Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

tweede college: 8 september 2011

4.3 Sequential Configuration Graph

4.4 Concurrency

eerste werkgroep: 15 september 2011

opgaven bij 4. EN Systems

installatie pipe2

Definition 8 Ctd. Let $M = (P, T, F, C_{in})$ be an EN system.

(4) $C \subseteq P$ is a *reachable configuration of* M
if there exists an $x \in \mathsf{FS}(M)$ with $C_{in}[x\rangle C$.
The set of all reachable configurations of M is denoted by $\mathbb{C}_M$.

(5) $t \in T$ is a *useful transition of* M
if there exists a reachable configuration C of M such that $t \ \mathbf{con}\ C$.
The set of useful transitions of M is denoted by $\mathbf{use}_M(T)$, or just $\mathbf{use}(T)$ when M is clear from the context.

(6) $t \in T$ is a *live transition of* M
if for each $C \in \mathbb{C}_M$ there exists an $x \in T^*$ with $xt \ \mathbf{con}\ C$.

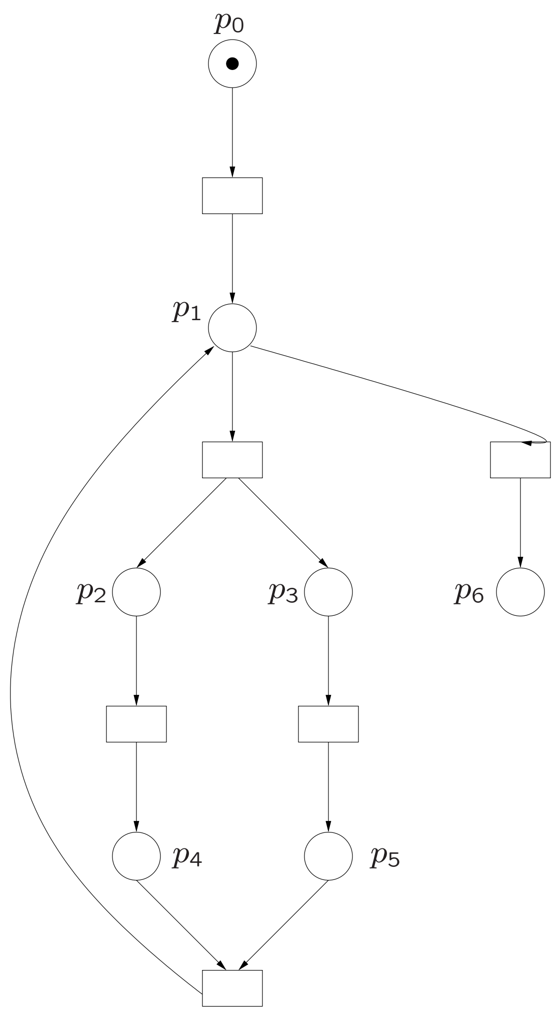


Fig. 15. All transitions are useful, but none is live.

Excercise 4.7 (Hint: use Lemma 7 (and Definition 8))

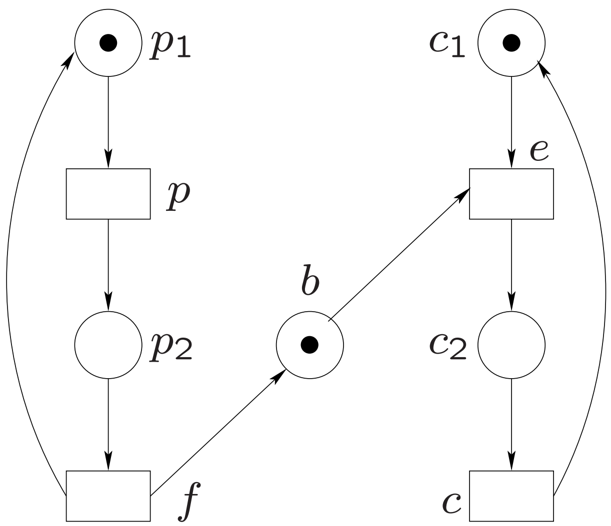
Lemma 7. Let $M = (P, T, F, C_{in})$ be an EN system.
Let $t \in T$ and let $C, D \subseteq P$.

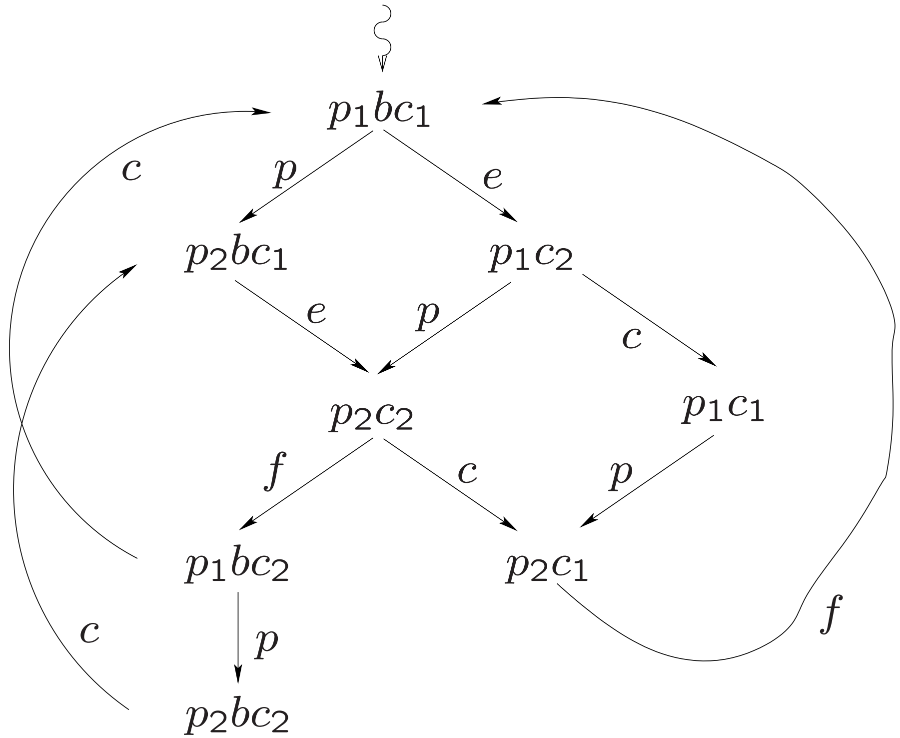
Then $C[t\rangle D$ iff $C - D = {}^\bullet t$ and $D - C = t^\bullet$.

Definition 8. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $t_1 \cdots t_n \in T^*$, with $n \geq 0$ and $t_1, \ldots, t_n \in T$. Let $C, D \subseteq P$. Then $t_1 \cdots t_n$ *fires from* C *to* D if there exist configurations $C_0, C_1, \ldots, C_n \subseteq P$ with $C_0 = C$, $C_n = D$ and $C_{i-1}[t_i\rangle C_i$ for all $1 \leq i \leq n$, written as $C[t_1 \cdots t_n\rangle D$.

(2) Let $x \in T^*$ and $C \subseteq P$.
Then x *has concession in* C
(or x *can be fired in* C, or x *is enabled in* C)
if there exists a $D \subseteq P$ such that $C[x\rangle D$, written as x **con** C.





An EN system **Fig. 12.** and its sequential configuration graph **Fig. 16.**

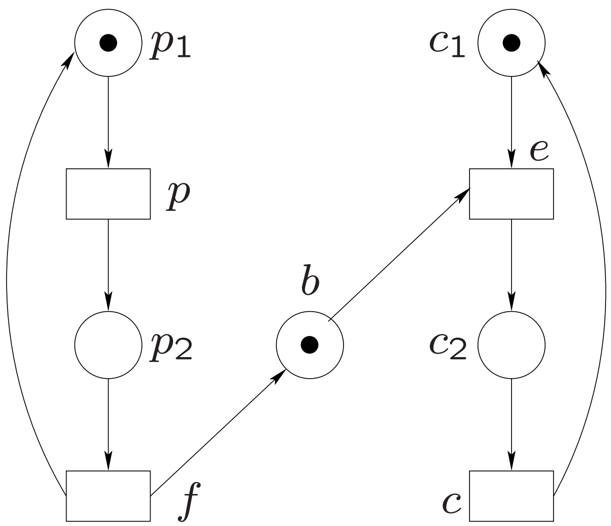
Definition 9. An (initialized) *edge-labelled graph* is a quadruple $(V, \Gamma, \Sigma, v_{in})$, where

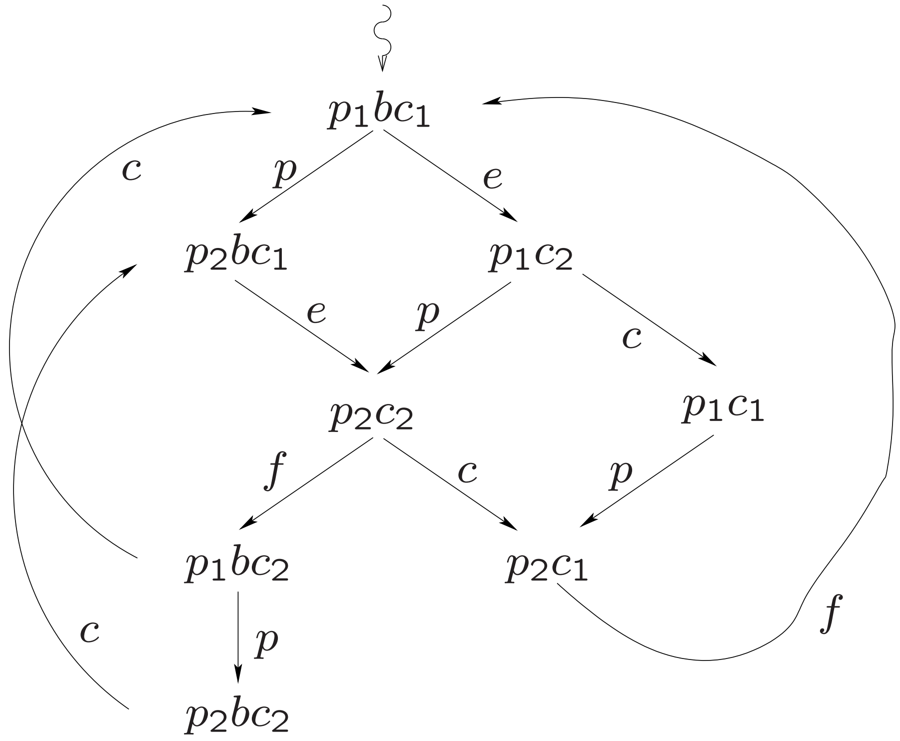
V is a finite set of *nodes*,
$v_{in} \in V$ is the *initial node*,
Σ is a finite set of *(edge-) labels*, and
$\Gamma \subseteq V \times \mathcal{P}(\Sigma) \times V$ is a set of *(labelled) edges*.

Definition 11. Let M be an EN system. The *sequential configuration graph of* M,
denoted by SCG(M),
is the edge-labelled graph $(V, \Gamma, \Sigma, v_{in})$, where

$V = \mathbb{C}_M$, $v_{in} = (C_{in})_M$, $\Sigma = \mathbf{use}(T_M)$, and

$\Gamma = \{(C, t, D) \mid C, D \in \mathbb{C}_M,\ t \in T_M,\ C[t\rangle_M D\}$.





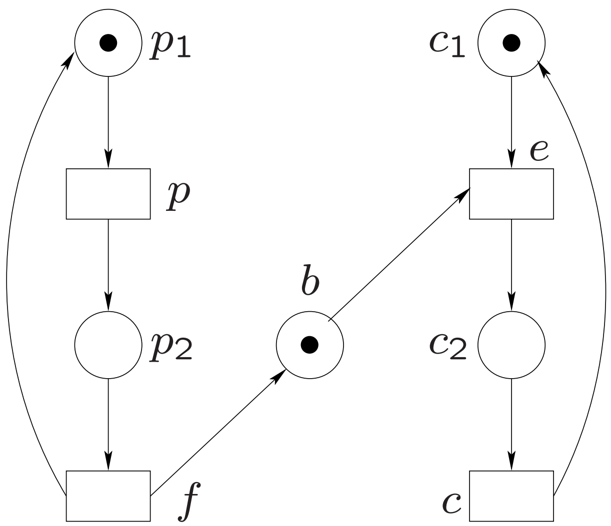
An EN system **Fig. 12.** and its sequential configuration graph **Fig. 16.**

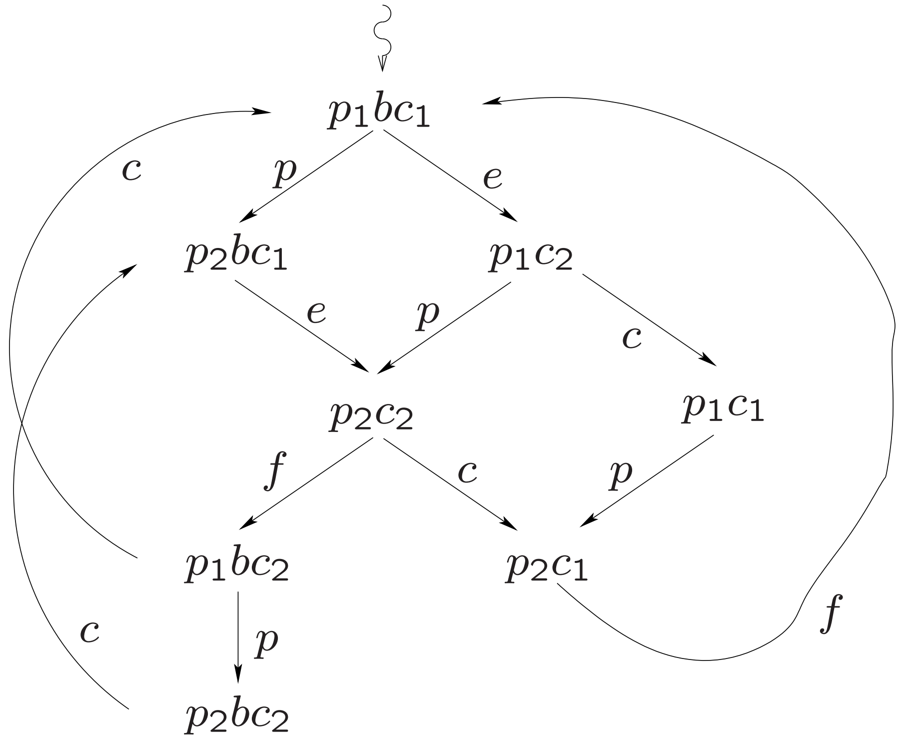
Definition 10. Let $G_1 = (V_1, \Gamma_1, \Sigma_1, v_1)$ and $G_2 = (V_2, \Gamma_2, \Sigma_2, v_2)$ be edge-labelled graphs.
Then G_1 and G_2 are *isomorphic*,
denoted by $G_1 \equiv G_2$, if there exist

two bijections $\alpha : V_1 \to V_2$ and $\beta : \Sigma_1 \to \Sigma_2$

such that $\alpha(v_1) = v_2$ and,

for all $v, w \in V_1$ and all $U \subseteq \Sigma_1$,
$(v, U, w) \in \Gamma_1$ iff $(\alpha(v), \beta(U), \alpha(w)) \in \Gamma_2$.





An EN system **Fig. 12.** and its sequential configuration graph **Fig. 16.**

Theorem 12. For every EN system M, FS(M) is a regular language.

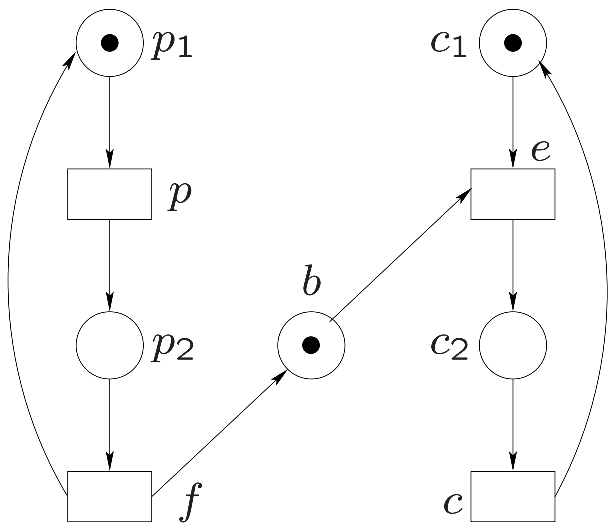


Fig. 12.

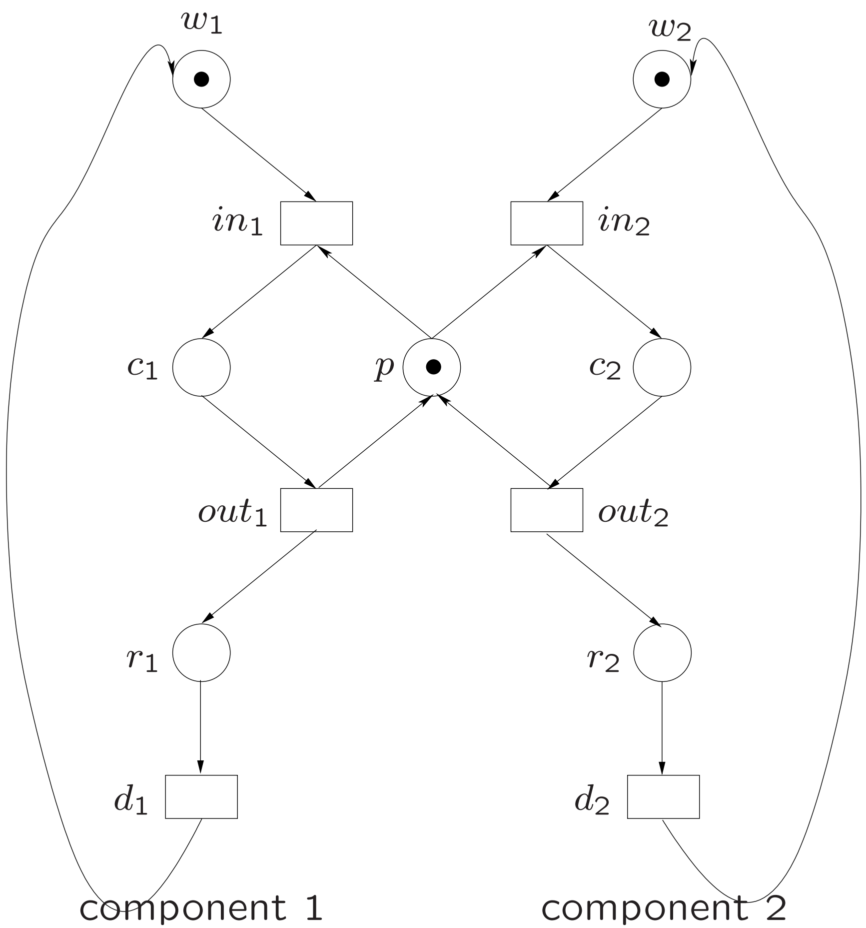


Fig. 5. The mutual exclusion problem.

Definition 13. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $U \subseteq T$. U is a *disjoint set of transitions*, notation $\mathbf{disj}(U)$,
if **1.** $U \neq \varnothing$
and **2.** for all transitions $t_1 \neq t_2 \in U$: $\mathbf{nbh}(t_1) \cap \mathbf{nbh}(t_2) = \varnothing$.

(2) Let $U \subseteq T$ and let $C \subseteq P$. Then U *has concession in* C
(or U *can be fired in* C, or U *is enabled in* C) if
1. $\mathbf{disj}(U)$, **2.** ${}^{\bullet}U \subseteq C$, and **3.** $U^{\bullet} \cap C = \varnothing$.
Notation: $U\ \mathbf{con}\ C$.

(3) Let $U \subseteq T$ and let $C, D \subseteq P$.
Then U *fires from* C *to* D, written as $C[U\rangle D$, if
1. $U\ \mathbf{con}\ C$ and **2.** $D = (C - {}^{\bullet}U) \cup U^{\bullet}$.
If $\#U \geq 2$, then U is a *concurrent step from* C *to* D.

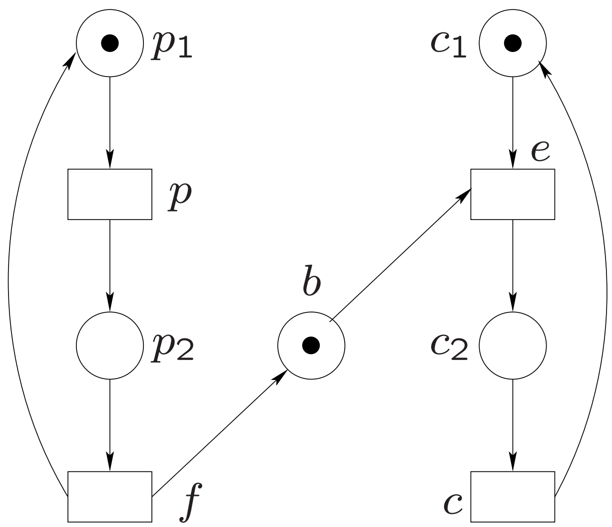


Fig. 12.

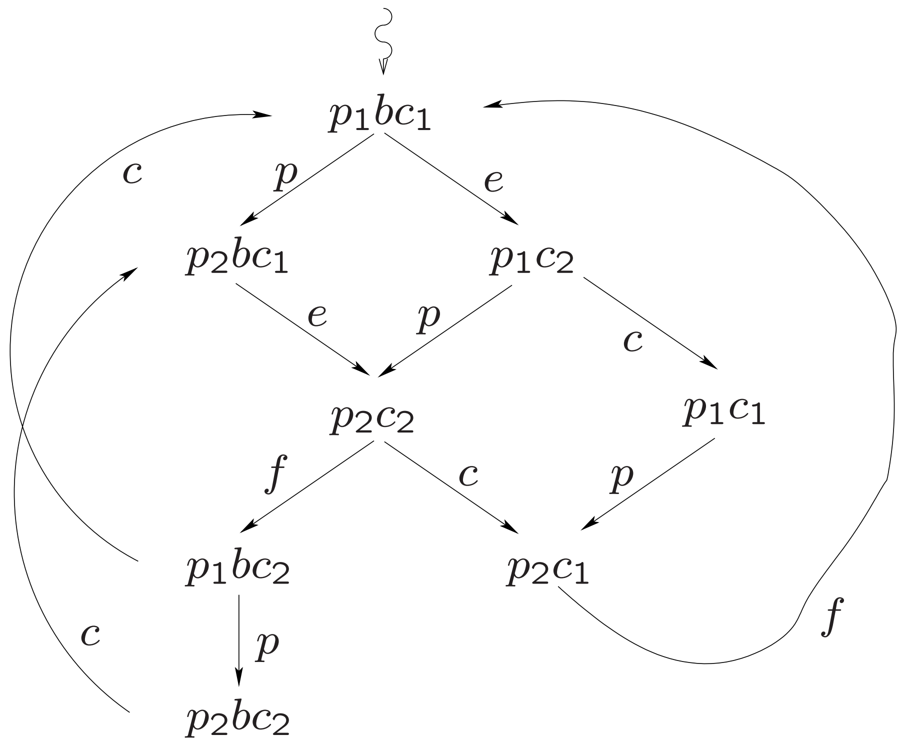


Fig. 16. A sequential configuration graph.

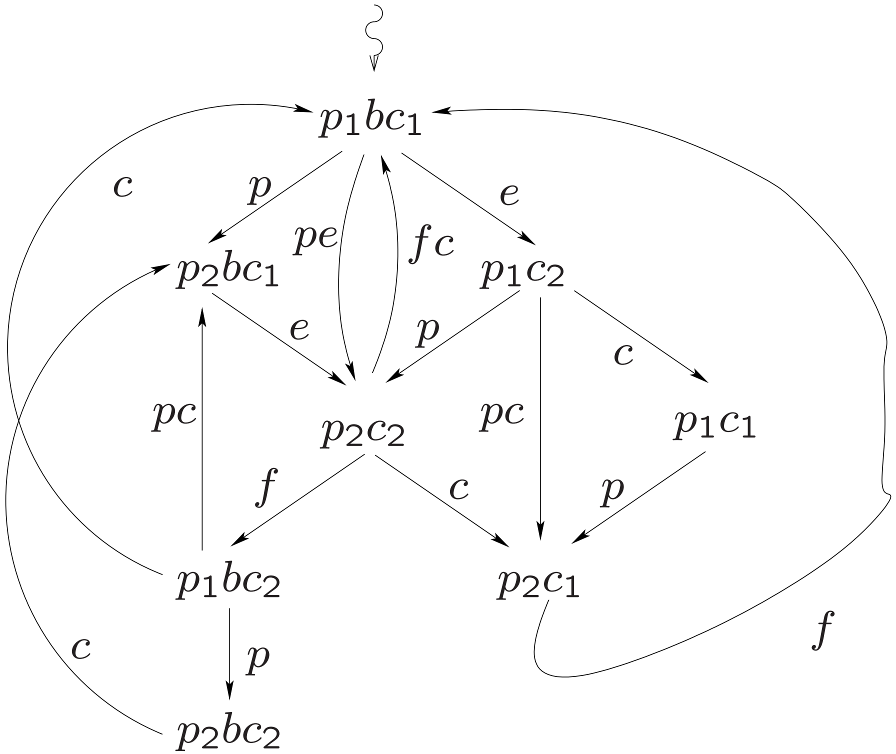


Fig. 18. A configuration graph.

Definition 18. Let M be an EN system. The *configuration graph of M*, denoted by $\mathsf{CG}(M)$, is the edge-labelled graph $(V, \Gamma, \Sigma, v_{in})$, where

$V = \mathbb{C}_M$,

$v_{in} = (C_{in})_M$,

$\Sigma = \mathbf{use}(T_M)$, and

$\Gamma = \{(C, U, D) \mid C, D \in \mathbb{C}_M,\ U \subseteq T_M,\ C[U\rangle_M D\}$.

Lemma 14. Let $M = (P, T, F, C_{in})$ be an EN system. Let $U \subseteq T$ and let $C, D \subseteq P$.
Then $C[U\rangle D$ holds iff $\mathbf{disj}(U)$, $C - D = {}^{\bullet}U$, and $D - C = U^{\bullet}$.