Theorie van Concurrency

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9. P/T Systems
9.1. Informal Introduction
9.2. Definitions
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9.5. Place Invariants

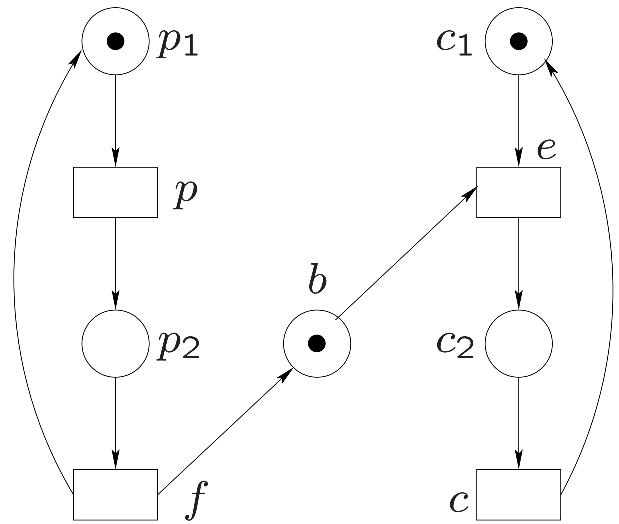
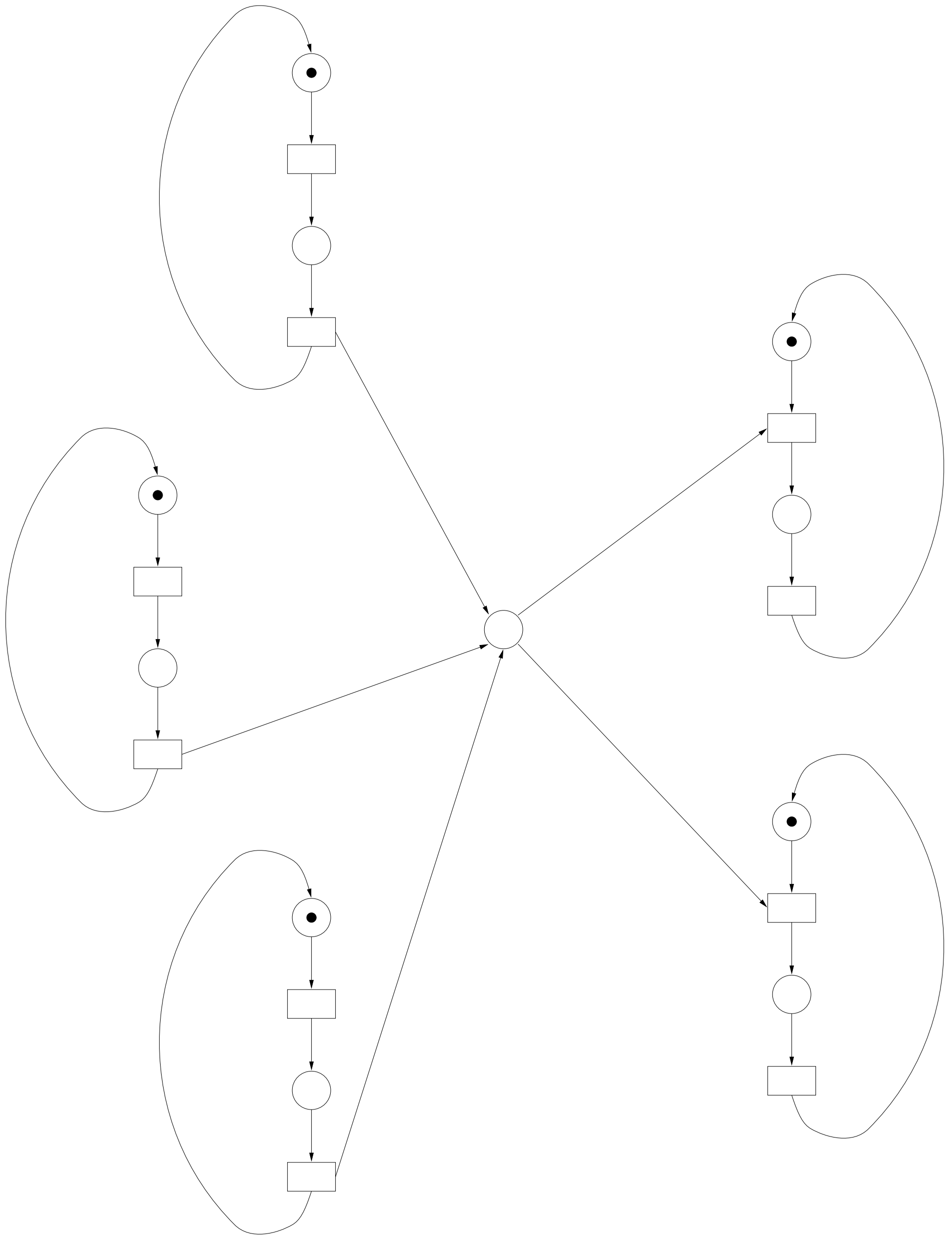


Fig. 12. The producer/consumer problem



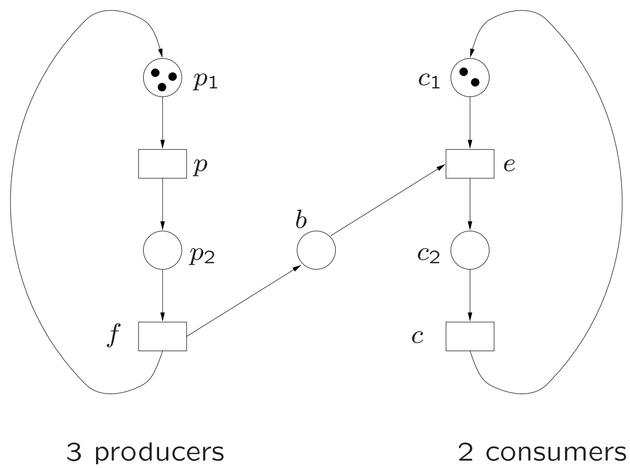


Fig. 76. P/T system for 3 producers and 2 consumers.

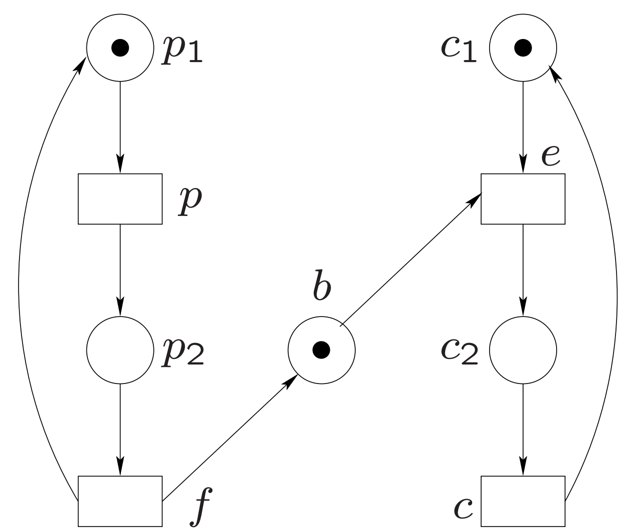


Fig. 12. The producer/consumer problem (with/without buffer capacity. . .).

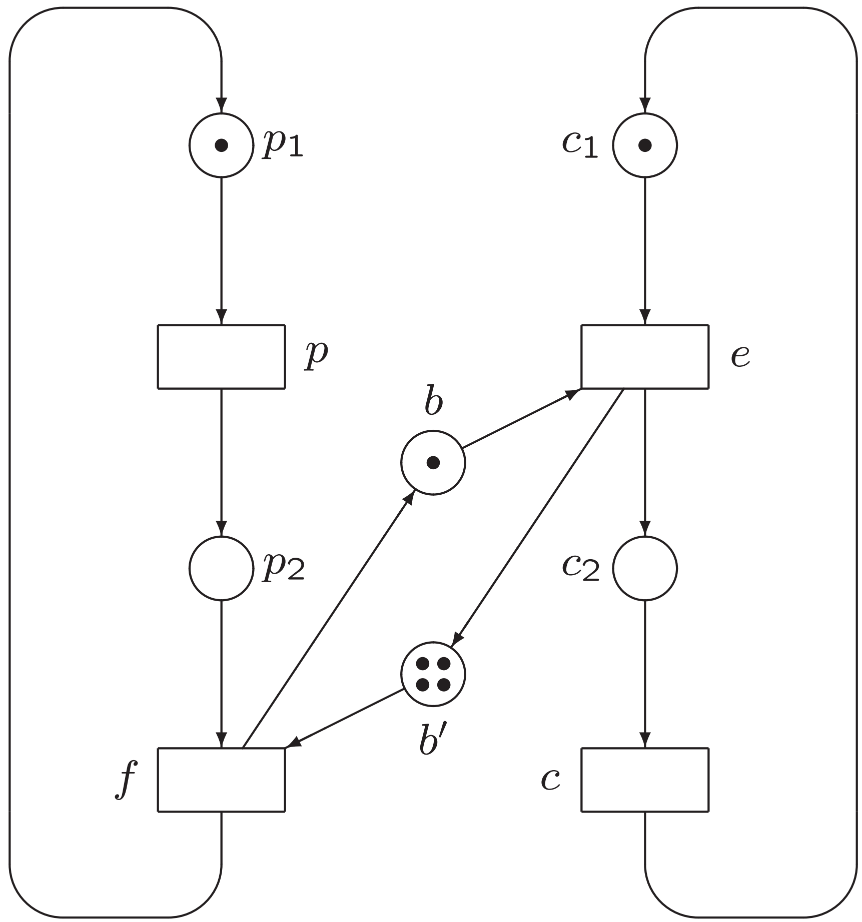


Fig. 77. A buffer with capacity 5.

Now, play the token game. . .

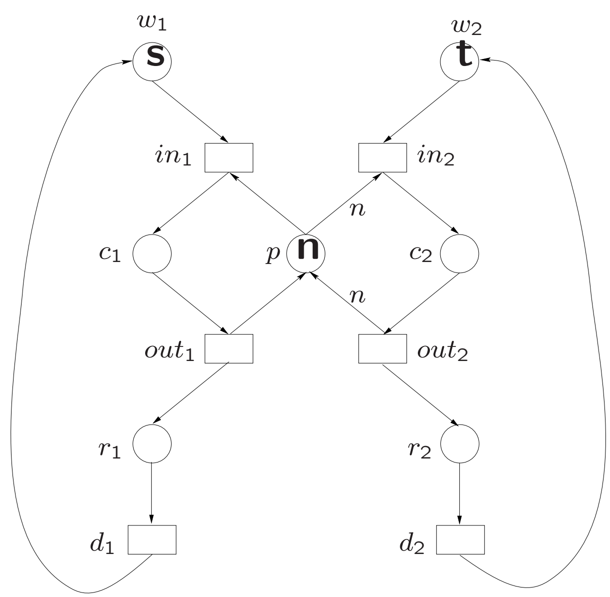


Fig. 78. The readers/writers problem.

Definition 152. A *multi-net* is a tuple $N = (P, T, F, W, K)$, where

(1) (P, T, F) is a net,

(2) $W : F \to \mathbb{N}_+$ is the *weight function*, and

(3) $K : P \to \mathbb{N}_+ \cup \{\omega\}$ is the *capacity function*.

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Definition 153. A *configuration* of a multi-net $N = (P, T, F, W, K)$ is a function $C : P \to \mathbb{N}$, such that
$\forall p \in P$: if $K(p) \in \mathbb{N}_+$, then $C(p) \leq K(p)$.

Definition 154. A *place/transition system*, P/T system for short, is a tuple $M = (P, T, F, W, K, C_{in})$, where

(1) (P, T, F, W, K) is a multi-net, and

(2) $C_{in} : P \to \mathbb{N}$ is a configuration of that multi-net:
the *initial configuration*.

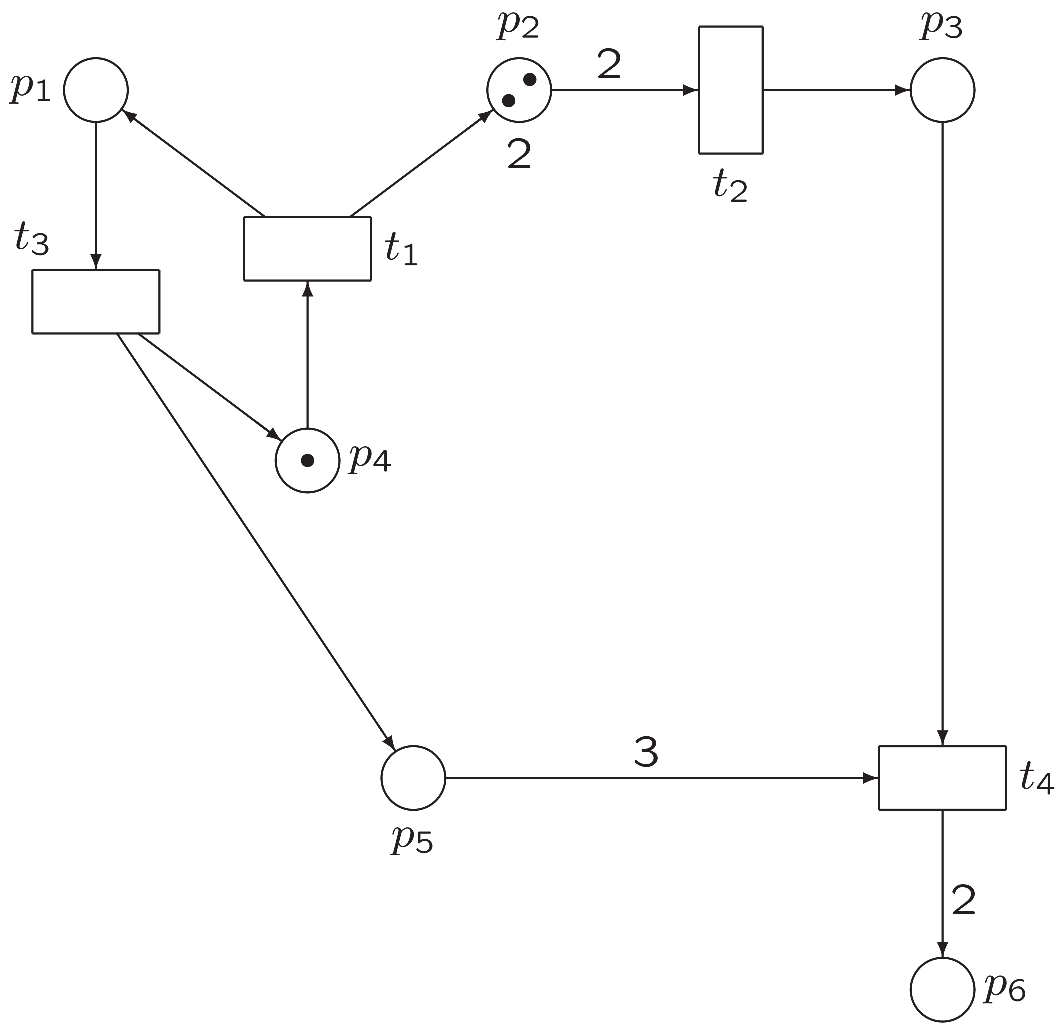


Fig. 79. A P/T system.

Definition 155. Let $M = (P, T, F, W, K, C_{in})$ be a P/T system and let $t \in T$

(1) Let $C : P \to \mathbb{N}$ be a configuration of M.
Then t *has concession in* C (or t *can be fired in* C, or t *is enabled in* C), written as $t \; \mathbf{con} \; C$, if:
$\forall p \in {}^{\bullet}t$: $W(p,t) \leq C(p)$ and
$\forall p \in t^{\bullet}$: if $K(p) \in \mathbb{N}_{+}$, then $C(p) + W(t,p) \leq K(p)$.

(2) Let C and D be configurations of M.
Then t *fires from* C *to* D, written as $C[t\rangle D$, if $t \; \mathbf{con} \; C$ and, for every $p \in P$,

$D(p) = C(p) - W(p,t)$	if $p \in {}^{\bullet}t$,
$D(p) = C(p) + W(t,p)$	if $p \in t^{\bullet}$, and
$D(p) = C(p)$	otherwise.

Definition 8. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) Let $t_1 \cdots t_n \in T^*$, with $n \geq 0$ and $t_1, \ldots, t_n \in T$. Let $C, D \subseteq P$. Then $t_1 \cdots t_n$ *fires from* C *to* D if there exist configurations $C_0, C_1, \ldots, C_n \subseteq P$ with $C_0 = C$, $C_n = D$ and $C_{i-1}[t_i\rangle C_i$ for all $1 \leq i \leq n$, written as $C[t_1 \cdots t_n\rangle D$.

(2) Let $x \in T^*$ and $C \subseteq P$.
Then x *has concession in* C
(or x *can be fired in* C, or x *is enabled in* C)
if there exists a $D \subseteq P$ such that $C[x\rangle D$, written as $x \ \mathbf{con} \ C$.

(3) $x \in T^*$ is a *firing sequence of* M if $x \ \mathbf{con} \ C_{in}$. The set of all firing sequences of M is denoted by FS(M).

Definition 8 Ctd. Let $M = (P, T, F, C_{in})$ be an EN system.

(4) $C \subseteq P$ is a *reachable configuration of* M
if there exists an $x \in \mathsf{FS}(M)$ with $C_{in}[x\rangle C$.
The set of all reachable configurations of M is denoted by $\mathbb{C}_M$.

(5) $t \in T$ is a *useful transition of* M
if there exists a reachable configuration C of M such that t **con** C.
The set of useful transitions of M is denoted by $\mathbf{use}_M(T)$, or just $\mathbf{use}(T)$ when M is clear from the context.

(6) $t \in T$ is a *live transition of* M
if for each $C \in \mathbb{C}_M$ there exists an $x \in T^*$ with xt **con** C.

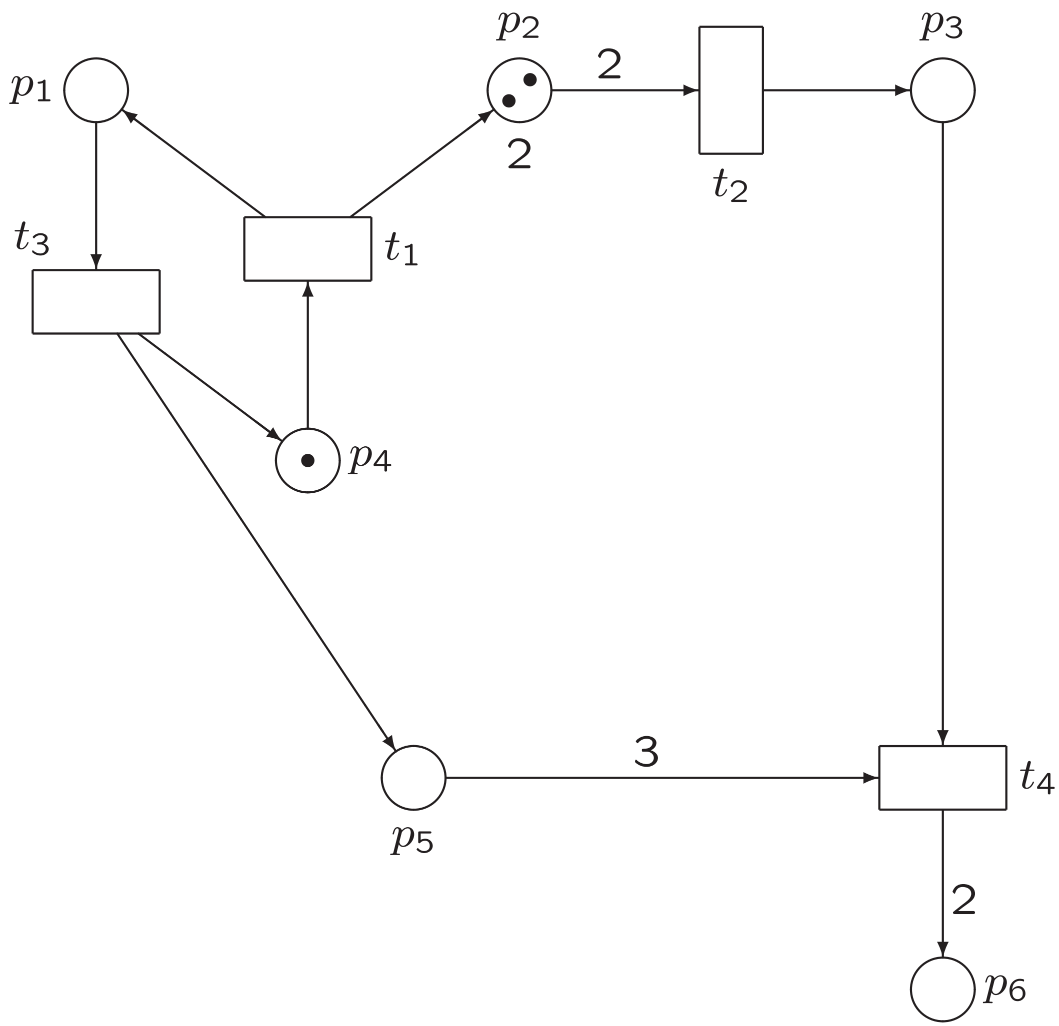


Fig. 79. A P/T system.

Now, play the token game. . .

For a P/T system M,
the set FS(M) is not necessarily a regular language.

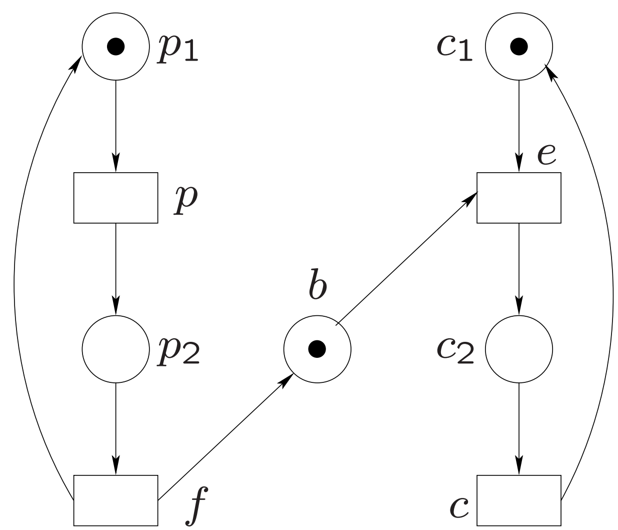


Fig. 12. The producer/consumer problem

Definition 156. (alternative) An *EN system* is a P/T system M with $K_M(p) = 1$ for all $p \in P_M$.

Definition 157. Let $M = (P, T, F, K, W, C_{in})$ be a P/T system. M is *contact-free* if for all $t \in T$ and $C \in \mathbb{C}_M$,
if $W(p,t) \leq C(p)$ for all $p \in {}^\bullet t$,
then $C(p) + W(t,p) \leq K(p)$ for all $p \in t^\bullet$ with $K(p) \in \mathbb{N}_+$.

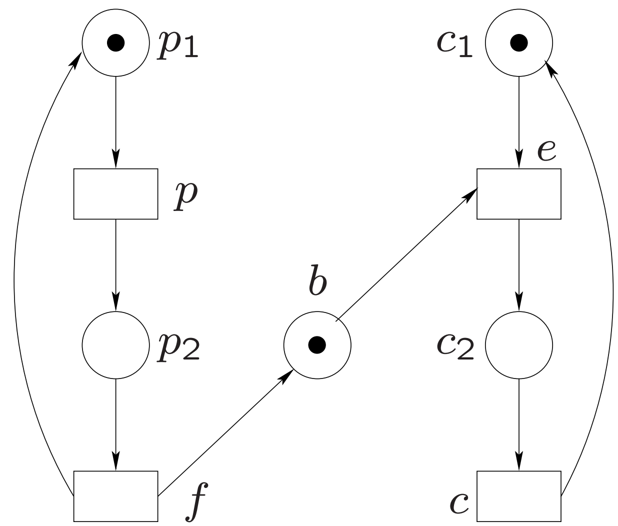


Fig. 12. The producer/consumer problem
(with buffer capacity 5).

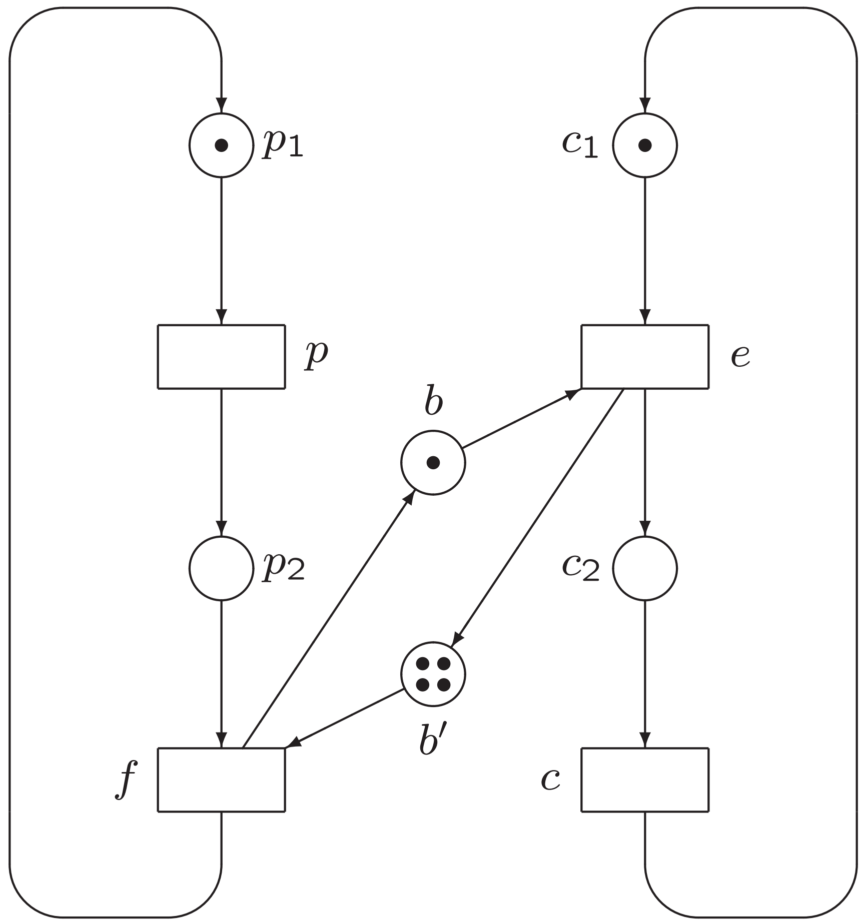


Fig. 77. A buffer with capacity 5.

Complement construction justifies assumption:
for every P/T system M: $K_M(p) = \omega$ for all $p \in P_M$.

We denote M by (P, T, F, W, C_{in}).
A configuration is any function $C : P \to \mathbb{N}$ and
$t \textbf{ con } C \Leftrightarrow \forall p \in {}^{\bullet}t : W(p, t) \leq C(p)$.

Definition 158. A P/T system $M = (P, T, F, W, C_{in})$ is *safe* if $C(p) \in \{0, 1\}$ for all $C \in \mathbb{C}_M$ and $p \in P$.

Definition 159. (alternative) A *contact-free EN system* is a safe P/T system.

A configuration is any function $C : P \to \mathbb{N}$.

In other words: C is a vector in $\mathbb{N}^n$.

A partial order for vectors:
For $C, D : P \to \mathbb{Z}$, we say $C \leq D$ iff $C(p) \leq D(p)$ for every $p \in P$.
We say $C < D$ if $C \leq D$ and $C \neq D$.

Vectors can be added and multiplied by a scalar.

9.3. A Finiteness Algorithm

Lemma 160. Let $M = (P, T, F, W, C_{in})$ be a P/T system, $x \in T^*$, and C, D, E configurations of M.

If $C[x\rangle D$, then $C + E[x\rangle D + E$.

Lemma 161. Let $M = (P, T, F, W, C_{in})$ be a P/T system, $x \in T^*$, and C, D configurations of M.

If $C[x\rangle D$ and $C \leq D$ then, for every $n \in \mathbb{N}$, $C[x^n\rangle D_n$, where $D_n(p) = C(p) + n \cdot (D(p) - C(p))$ for every $p \in P$.

Lemma 162. Let $M = (P, T, F, W, C_{in})$ be a P/T system, $x \in T^*$, and $C, D \in \mathbb{C}_M$.

If $C[x\rangle D$ and $C < D$, then $\mathbb{C}_M$ is infinite.

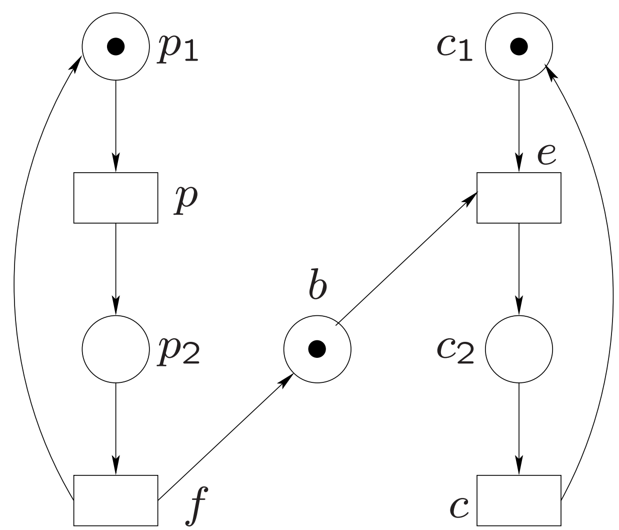


Fig. 12. The producer/consumer problem

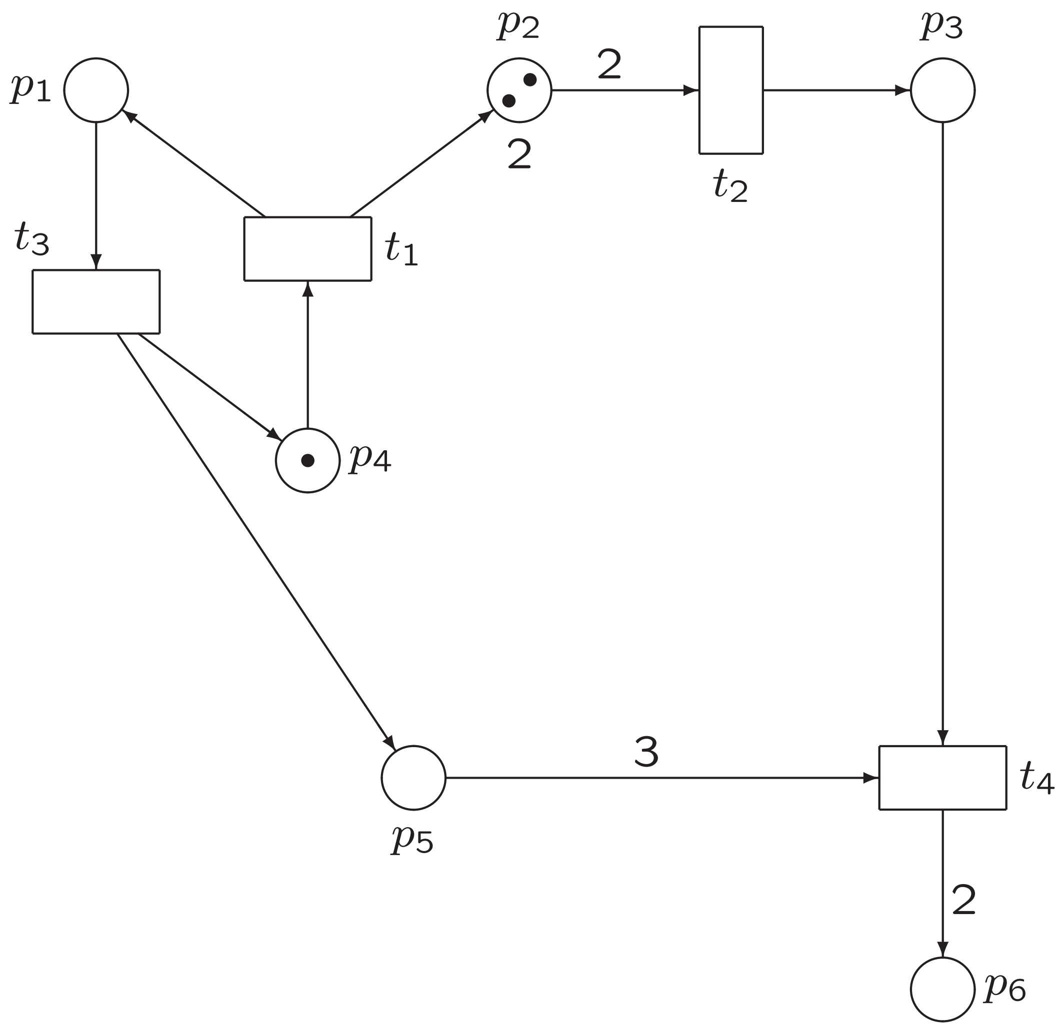


Fig. 79. A P/T system.

Lemma 162. Let $M = (P, T, F, W, C_{in})$ be a P/T system, $x \in T^*$, and $C, D \in \mathbb{C}_M$.

If $C[x\rangle D$ and $C < D$, then $\mathbb{C}_M$ is infinite.

TO BE PROVED (from left to right):

Theorem 163. Let $M = (P, T, F, W, C_{in})$ be a P/T system.

Then $\mathbb{C}_M$ infinite iff

there exist $x \in T^*$ and $C, D \in \mathbb{C}_M$ such that $C[x\rangle D$ and $C < D$.

An infinite sequence of configurations $C_1, C_2, C_3, \ldots$ of a P/T system is *increasing* if $C_i \leq C_{i+1}$ for all $i \geq 0$.

Lemma 164. (Dickson's Lemma)
Every infinite sequence of configurations of a P/T system has an infinite increasing subsequence.

Definition 165. Language L is *prefix-closed* if for every $x \in L$ also all prefixes of x are element of L.

Lemma 166. (König's Lemma)
If $L \subseteq \Sigma^*$ is an infinite prefix-closed language, then there is an infinite sequence $t_1, t_2, t_3, \ldots$ with $t_i \in \Sigma$, such that, for all $n \geq 0$, $t_1 \cdots t_n \in L$.

Theorem 167 It is decidable, for an arbitrary P/T system M, whether $\mathbb{C}_M$ is finite or infinite.

The algorithm uses a (finite) edge-labelled graph $G = (V, \Gamma, \Sigma, v_{in})$, with $V \subseteq \mathbb{C}_M$, . . . The algorithm works as follows.

1. Initialization: $V := \{C_{in}\}$, $\Gamma := \varnothing$, $\Sigma := \varnothing$ and $v_{in} := C_{in}$.

2. For all $C \in V$ and $t \in T$ do:
if $C[t\rangle D$ then $V := V \cup \{D\}$, $\Gamma := \Gamma \cup \{(C, t, D)\}$, and $\Sigma := \Sigma \cup \{t\}$.

3. If G did not change in Step (2),
— then $\mathbb{C}_M$ is finite and $G = \mathrm{SCG}(M)$.
If G did change in Step (2),
— $\exists$? $C, D \in V$: $C < D$ and there is a path in G from C to D.
If this is the case, then $\mathbb{C}_M$ is infinite.
If not, then goto (2).

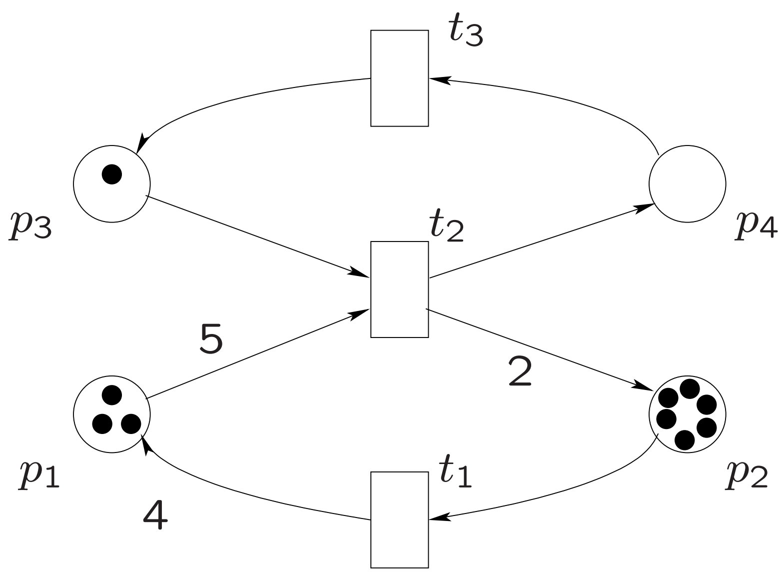
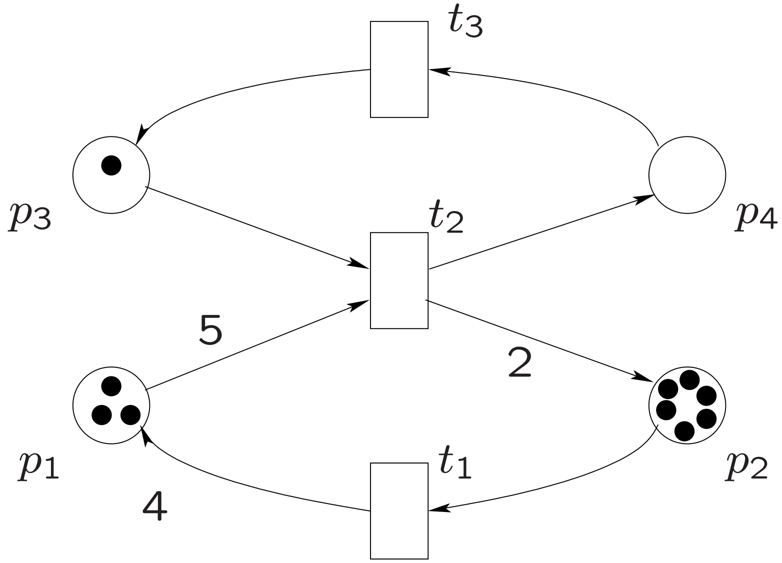


Fig. 80. A P/T system M with infinite SCG(M).



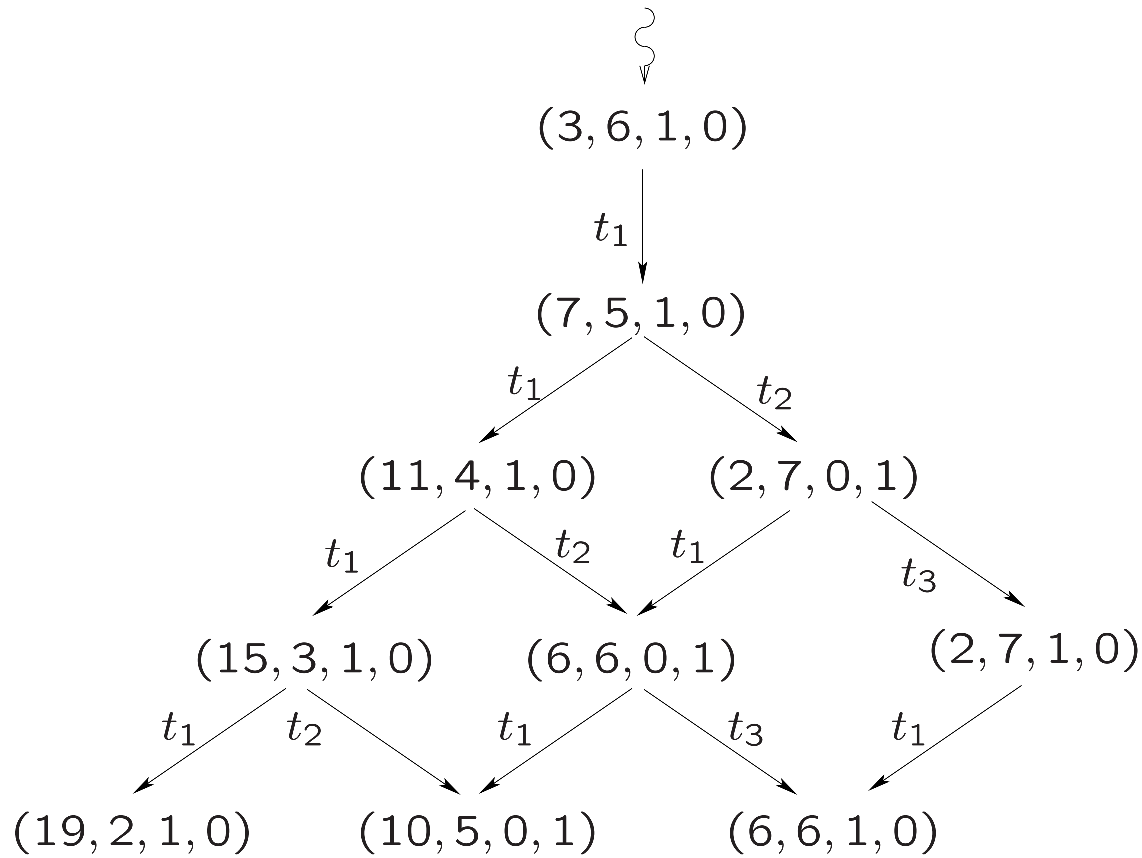
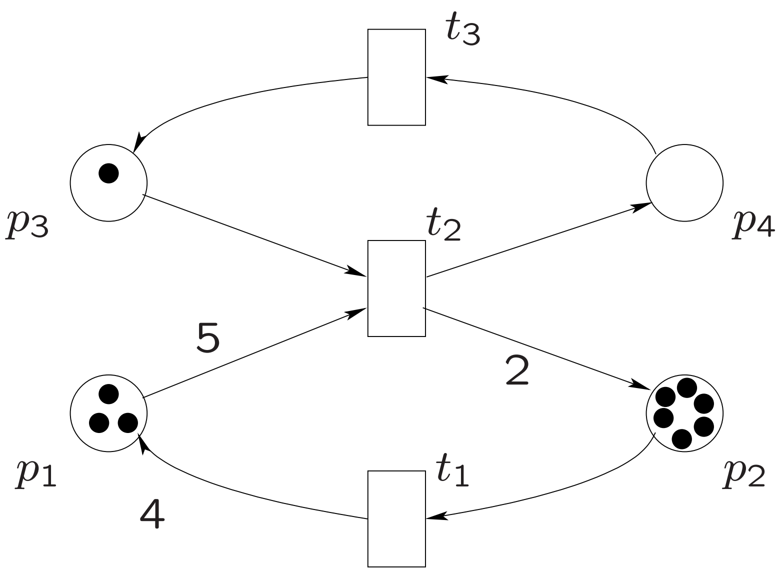
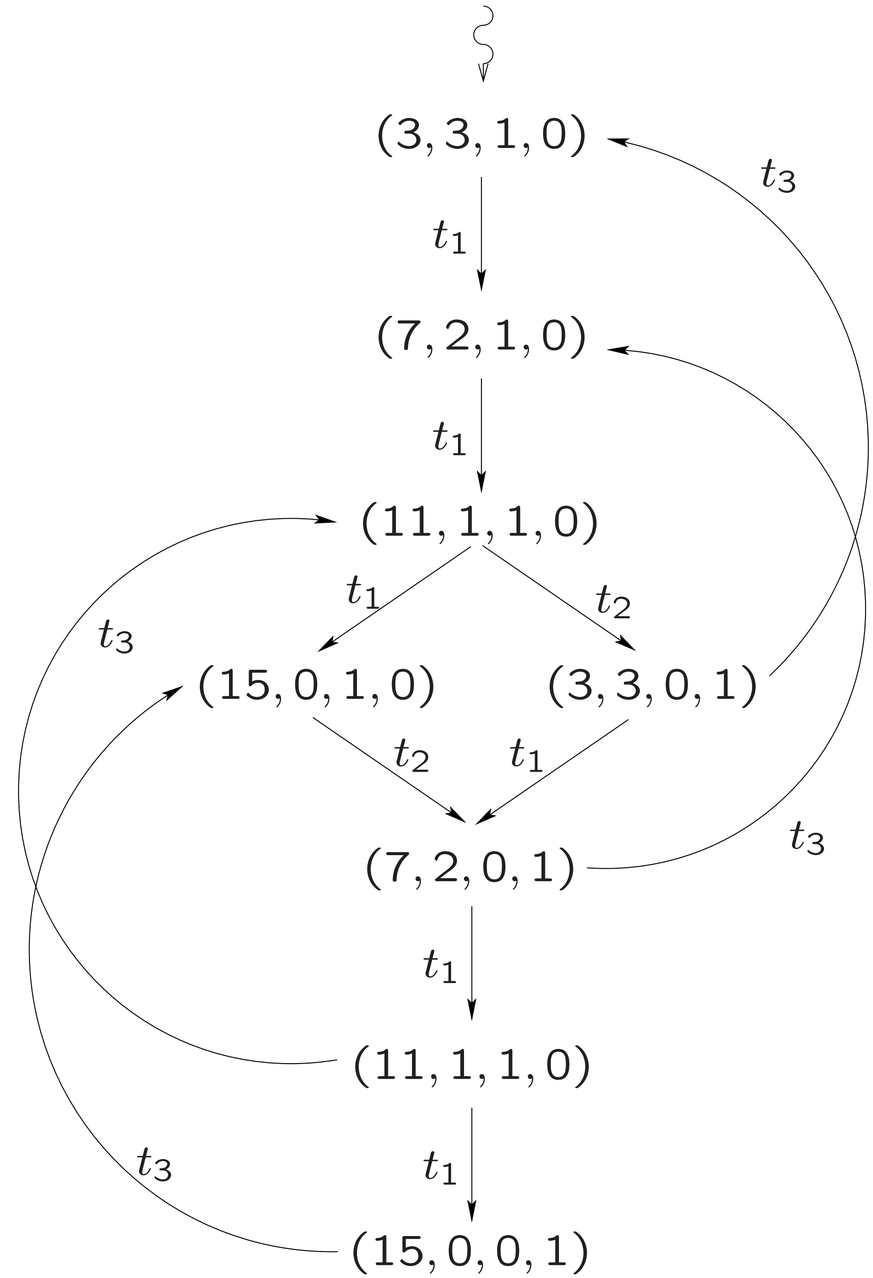


Fig. 81. The computed part of SCG(M).



Let $W(p_1, t_2) = 8$
and $C_{in}(p_2) = 3$.



Definition 168. A P/T system M is *bounded* if there exists a $k \in \mathbb{N}$ such that $C(p) \leq k$ for every $C \in \mathbb{C}_M$ and $p \in P_M$.

Theorem 169. It is decidable, for an arbitrary P/T system M, whether or not M is safe.

9.5. Place Invariants

An example of an invariant:

Theorem 49. Let $M = (P, T, F, C_{in})$ be a reduced EN system and let $S \subseteq P$.
Then the following statements are equivalent.

(1) There is a sequential component M' of M with $P_{M'} = S$,

(2) $\#(C \cap S) = 1$ for all $C \in \mathbb{C}_M$,

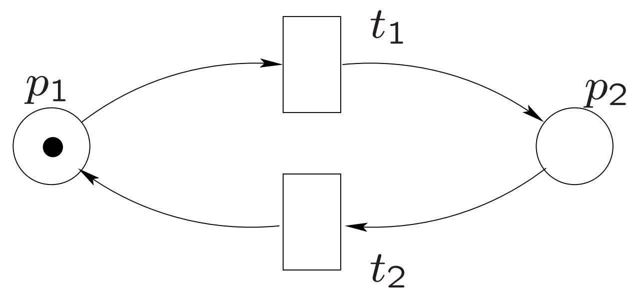


Fig. 88. P/T system M with invariant $C(p_1) + C(p_2) = 1$.

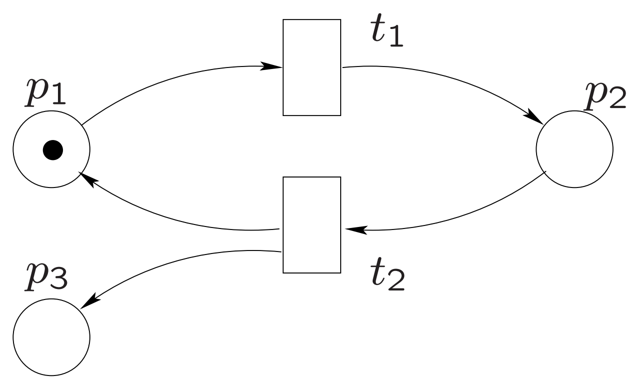


Fig. 89. P/T system M' with invariant $C(p_1) + C(p_2) = 1$.

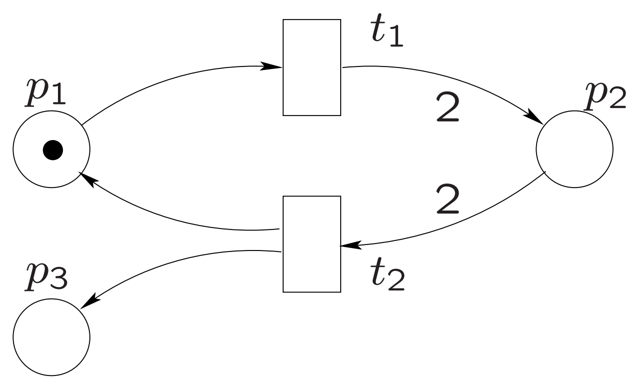


Fig. 90. P/T system M'' with invariant $2C(p_1) + C(p_2) = 2$.

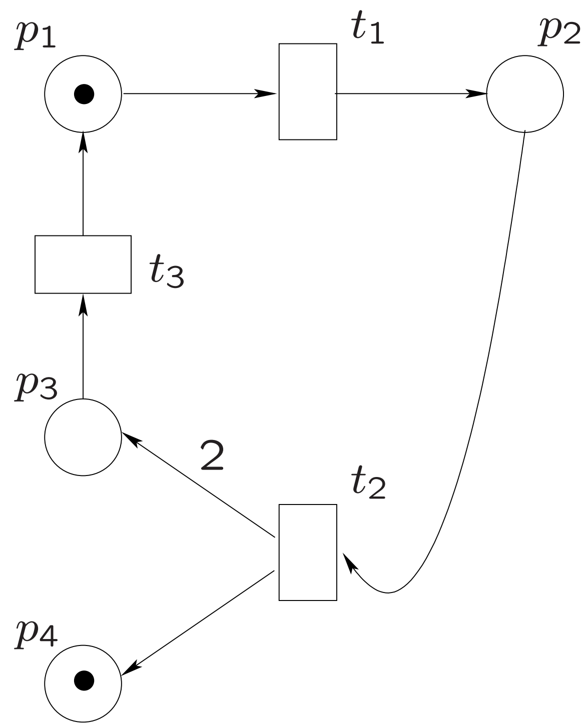


Fig. 91. P/T system M_1 with invariant
$C(p_1) + C(p_2) + C(p_3) - C(p_4) = 0$.

Definition 188. Let $M = (P, T, F, W, C_{in})$ be a P/T system.

A vector $i : P \to \mathbb{Z}$ is a *place invariant* (*p-invariant*) of M if for all configurations $C, D : P \to \mathbb{N}$ and all $t \in T$: if $C[t\rangle D$, then $C \cdot i = D \cdot i$. The *value* of i is the integer $C_{in} \cdot i$.

A p-invariant $i : P \to \mathbb{N}$ is a *positive* p-invariant of M.

A p-invariant $i : P \to \{0, 1\}$ is a *characteristic* p-invariant of M

A characteristic p-invariant with value 1 is a *sequential component* of M.

Lemma 189. Let $M = (P, T, F, W, C_{in})$ be a P/T system and let $i : P \to \mathbb{Z}$ be a p-invariant of M.

Then $C \cdot i = C_{in} \cdot i$ for all $C \in \mathbb{C}_M$.

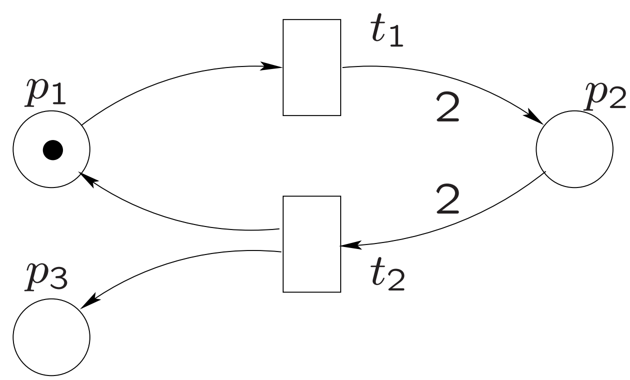


Fig. 90. P/T system M'' with invariant $2C(p_1) + C(p_2) = 2$.