Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

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Theorie van Concurrency — najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

• hoorcollege/werkgroep $\sim$ 2/1
dinsdag 6 september - 8 november, zaal 403, 11.15–13.00
donderdag 8 september - 10 november, zaal 403, 11.15–13.00
donderdag 17 november - 8 december, zaal 403, 10.00–13.00
• **eerste wg 15 september:** laptop, Java platform, ...
inlichtingen: Wouter

• dictaat en opgavenbundel
tutorial article (recommended)

• tentamens: vrijdag 13 januari 2012; vrijdag 17 februari 2012
• modelleertoets, . . . november 2011

Petri Net C.A. Petri 1962
model voor *concurrente/parallelle* systemen
d.w.z. niet sequentiele systemen,
vaak bestaand uit parallelle componenten:
samenwerkend, concurrerend, communicerend
complex!!!

- veel soorten Petri netten:
wij: 1) EN systemen, 2) P/T systemen
verder: stochastisch, timed

- bachelorprojecten . . . , vraag Jetty Kleijn

- dit college: niet modelleren, maar **theorie** van het model

- andere modellen voor concurrency: proces algebra, . . .

toepassingen:
operating systems
distributed algorithms
manufacturing systems (industrial production)
computer supported cooperative work (cscw)
protocol specification (e.g. telecom)
semantics of parallel programming languages
hardware design (asynchronous circuits, multi core chips)
biomodeling ...

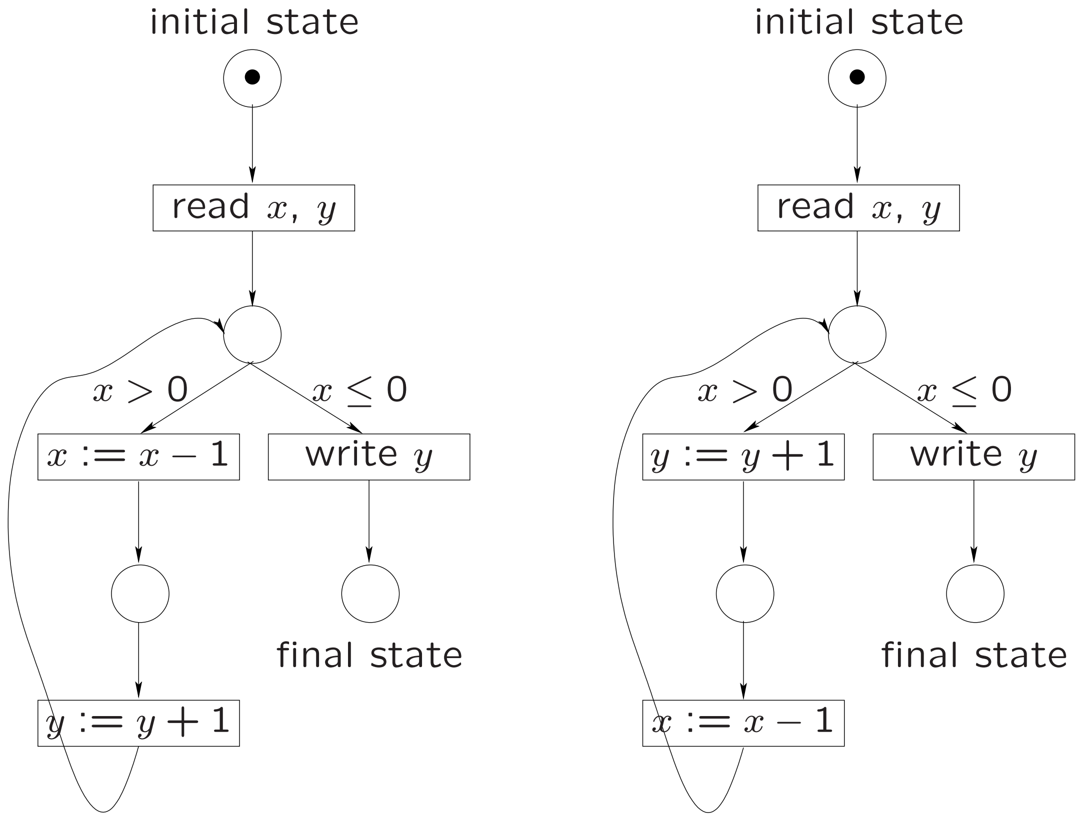


Fig. 1. Sequential addition programs.

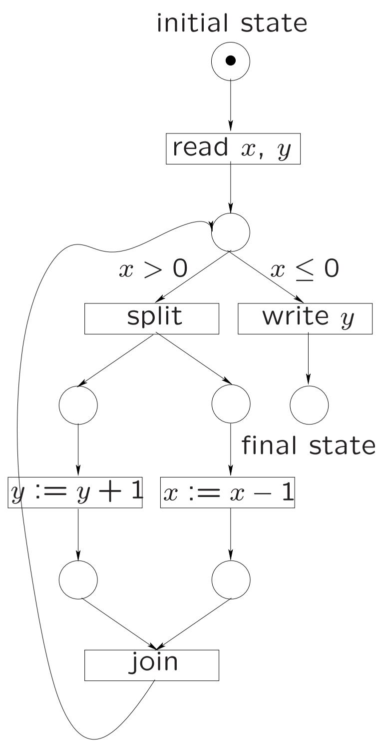


Fig. 2. A concurrent addition program.

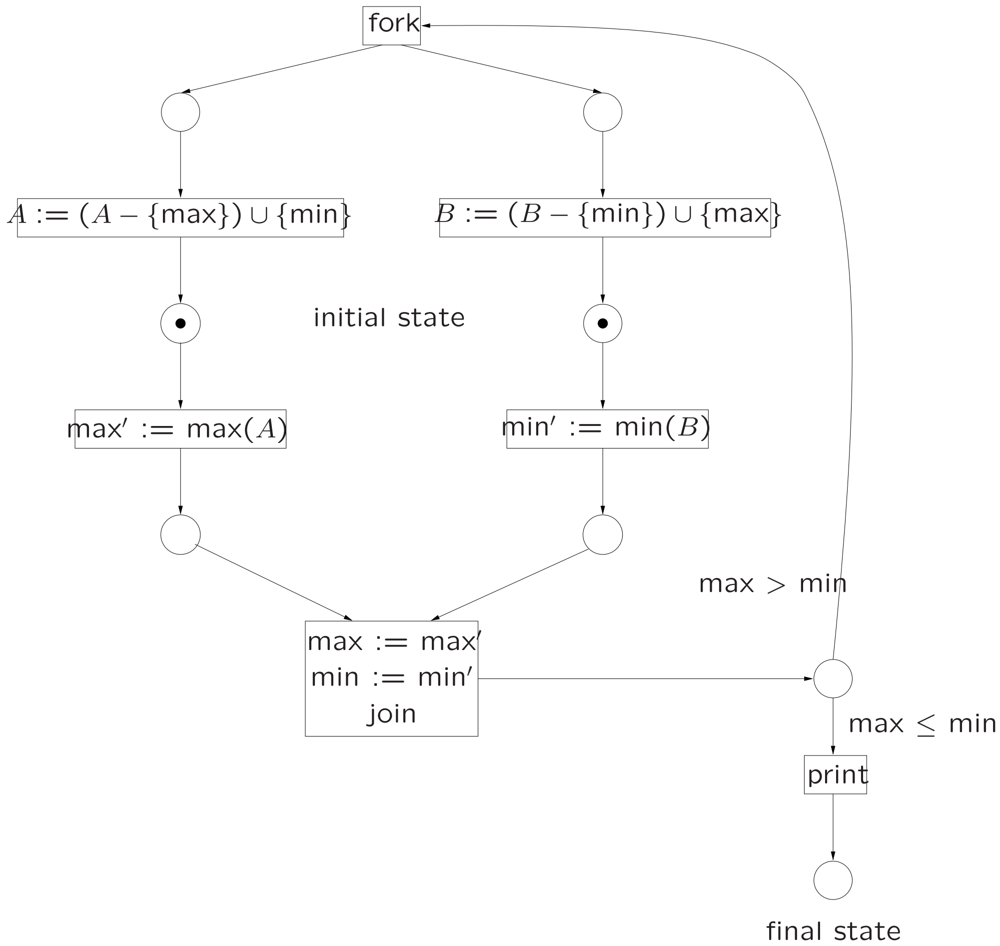


Fig. 3. Concurrent operations on sets.

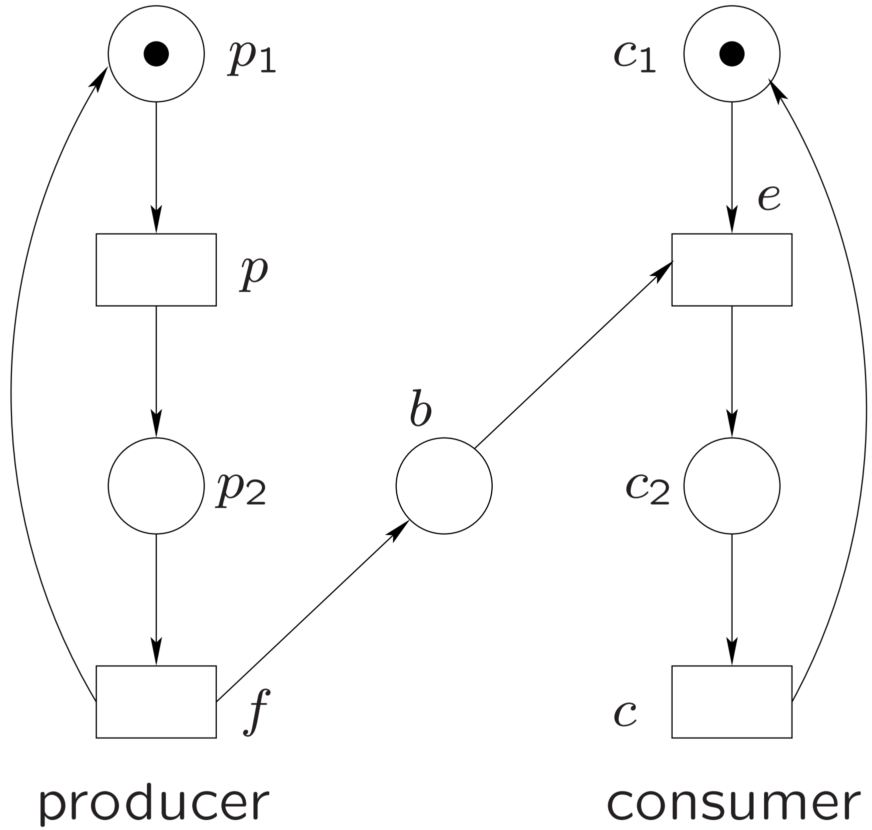


Fig. 4. The producer/consumer problem.

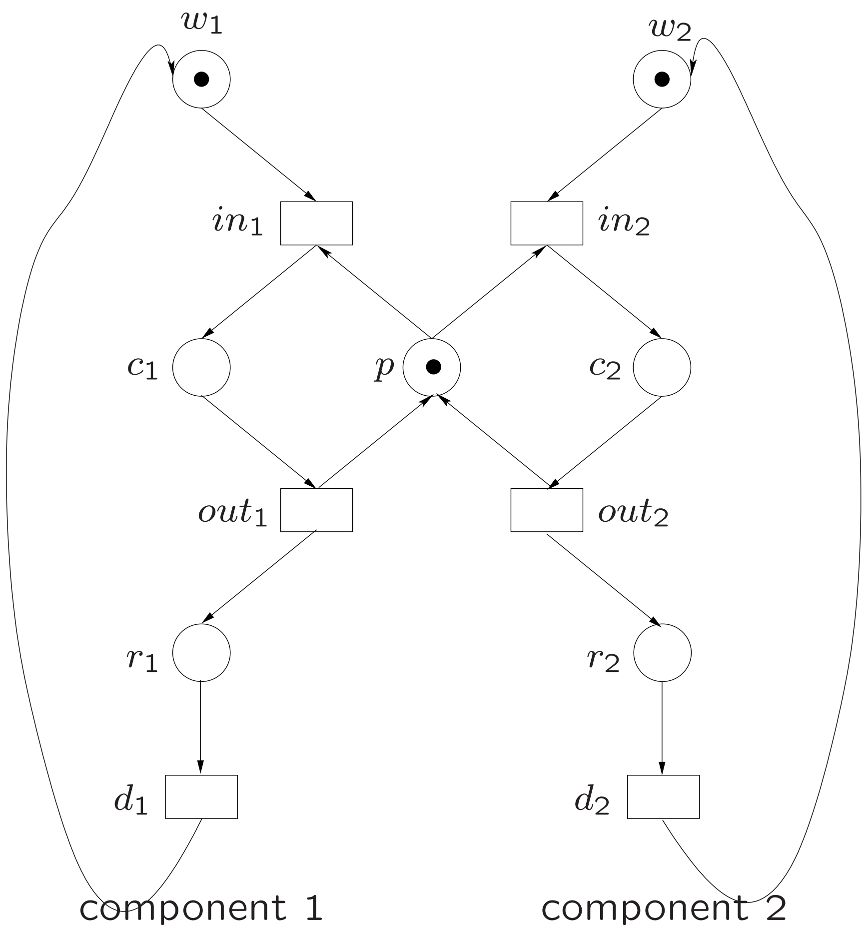
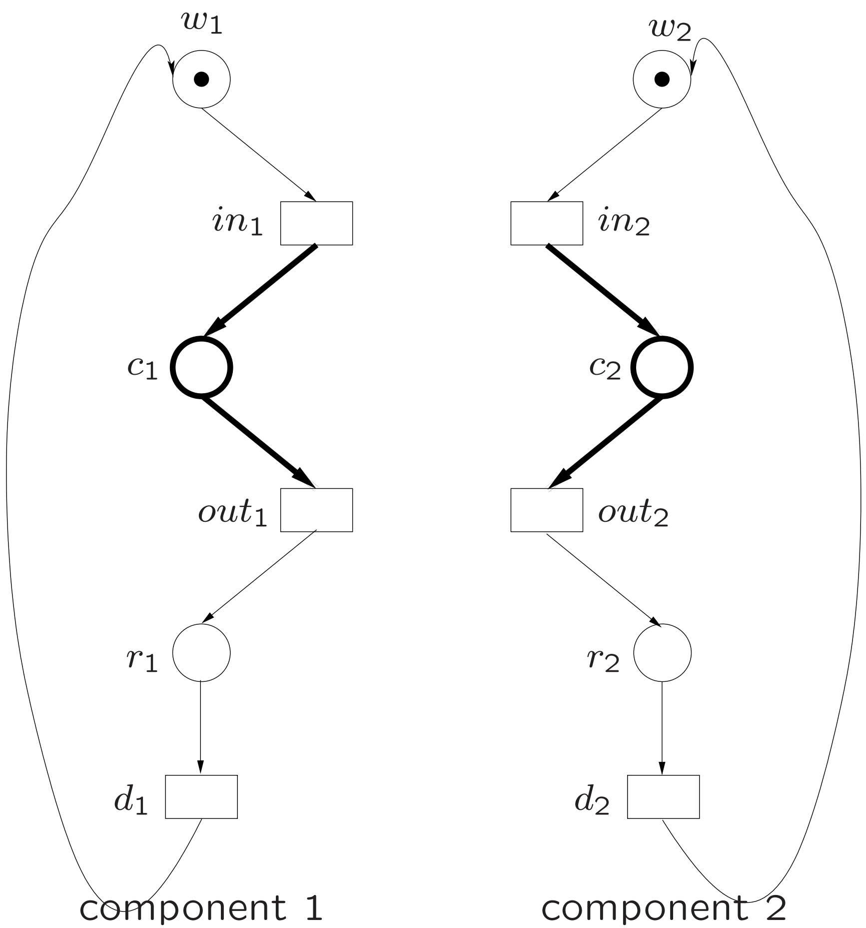
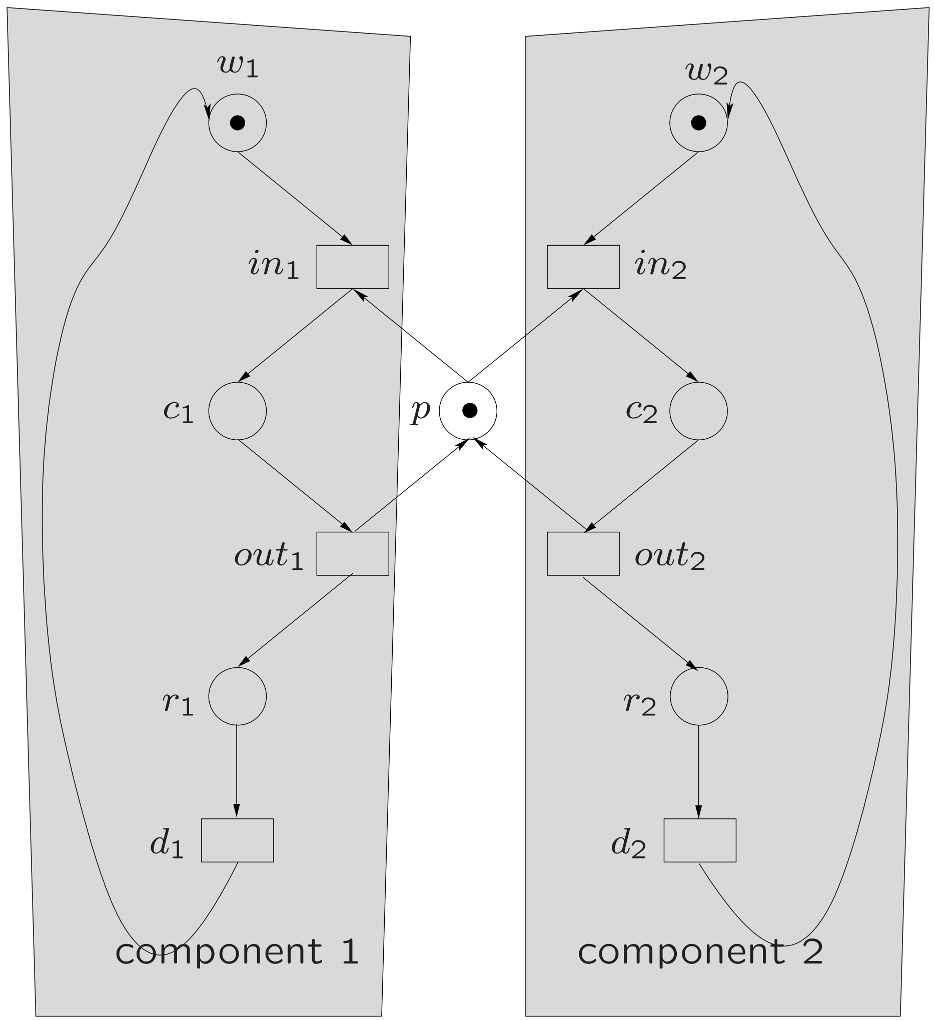


Fig. 5. The mutual exclusion problem.



mutual exclusion



competition

Definition 1. A *net* is a triple $N = (P, T, F)$, where:

(1) P and T are finite sets with $P \cap T = \varnothing$,

(2) $F \subseteq (P \times T) \cup (T \times P)$,

(3) for every $t \in T$ there exist $p, q \in P$ such that $(p, t), (t, q) \in F$, and

(4) for every $t \in T$ and $p, q \in P$,
if $(p, t), (t, q) \in F$, then $p \neq q$.

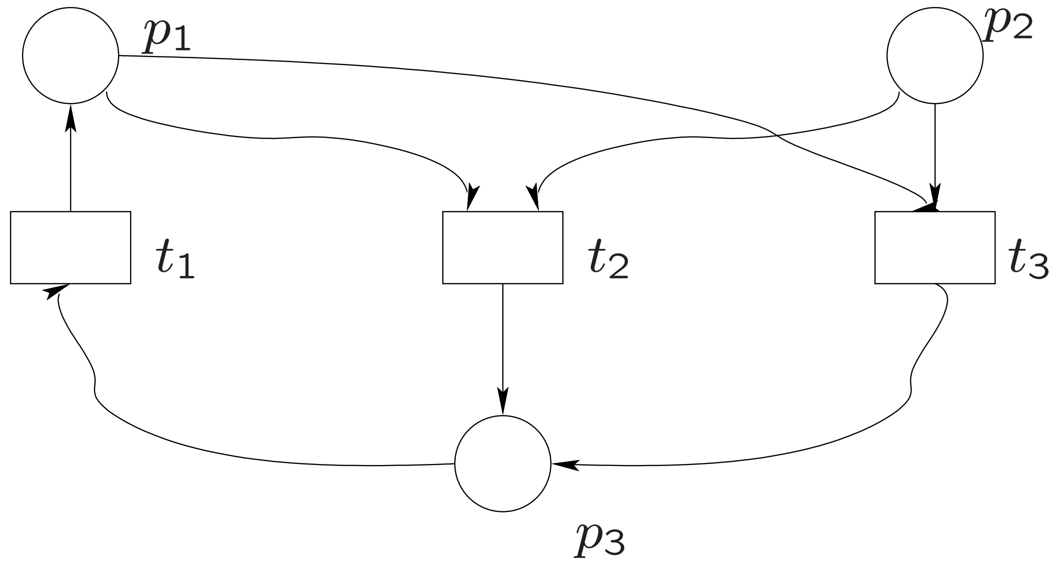


Fig. 6. A net.

$X = P \cup T$
(elementen / 'knopen')

Voor $x \in X$,

${}^{\bullet}x = \{y \in X | (y, x) \in F\}$
(input-set / pre-set)

$x^{\bullet} = \{y \in X | (x, y) \in F\}$
(output-set / post-set)

nbh(x) $= {}^{\bullet}x \cup x^{\bullet}$
(neighbourhood / omgeving)

uit te breiden tot verzamelingen

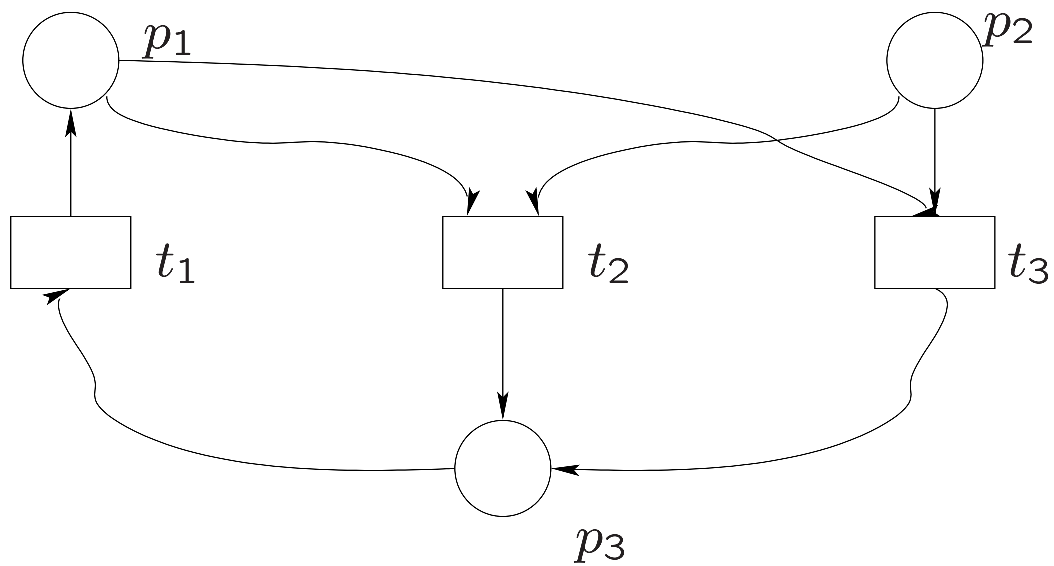


Fig. 6. A net.

Definition 2. A net $N = (P, T, F)$

(1) is *acyclic* if, for every $x \in X$, $(x, x) \notin F^+$,

(2) is *P-simple* if, for all $p, q \in P$,
$({}^\bullet p = {}^\bullet q$ and $p^\bullet = q^\bullet)$ implies $p = q$,

(3) is *T-simple* if, for all $s, t \in T$,
$({}^\bullet s = {}^\bullet t$ and $s^\bullet = t^\bullet)$ implies $s = t$,

(4) has *no isolated places* if, for all $p \in P$, $\mathbf{nbh}(p) \neq \varnothing$.

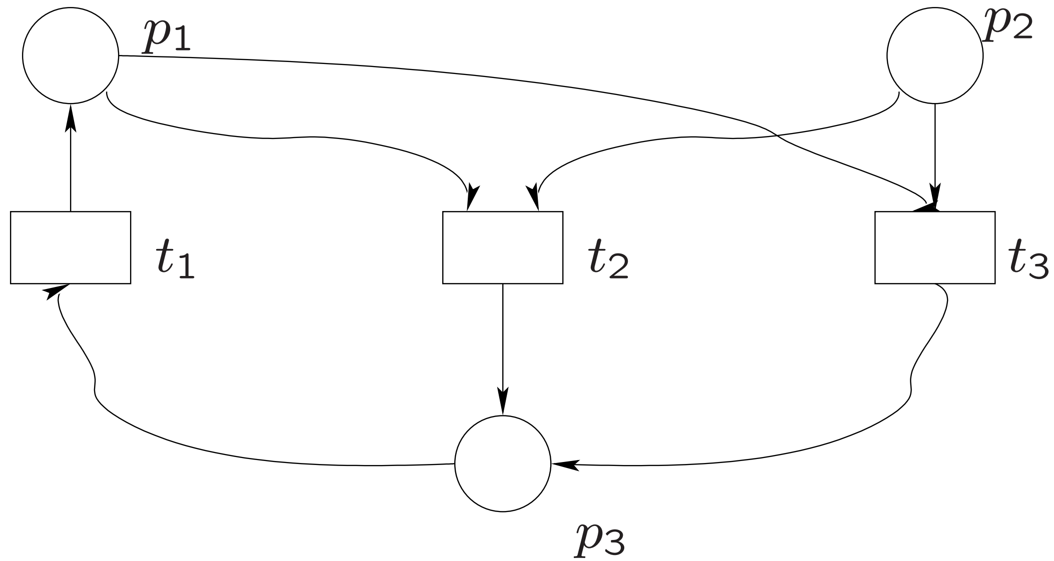


Fig. 6. Wel P-simpel, niet T-simpel.

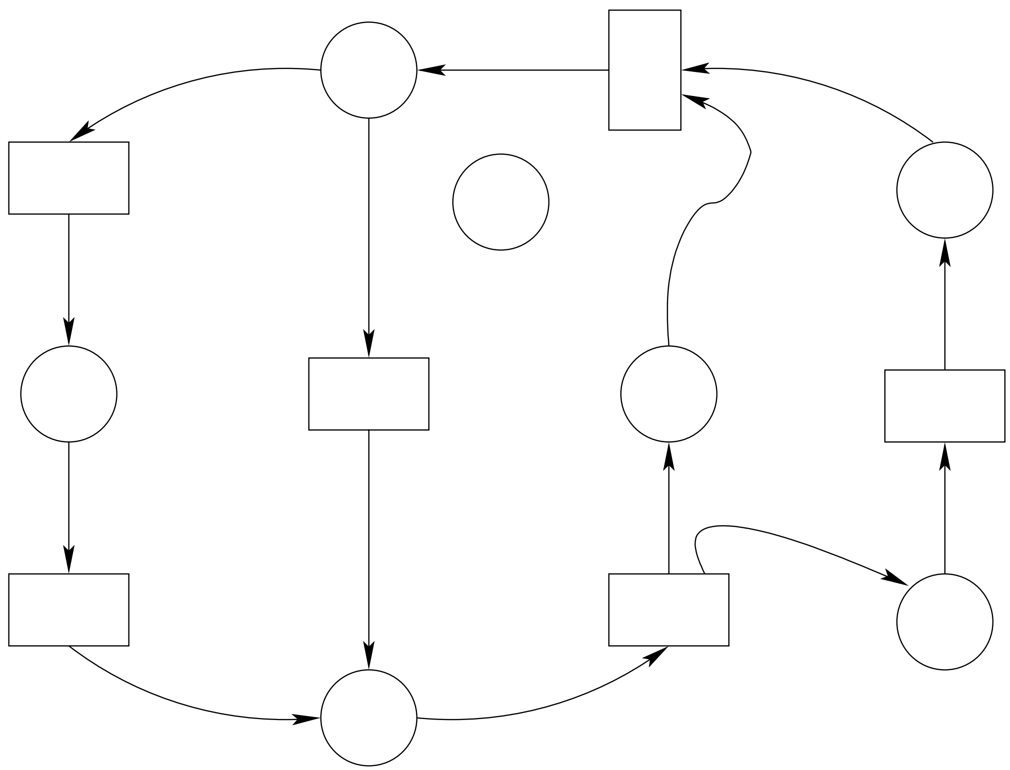
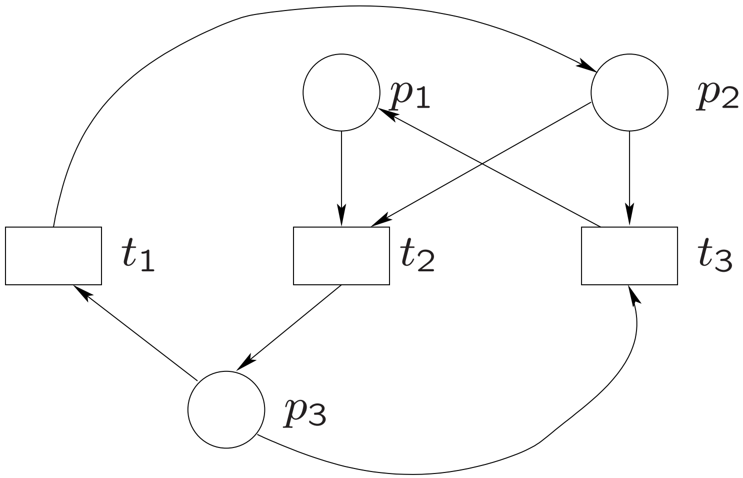


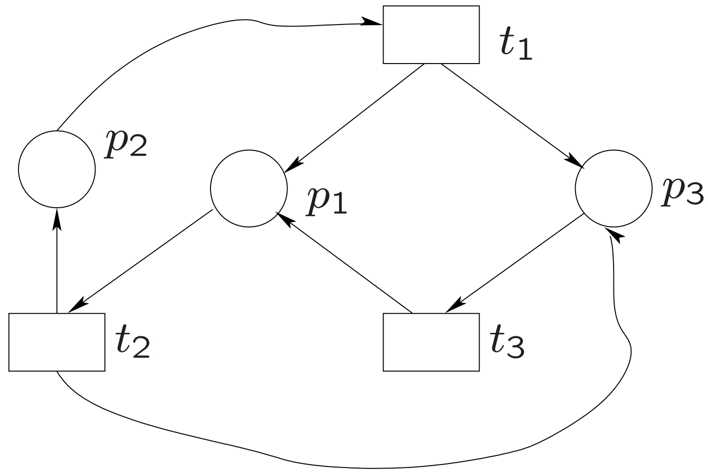
Fig. 7. A net that is P-simple and T-simple.

Definition 3. Two nets
$N = (P, T, F)$ and $N' = (P', T', F')$
are *isomorphic*, denoted by $N \equiv N'$,

if there exist two bijections
$\alpha : P \to P'$ and $\beta : T \to T'$,

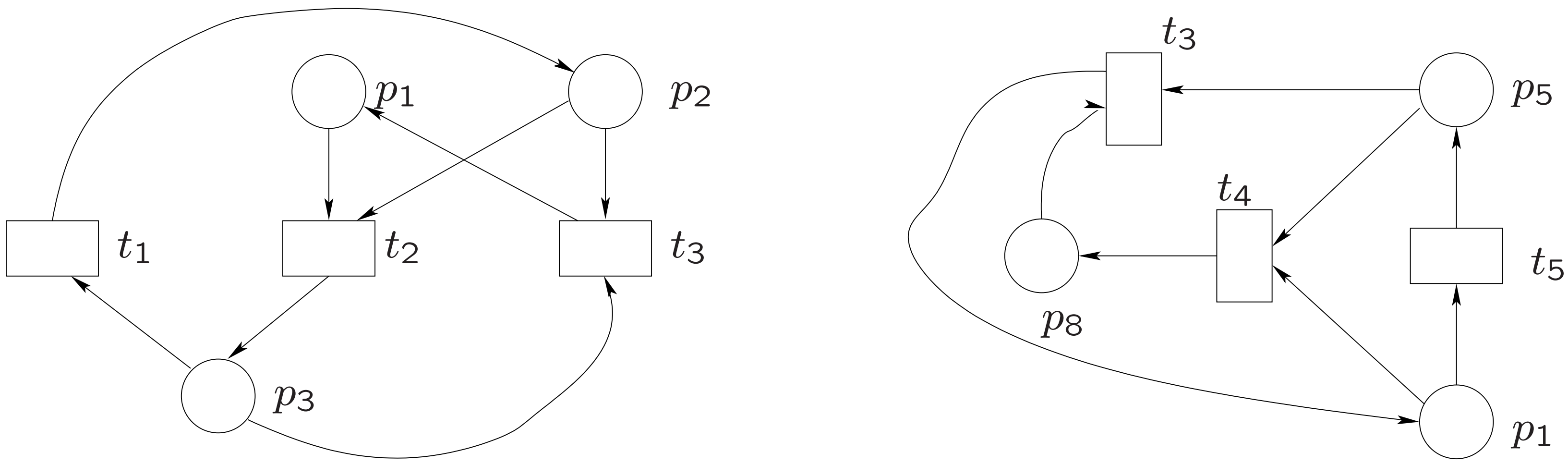
such that for every $p \in P$ and $t \in T$,
$(p, t) \in F$ iff $(\alpha(p), \beta(t)) \in F'$
and
$(t, p) \in F$ iff $(\beta(t), \alpha(p)) \in F'$.





A net N **Fig. 8** and a net N' **Fig. 9**, not isomorphic.

G_N and $G_{N'}$ are isomorphic graphs!



A net N **Fig. 8** and a net N'' **Fig. 10**, isomorphic.

Definition 4. A *configuration* of a net $N = (P, T, F)$ is a subset of P.

Definition 5. An *elementary net system*, EN system for short, is a quadruple $M = (P, T, F, C_{in})$, where:

(1) (P, T, F) is a net and

(2) $C_{in} \subseteq P$ is the *initial configuration*.

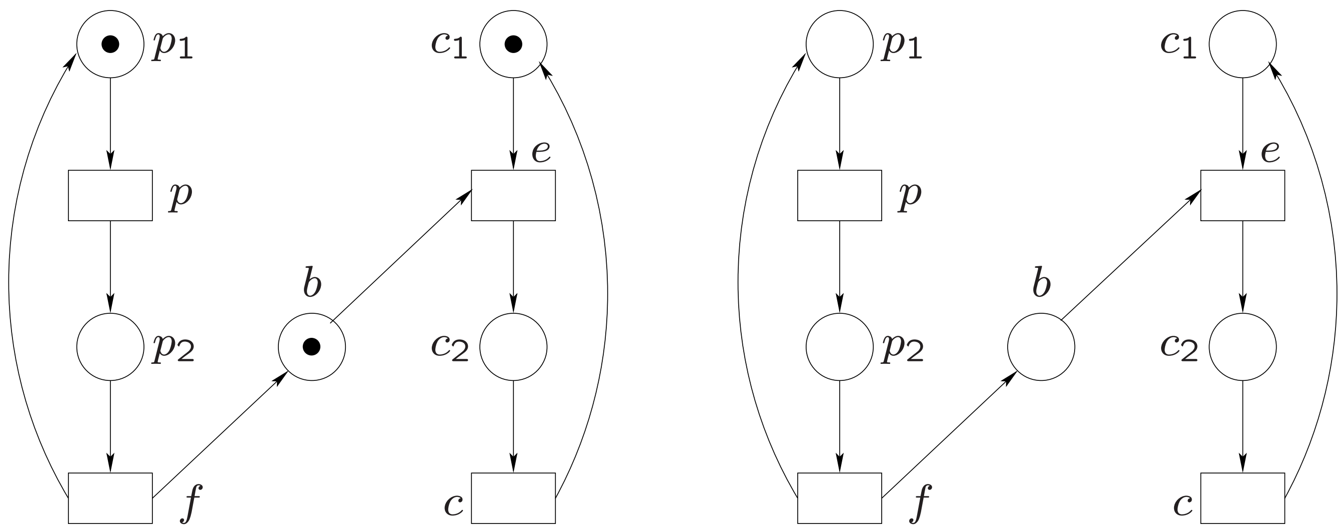


Fig. 12. An EN system and its underlying net **Fig. 11.**

Definition 6. Let $M = (P, T, F, C_{in})$ be an EN system and let $t \in T$.

(1) Let $C \subseteq P$ be a configuration.
Then t *has concession in* C
(or t *can be fired in* C, or t *is enabled in* C)
if ${}^\bullet t \subseteq C$ and $t^\bullet \cap C = \varnothing$, written as $t \ \mathbf{con} \ C$.

(2) Let $C, D \subseteq P$. Then t *fires from* C *to* D if $t \ \mathbf{con} \ C$ and $D = (C - {}^\bullet t) \cup t^\bullet$, written as $C[t\rangle D$;
t is also called a *sequential step from* C *to* D.

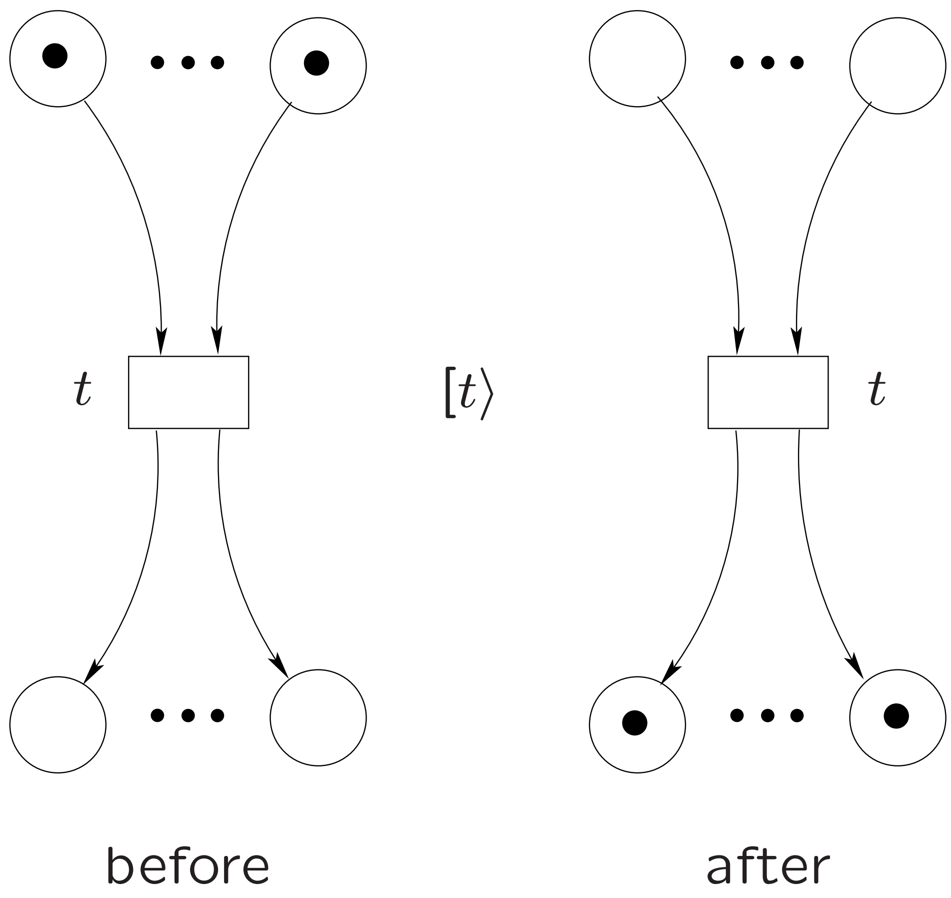
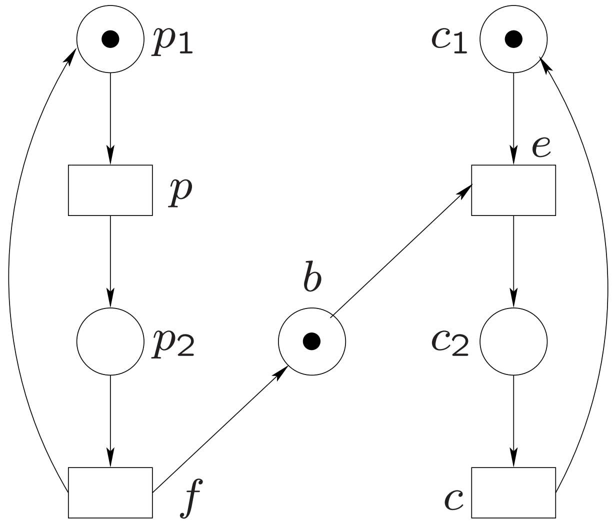
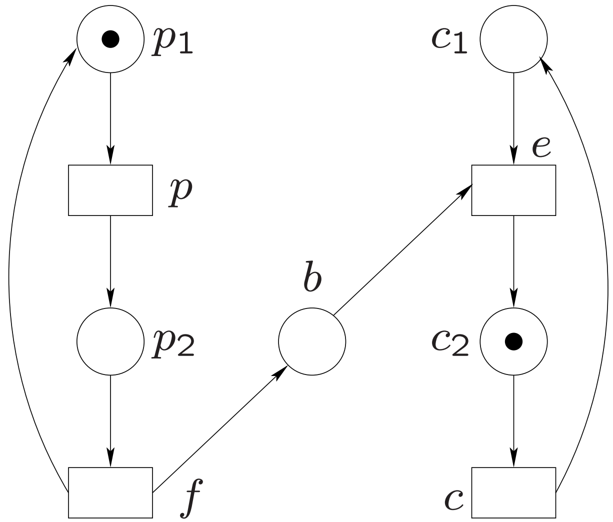


Fig. 13. Firing of transition t.





Before **Fig. 12.** and after **Fig. 14.** firing transition e.

Lemma 7. Let $M = (P, T, F, C_{in})$ be an EN system.
Let $t \in T$ and let $C, D \subseteq P$.

Then $C[t\rangle D$ iff $C - D = {}^\bullet t$ and $D - C = t^\bullet$.

Definition 8. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $t_1 \cdots t_n \in T^*$, with $n \geq 0$ and $t_1, \ldots, t_n \in T$. Let $C, D \subseteq P$. Then $t_1 \cdots t_n$ *fires from* C *to* D if there exist configurations $C_0, C_1, \ldots, C_n \subseteq P$ with $C_0 = C$, $C_n = D$ and $C_{i-1}[t_i\rangle C_i$ for all $1 \leq i \leq n$, written as $C[t_1 \cdots t_n\rangle D$.

(2) Let $x \in T^*$ and $C \subseteq P$.
Then x *has concession in* C
(or x *can be fired in* C, or x *is enabled in* C)
if there exists a $D \subseteq P$ such that $C[x\rangle D$, written as $x \ \mathbf{con} \ C$.

(3) $x \in T^*$ is a *firing sequence of* M if $x \ \mathbf{con} \ C_{in}$. The set of all firing sequences of M is denoted by FS(M).

Definition 8 Ctd. Let $M = (P, T, F, C_{in})$ be an EN system.

(4) $C \subseteq P$ is a *reachable configuration of* M
if there exists an $x \in \mathsf{FS}(M)$ with $C_{in}[x\rangle C$.
The set of all reachable configurations of M is denoted by $\mathbb{C}_M$.

(5) $t \in T$ is a *useful transition of* M
if there exists a reachable configuration C of M such that $t\ \mathbf{con}\ C$.
The set of useful transitions of M is denoted by $\mathbf{use}_M(T)$, or just $\mathbf{use}(T)$ when M is clear from the context.

(6) $t \in T$ is a *live transition of* M
if for each $C \in \mathbb{C}_M$ there exists an $x \in T^*$ with $xt\ \mathbf{con}\ C$.

behandeld eerste college:

1. Preface, 2. Introduction
(3. Preliminaries — zelf lezen)
4. EN systems: 4.1, 4.2, 4.3 tot blz. 18

tweede college: 8 september 2011
4.3 afmaken
4.4 Concurrency
4.5 Fundamental Situations

eerste werkgroep: 15 september 2011
laptop, Java platform, ..., inlichtingen: website TvC en Wouter
alle opgaven bij 4. EN Systems