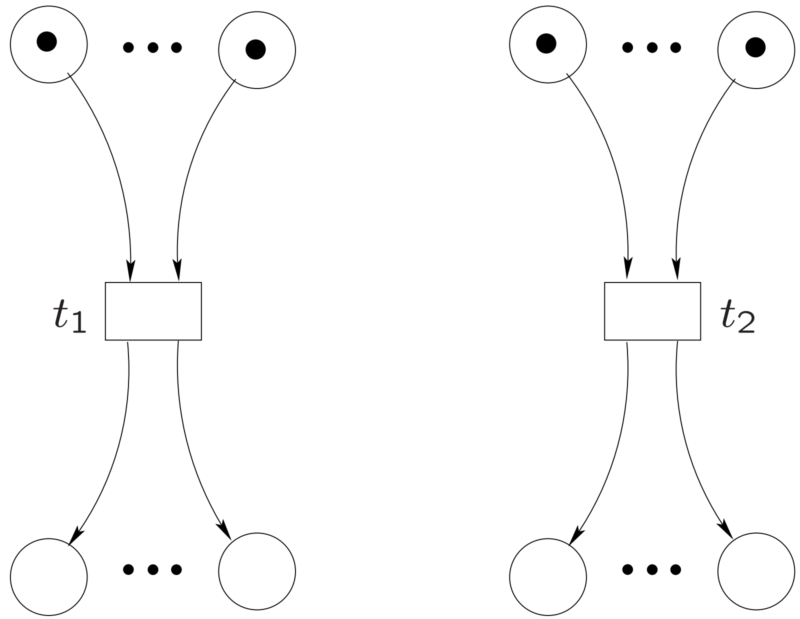

Fig. 21. Concurrency.

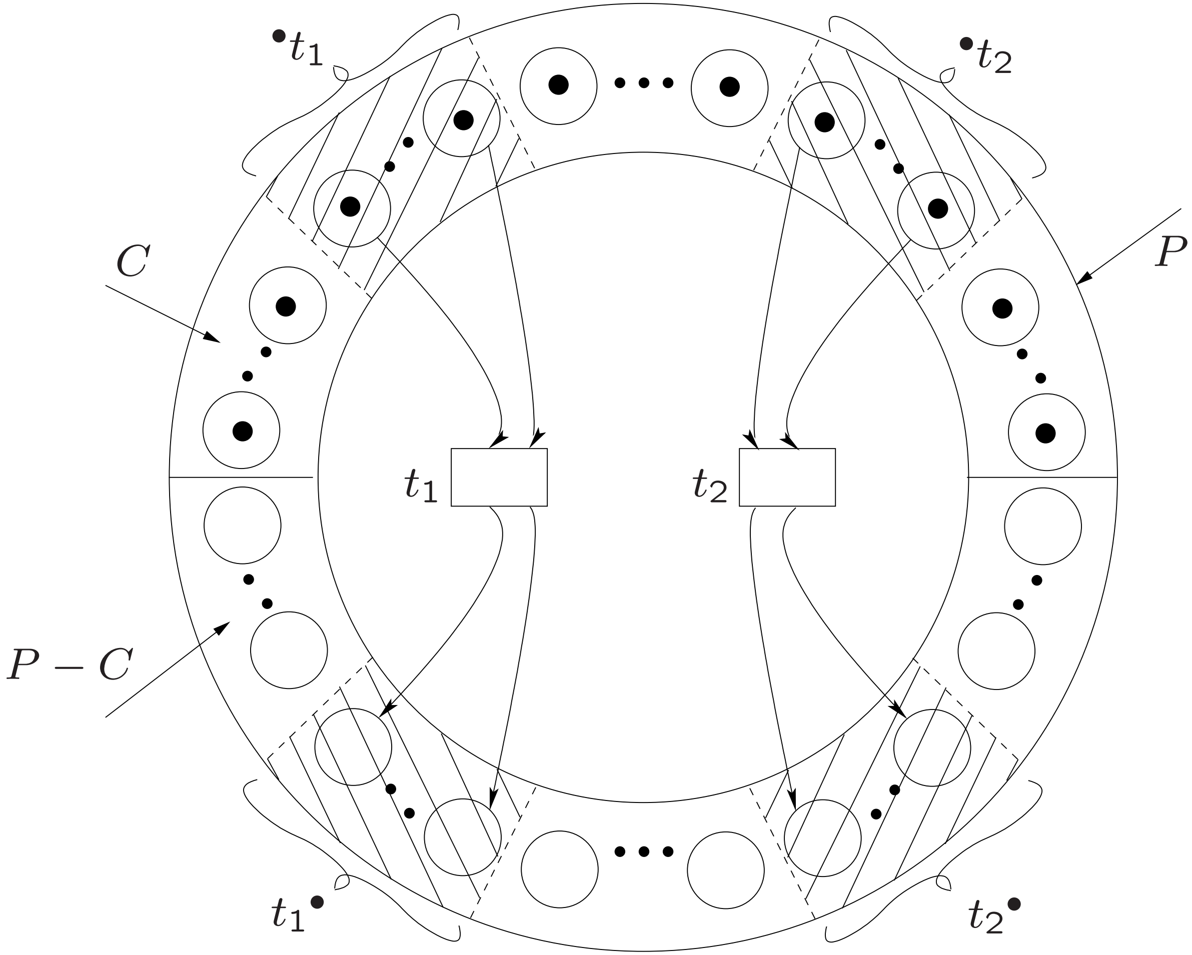

Fig. 22. Concurrency, the complete picture.

Causality: $t_1 t_2$ **con** C, but not t_2 **con** C.

Concurrency: $t_1 t_2$ **con** C, and t_2 **con** C (Lemma 17 and Lemma 19).

Hence, if $t_1 t_2$ **con** C, then either causality or concurrency.

Definition: Transitions t_1 and t_2 are in conflict in configuration C, if t_1 **con** C and t_2 **con** C, but **not** $\{t_1, t_2\}$ **con** C.

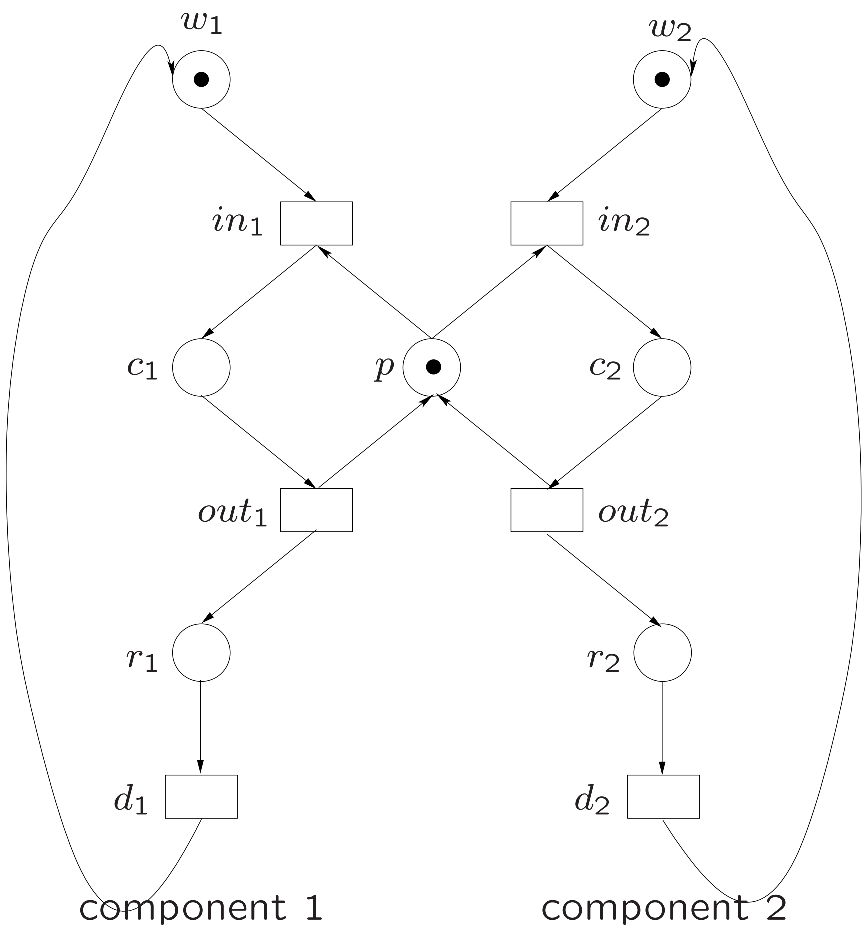

Fig. 5. The mutual exclusion problem.

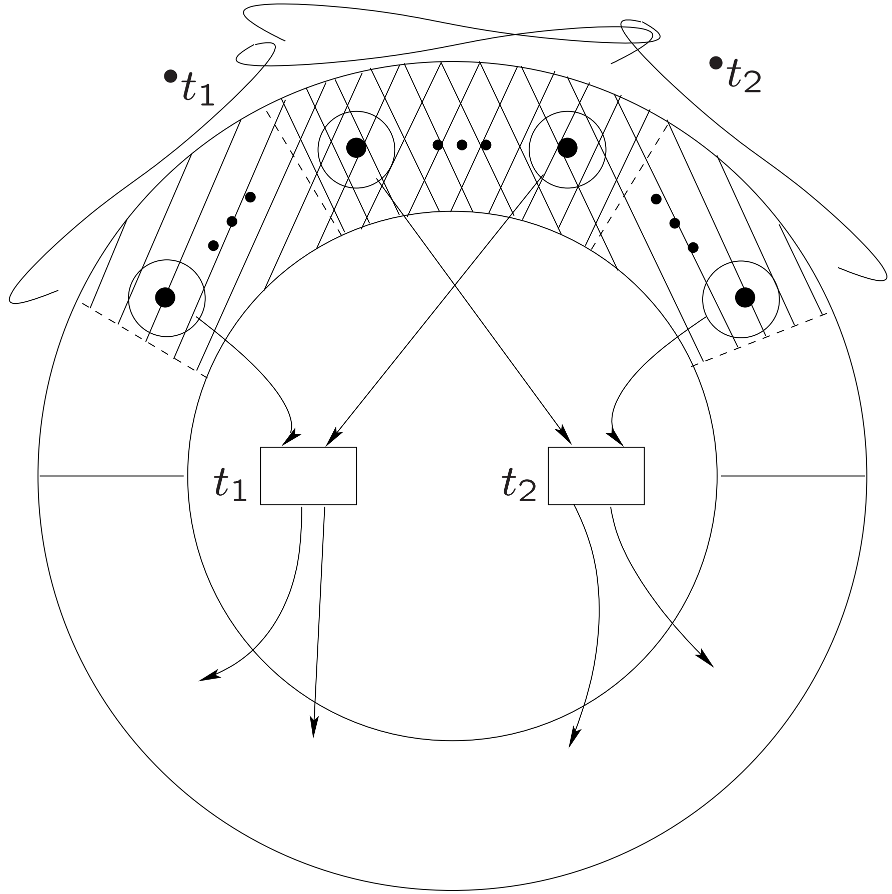

Fig. 23. Input-conflict.

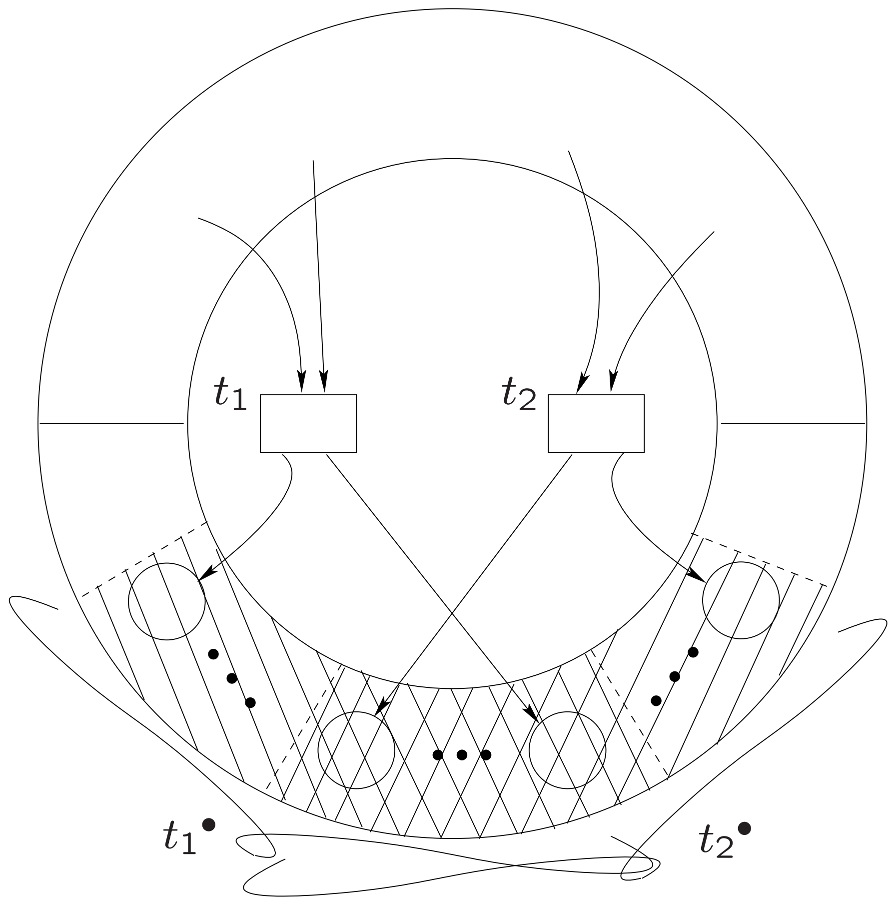

Fig. 24. Output-conflict.

Concurrency: $\{t_1, t_2\}$ **con** C, hence t_1, t_2 **con** C (Lemma 15).

Conflict: t_1, t_2 **con** C, but not $\{t_1, t_2\}$ **con** C.

Hence, if t_1, t_2 **con** C, then either concurrency or conflict.

Definition 23. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$, and let $t \in T$ be such that $t \ \mathbf{con} \ C$.
Then $\mathbf{cfl}(t, C) = \{t' \in T \mid t' \ \mathbf{con} \ C$ and $\neg \ \{t, t'\} \ \mathbf{con} \ C\}$ is the *conflict set of* t *in* C.

Definition 22. An EN system $M = (P, T, F, C_{in})$ is *conflict-free* if, for every $C \in \mathbb{C}_M$ and all transitions $t_1, t_2 \in T$: $\{t_1, t_2\}$ $\mathbf{con}$ C whenever t_1 $\mathbf{con}$ C and t_2 $\mathbf{con}$ C.

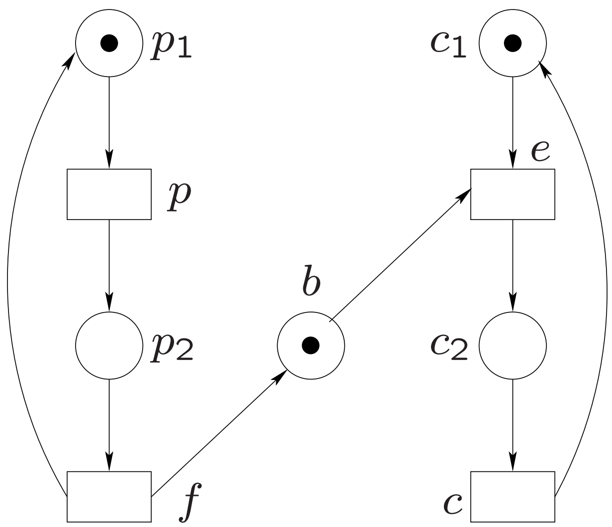

Fig. 12. Conflict-free.

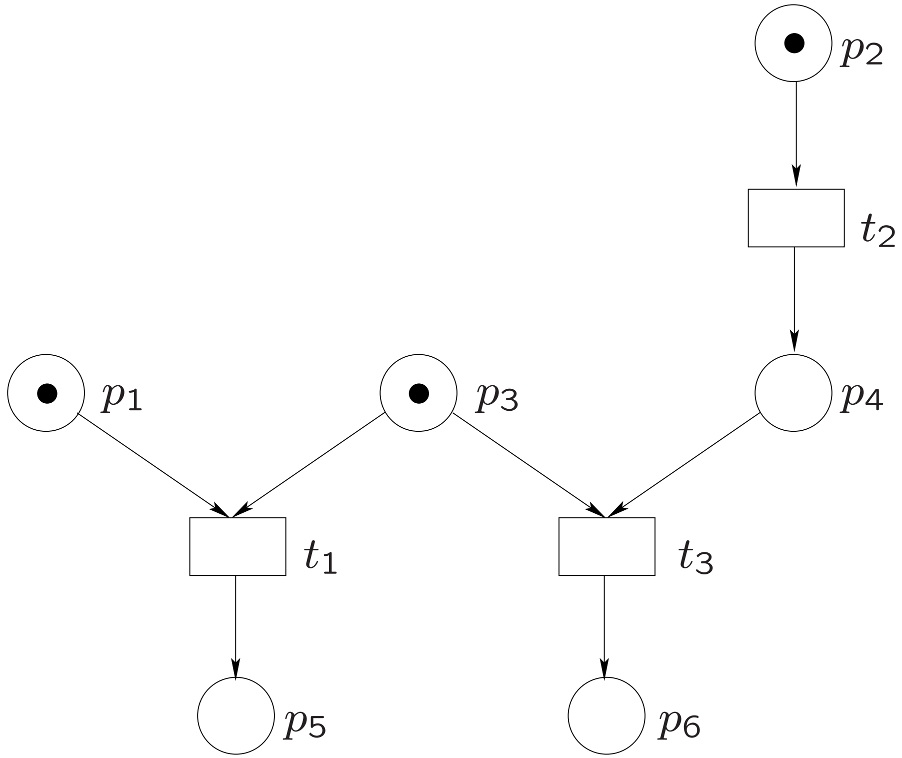

Fig. 25. A conflict-increasing confusion.

Definition 24. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$, and let $t_1, t_2 \in T$.
The triple (C, t_1, t_2) is called a *confusion* (*in* C) if

1. $t_1 \neq t_2$,

2. $\{t_1, t_2\}$ $\mathbf{con}$ C, and

3. $\mathbf{cfl}(t_1, C) \neq \mathbf{cfl}(t_1, D)$, where $C[t_2\rangle D$.

M *is confused in* C if there is a confusion in C.

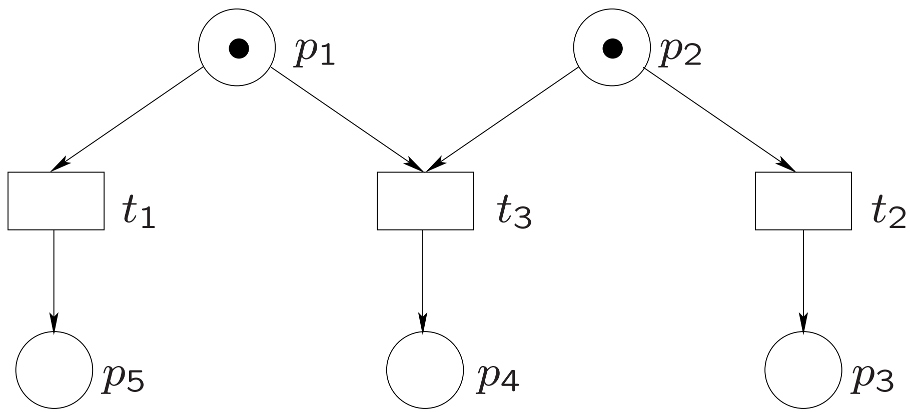

Fig. 26. A conflict-decreasing confusion.

Definition 25 Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$ and $t_1, t_2 \in T$.
Let $\gamma = (C, t_1, t_2)$ be a confusion and $C[t_2\rangle D$.

(1) γ is a *conflict-increasing confusion*, *ci confusion* for short, if $\mathbf{cfl}(t_1, D) \supsetneq \mathbf{cfl}(t_1, C)$.

(2) γ is a *conflict-decreasing confusion*, *cd confusion* for short, if $\mathbf{cfl}(t_1, D) \subsetneq \mathbf{cfl}(t_1, C)$.

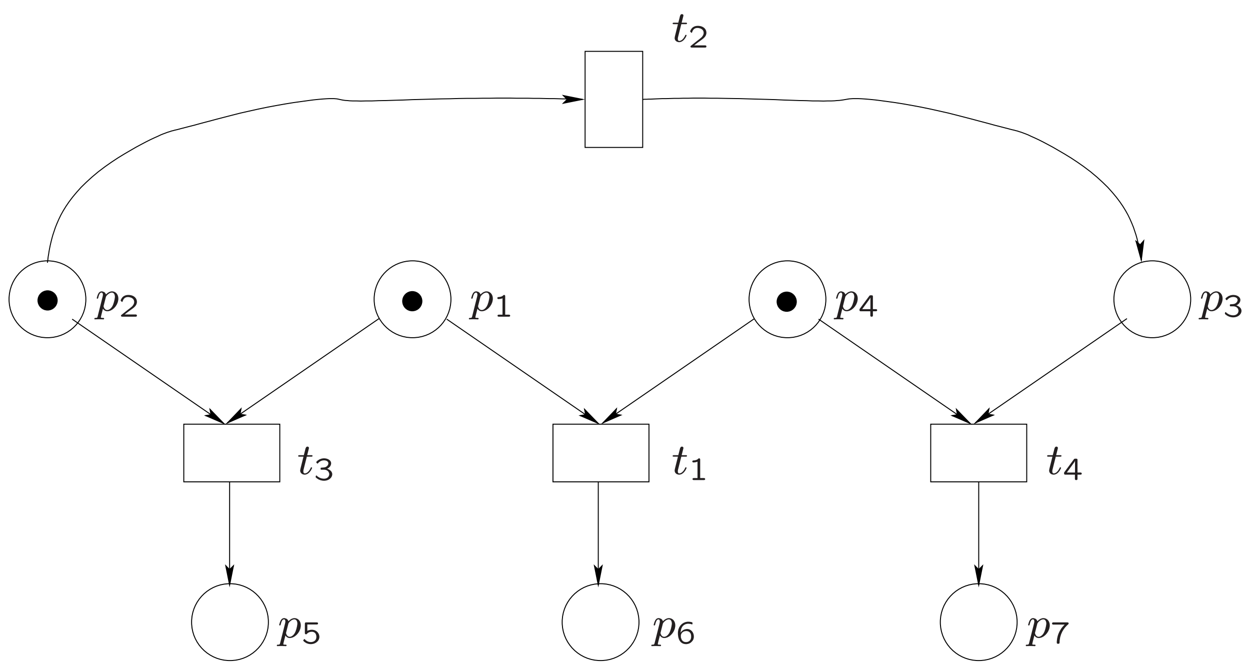

Fig. 27. A confusion which is neither ci nor cd.

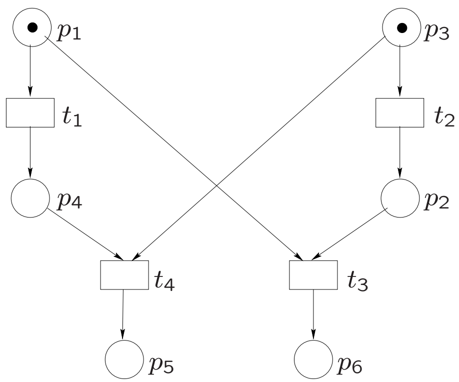

Fig. 28. A symmetric confusion.

Definition 26. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \in \mathbb{C}_M$ and $t_1, t_2 \in T$. Let $\gamma = (C, t_1, t_2)$ be a confusion.

γ is *symmetric* if (C, t_2, t_1) is also a confusion, otherwise γ is *asymmetric*.

Consider the EN system Mutex (Figure 5).
Give CG(Mutex) and
determine all confusions (C, t_1, t_2) with $C \in \mathbb{C}_{\mathsf{Mutex}}$.

Give - if possible - examples of confusions which are conflict-increasing, conflict-decreasing, neither and in addition (a)symmetric.

Prove: every confusion which is not ci is symmetric.