Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

dertiende college: 3 november 2011

7.1. LPO-Equivalence and Firing Sequence Equivalence

7.2. Equivalence of Firing Sequences

vijfde werkcollege: 10 november 2011

For a contact-free EN system M
$\mathsf{LPO}(M)$ is the set of all pruned contracted processes of M:
$\mathsf{LPO}(M) = \{\mathbf{pru}(\mathbf{ctr}(N)) \mid N \in \mathsf{PROC}(M)\}$

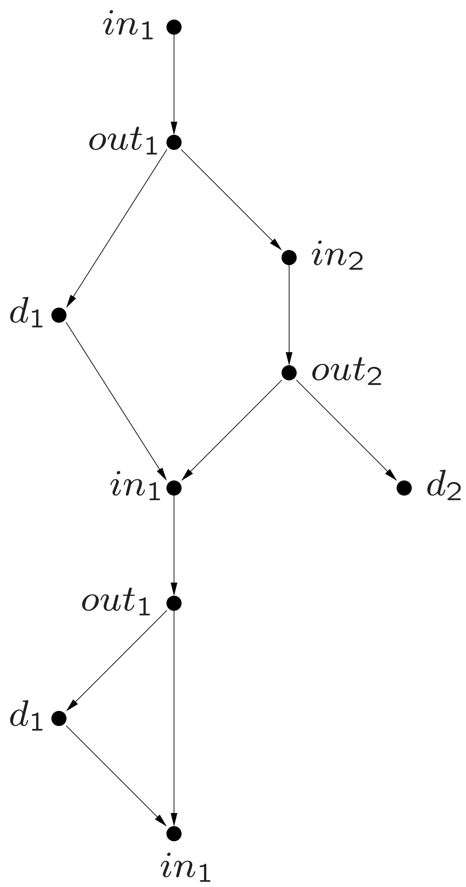
Definition 105. Two contact-free EN systems M and M' are *lpo-equivalent* if $\mathsf{LPO}(M) \equiv \mathsf{LPO}(M')$.

$M \equiv M'$	$\Longrightarrow$	$M \approx M'$	$\Longrightarrow$	$M \approx_w M'$	$\Longleftrightarrow$	$M \approx_{fs} M'$
isomorphic		config. equivalent		weakly config. equivalent		firing seq. equivalent

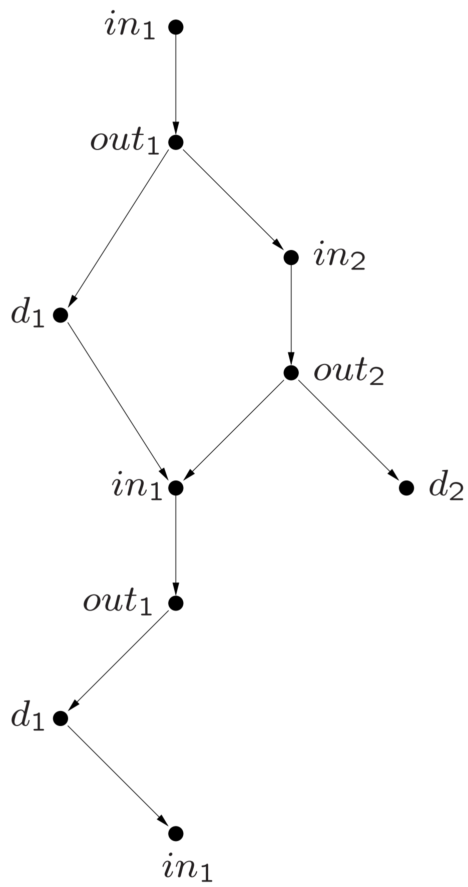
$$\mathsf{SCG}(M) \equiv \mathsf{SCG}(M')$$
$$\mathsf{CG}(M) \equiv \mathsf{CG}(M')$$

Theorem 115. Let M and M' be two contact-free EN systems and let β be a bijection from $\mathbf{use}(T_M)$ to $\mathbf{use}(T_{M'})$.

If $\mathsf{LPO}(M) \equiv_\beta \mathsf{LPO}(M')$ then $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$.



ctr(N)



pru(ctr(N))

Next (**Thm 125**):
M and M' contact-free EN systems.
If $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$ then $\mathsf{LPO}(M) \equiv_\beta \mathsf{LPO}(M')$.

Definition 117. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) The *independency relation of* M is the independency relation $\mathbf{ind}(M)$ over $\mathbf{use}(T)$ defined by
$\mathbf{ind}(M) = \{(s,t) \in T \times T \mid s \neq t$ and $\exists C \in \mathbb{C}_M : \{s,t\}\ \mathbf{con}\ C\}$.

(2) The *dependency relation of* M is the relation $\mathbf{dep}(M)$ defined by $\mathbf{dep}(M) = (\mathbf{use}(T) \times \mathbf{use}(T)) - \mathbf{ind}(M)$.

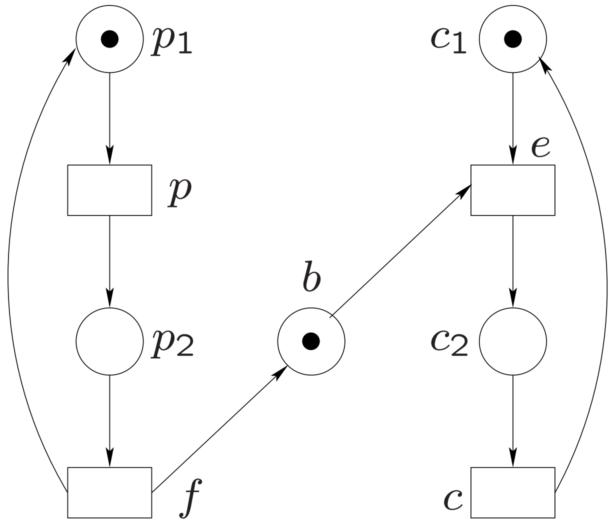


Fig. 12. The producer/consumer problem

(not contact-free!)

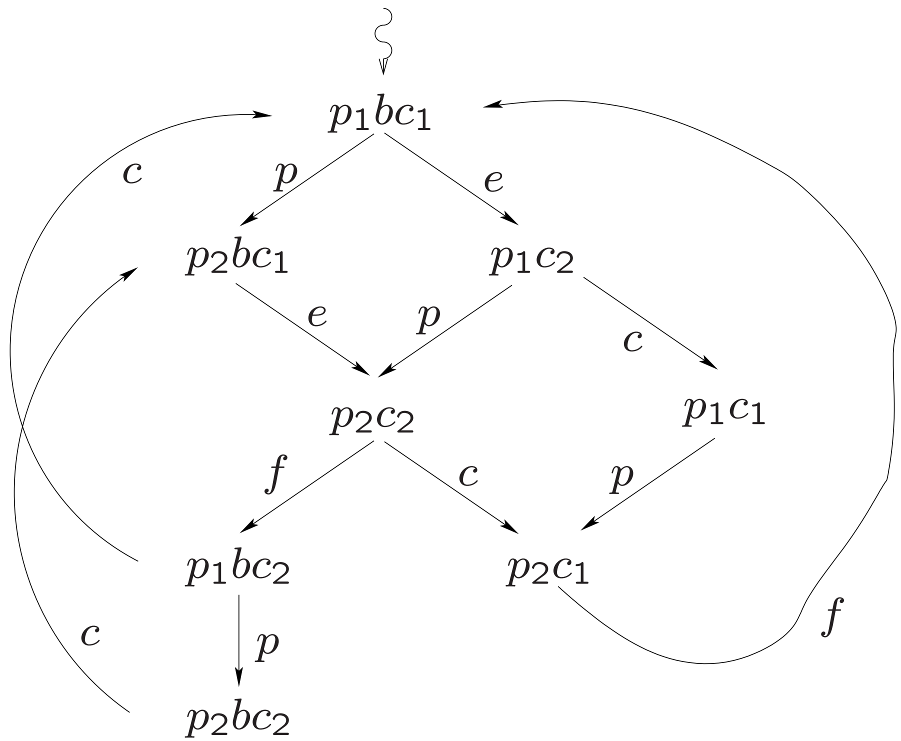


Fig. 16. A sequential configuration graph.

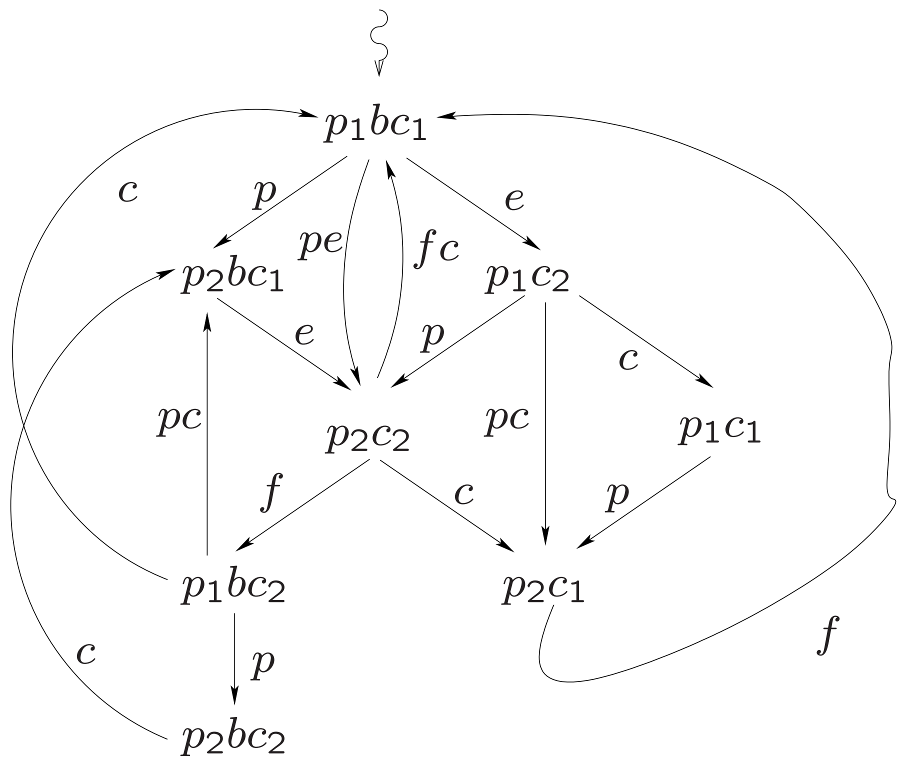


Fig. 18. A configuration graph.

Definition 116. Let Σ be an alphabet.
A relation $I \subseteq \Sigma \times \Sigma$ is an *independency relation* (*over* Σ)
if I is irreflexive and symmetric.

Definition 119. Let Σ be an alphabet and I an independency relation over Σ.
Let $x = t_1 \cdots t_n \in \Sigma^*$, with $n \geq 0$ and $t_1, \ldots, t_n \in \Sigma$.

(1) The *dependency graph of* x (*over* I), denoted by $\mathbf{dep}_I(x)$, is the labelled graph $(V, \Gamma, \Sigma, \phi)$, where
$V = \{v_1, \ldots, v_n\}$,
$\phi(v_i) = t_i$ for all $1 \leq i \leq n$, and,

for all $1 \leq i, j \leq n$,
$(v_i, v_j) \in \Gamma$ iff $i < j$ and $(t_i, t_j) \notin I$.

(2) The *pruned dependency graph of* x (*over* I) is $\mathbf{pru}(\mathbf{dep}_I(x))$.

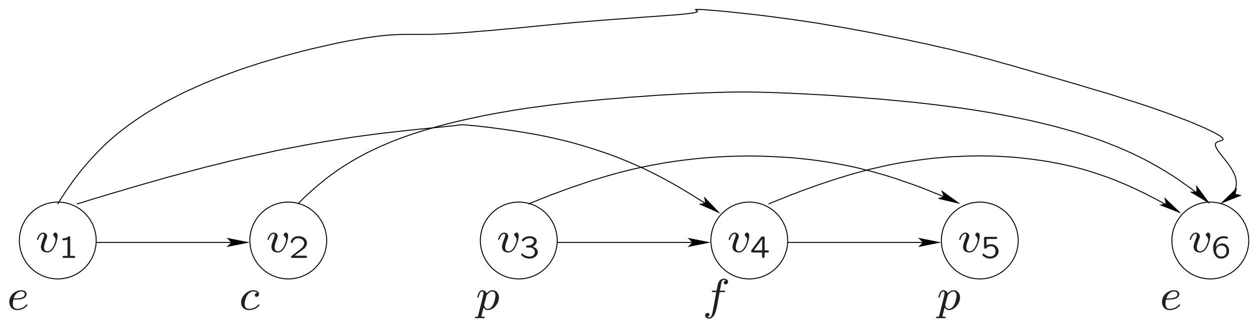


Fig. 67. A dependency graph.

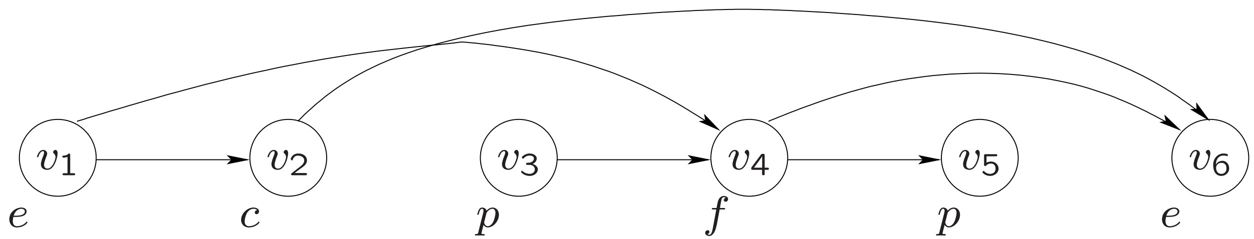


Fig. 68. A pruned dependency graph.

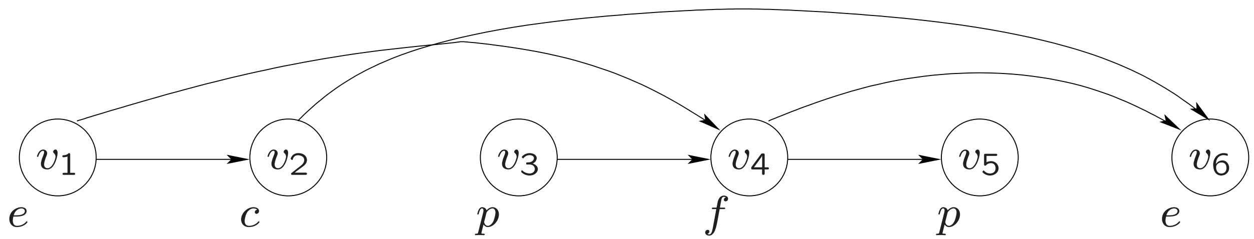


Fig. 68. A pruned dependency graph.

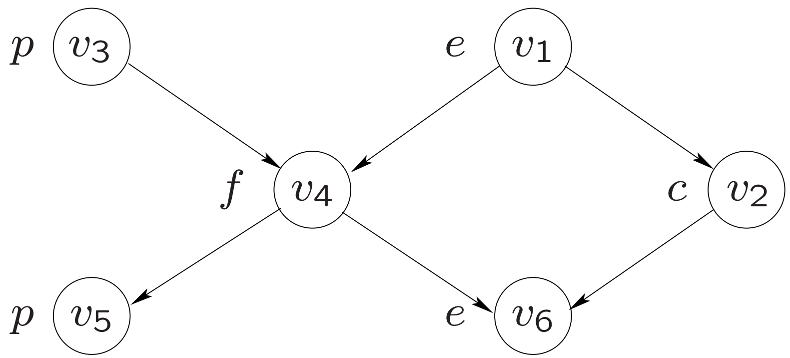


Fig. 66. An acyclic labelled graph.

Example of topological order:

$v_1v_2v_3v_4v_5v_6$ gives word $ecpfpe$

$\mathbf{ind}(M) = \{(s,t) \in T \times T \mid s \neq t$ and $\exists C \in \mathbb{C}_M : \{s,t\}$ $\mathbf{con}$ $C\}$

Lemma 118. Let $M = (P, T, F, C_{in})$ be an EN system. Then

$\mathbf{ind}(M) =$
$\{(s,t) \in T \times T \mid \exists x \in T^* : xst \in \mathsf{FS}(M)$ and $xts \in \mathsf{FS}(M)\}$.

Lemma 19. Let $M = (P, T, F, C_{in})$ be an EN system. Let $C \subseteq P$ and let $s, t \in T$.

If st **con** C and t **con** C, then $\{s, t\}$ **con** C.

The following two lemmas do not have to be known for the exam

$\mathbf{ind}(M) = \{(s,t) \in T \times T \mid s \neq t$ and $\exists C \in \mathbb{C}_M : \{s,t\}$ $\mathbf{con}$ $C\}$

* **Lemma 120.** Let $N = (P, T, F, \phi_1, \phi_2)$ be a process of a contact-free EN system M and let s, t be distinct elements of T. Then:

(1) if s $\mathbf{co}_N$ t, then $(\phi(s), \phi(t)) \in \mathbf{ind}(M)$,

(2) if $s^\bullet \cap {}^\bullet t \neq \varnothing$, then $(\phi(s), \phi(t)) \in \mathbf{dep}(M)$.

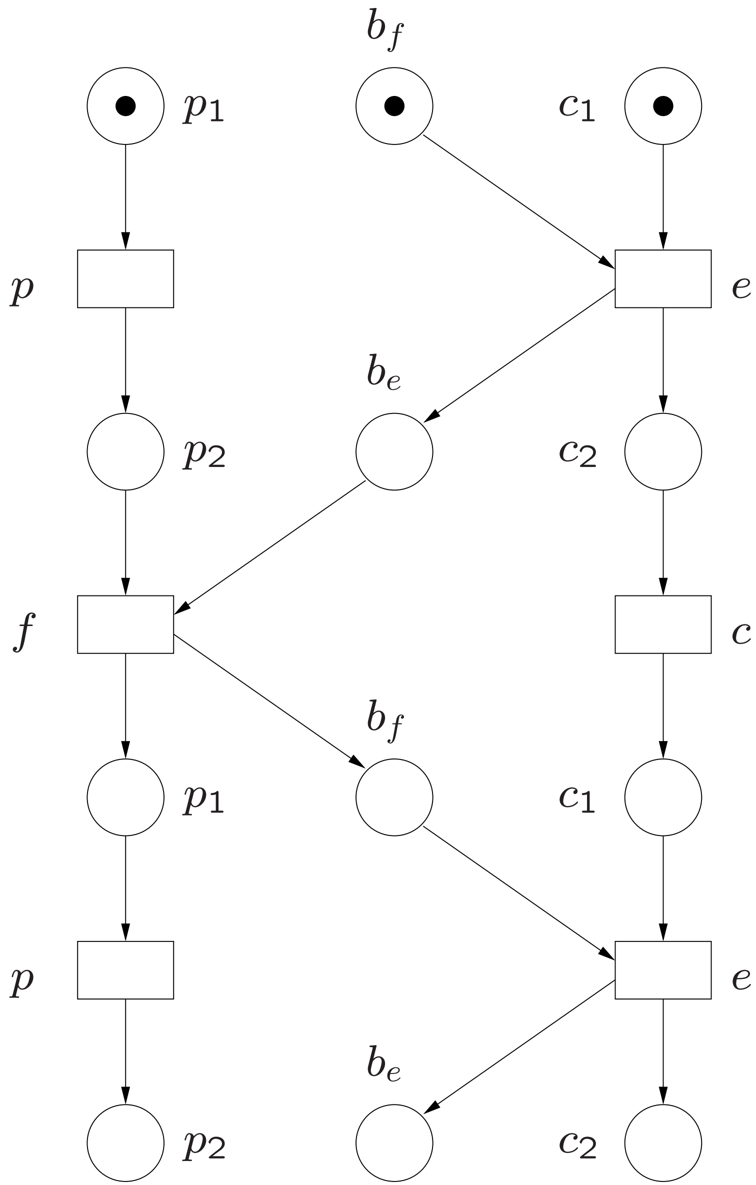
* **Lemma 121.** Let $N = (P, T, F, \phi_1, \phi_2)$ be a process of a contact-free EN system M.
Let ${}^\circ N[s_1 \cdots s_n\rangle N^\circ$ with $T = \{s_1, \ldots, s_n\}$, and let $G = \mathbf{dep}_M(\phi(s_1) \cdots \phi(s_n))$.

Then for all $1 \leq i, j \leq n$:
$(s_i, s_j) \in F_N^+$ iff $(v_i, v_j) \in \Gamma_G^+$.

Theorem 122. Let $N = (P, T, F, \phi_1, \phi_2)$ be a process of a contact-free EN system M and let ${}^\circ N[s_1 \cdots s_n\rangle N^\circ$.
Let β be the identity on $\mathbf{use}(T_M)$.

Then $\mathbf{pru}(\mathbf{ctr}(N)) \equiv_\beta \mathbf{pru}(\mathbf{dep}_M(\phi(s_1) \cdots \phi(s_n)))$.

The proof of Theorem 122 does not have to be known for the exam



b_f
p_1
c_1
p
e
b_e
p_2
c_2
f
c
b_f
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b_e
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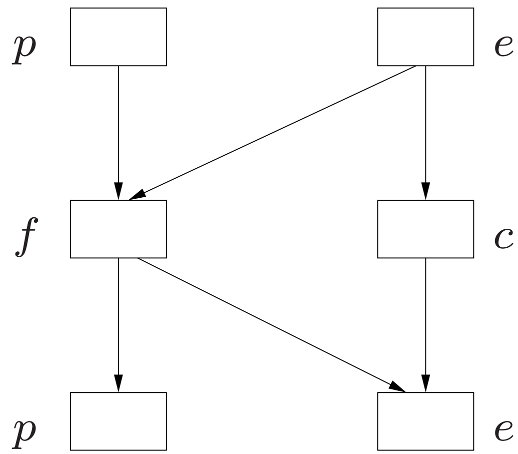
c_2



or

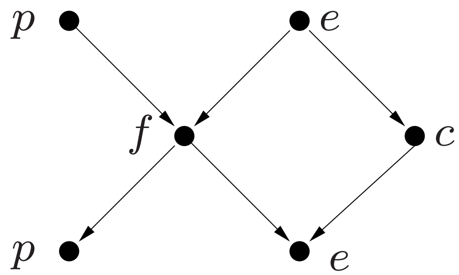


Fig. 62. Pruned/contracted version.

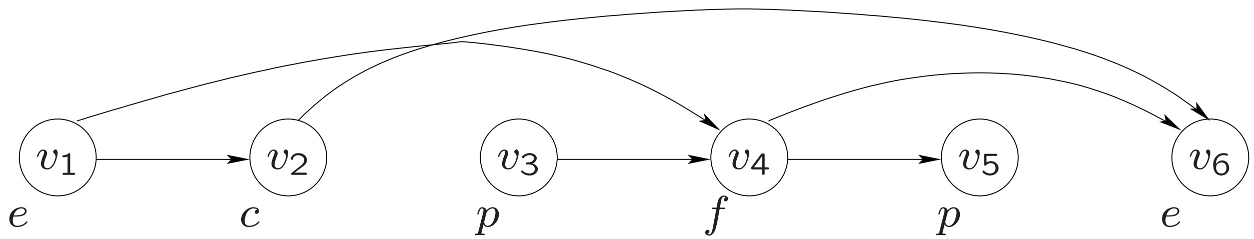


Fig. 68. A pruned dependency graph.

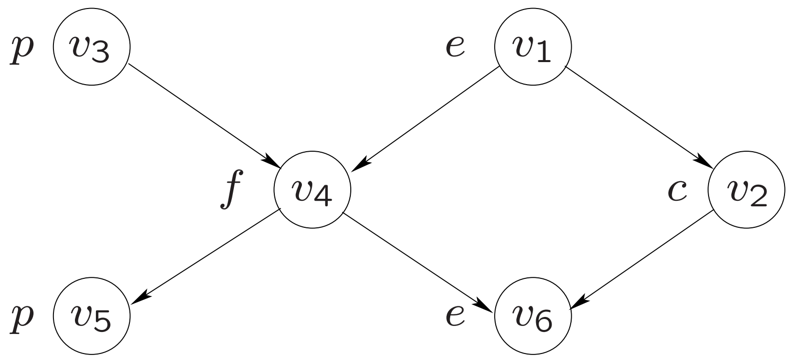


Fig. 66. An acyclic labelled graph.

Example of topological order:

$v_1 v_2 v_3 v_4 v_5 v_6$ gives word $ecpfpe$

Theorem 122. Let $N = (P, T, F, \phi_1, \phi_2)$ be a process of a contact-free EN system M and let ${}^\circ N[s_1 \cdots s_n\rangle N^\circ$.
Let β be the identity on $\mathbf{use}(T_M)$.

Then $\mathbf{pru}(\mathbf{ctr}(N)) \equiv_\beta \mathbf{pru}(\mathbf{dep}_M(\phi(s_1) \cdots \phi(s_n)))$.

Theorem 123. Let M be a contact-free EN system, and let β be the identity on $\mathbf{use}(T_M)$.

Then $\mathsf{LPO}(M) \equiv_\beta \{\mathbf{pru}(\mathbf{dep}_M(x)) \mid x \in \mathsf{FS}(M)\}$.

Theorem 92. Let $M = (P, T, F, C_{in})$ be a contact-free EN system, let $t_1, \ldots, t_n$ be transitions in T, and let $C \subseteq P$. Then

$C_{in}[t_1 \cdots t_n\rangle_M C$ iff

there exists a process $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ of M and there exist transitions $s_1, \ldots, s_n$ in T_N such that

(1) $\phi(s_i) = t_i$ for $1 \leq i \leq n$,
(2) $\phi(N^\circ) = C$, and
(3) ${}^\circ N[s_1 \cdots s_n\rangle_N N^\circ$.

Lemma 124. Let M and M' be EN systems and let $\beta : \mathbf{use}(T_M) \rightarrow \mathbf{use}(T_{M'})$ be a bijection.

If $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$ then

$$\{\mathbf{pru}(\mathbf{dep}_M(x)) \mid x \in \mathsf{FS}(M)\} \equiv_\beta \{\mathbf{pru}(\mathbf{dep}_{M'}(x)) \mid x \in \mathsf{FS}(M')\}.$$

Theorem 125. Let M and M' be two contact-free EN systems and let β be a bijection from $\mathbf{use}(T_M)$ to $\mathbf{use}(T_{M'})$.

If $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$ then $\mathsf{LPO}(M) \equiv_\beta \mathsf{LPO}(M')$.

Theorem 126. Two contact-free EN systems are lpo-equivalent iff they are firing sequence equivalent.

Corollary 127. If two contact-free EN systems are configuration equivalent, then they are lpo-equivalent.

Corollary 128. There is an algorithm that, for two arbitrary contact-free EN systems M and M', decides whether or not M and M' are lpo-equivalent.

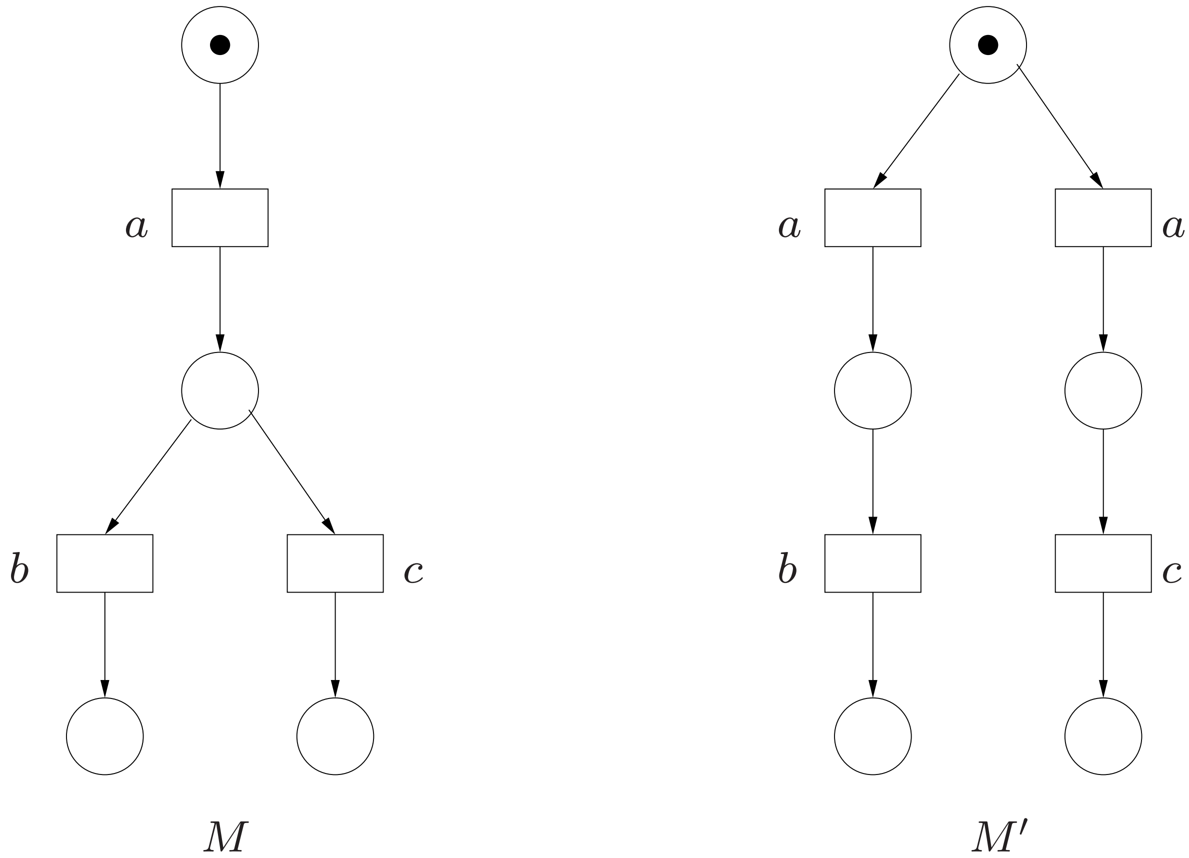


Fig. 69. Two labelled EN systems M and M' that are lpo-equivalent but not weakly configuration equivalent.

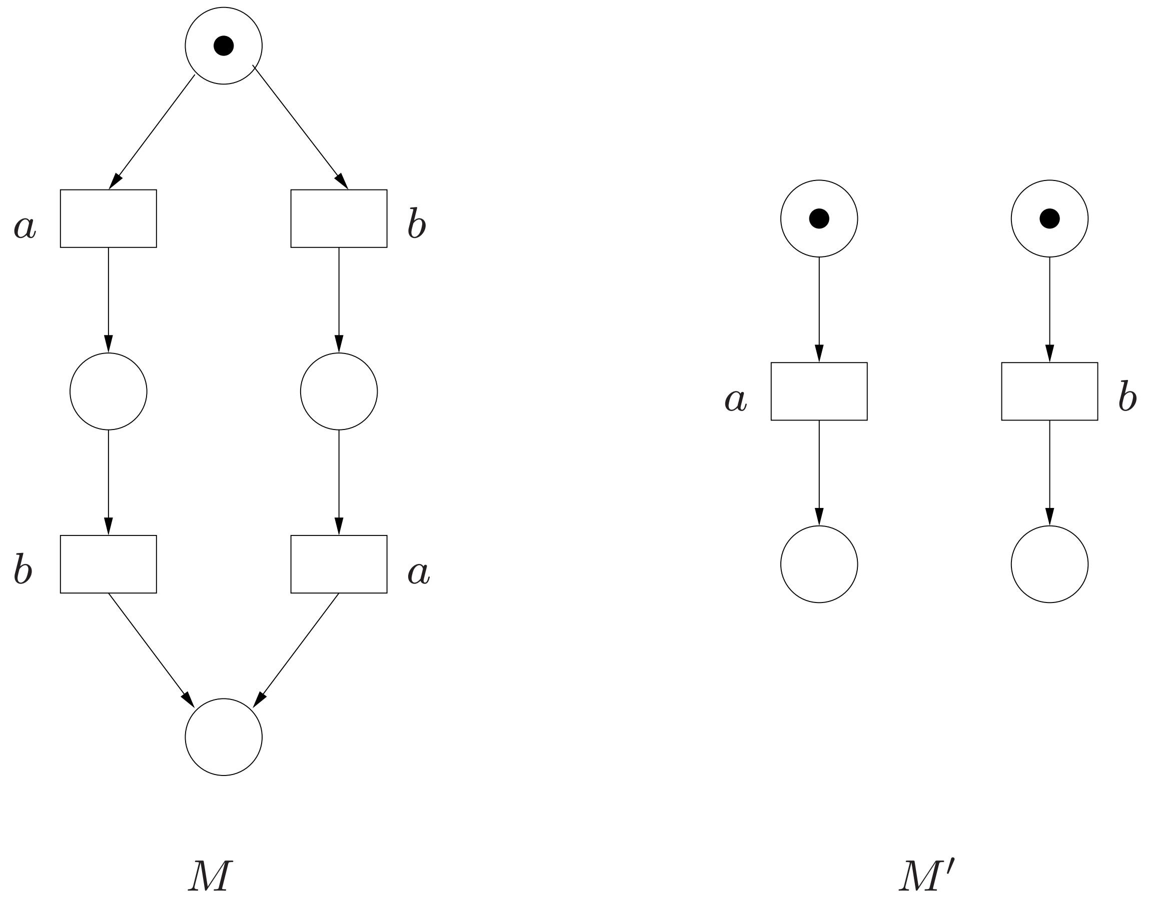
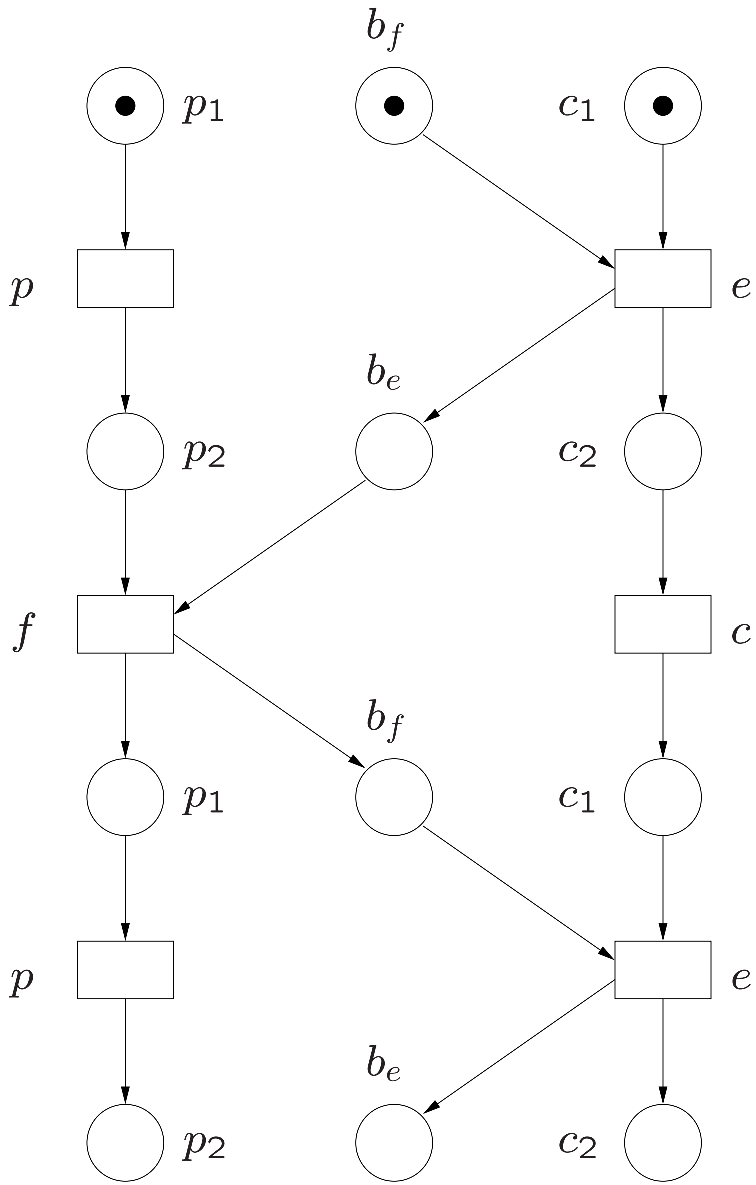


Fig. 70. Two labelled EN systems M and M' that are configuration equivalent but not lpo-equivalent.

7.2. Equivalence of Firing Sequences

Definition 129. Let M be a contact-free EN system and let $x, x' \in \mathsf{FS}(M)$.

Then x and x' are *lpo-equivalent*, $x \approx_{lpo} x'$,
if there exist a process N of M and two firing sequences $y, y' \in T_N^*$ such that
$^\circ N[y\rangle N^\circ$, $\phi_N(y) = x$, $^\circ N[y'\rangle N^\circ$, and $\phi_N(y') = x'$.



b_f
p_1
c_1
p
e
b_e
p_2
c_2
f
c
b_f
p_1
c_1
p
e
b_e
p_2
c_2

Theorem 130. Let M be a contact-free EN system and let $x, x' \in \mathsf{FS}(M)$. Then

$x \approx_{lpo} x'$ iff $\mathbf{pru}(\mathbf{dep}_M(x)) \equiv_\beta \mathbf{pru}(\mathbf{dep}_M(x'))$,

where β is the identity on $\mathbf{use}(T_M)$.

“$\Longrightarrow$”

Theorem 122. Let $N = (P, T, F, \phi_1, \phi_2)$ be a process of a contact-free EN system M and let ${}^\circ N[s_1 \cdots s_n\rangle N^\circ$.
Let β be the identity on $\mathbf{use}(T_M)$.

Then $\mathbf{pru}(\mathbf{ctr}(N)) \equiv_\beta \mathbf{pru}(\mathbf{dep}_M(\phi(s_1) \cdots \phi(s_n)))$.

“$\Longleftarrow$”

Theorem 123. Let M be a contact-free EN system, and let β be the identity on $\mathbf{use}(T_M)$.

Then $\mathsf{LPO}(M) \equiv_\beta \{\mathbf{pru}(\mathbf{dep}_M(x)) \mid x \in \mathsf{FS}(M)\}$.

Theorem 112. Let $N = (P, T, F)$ be a process net and let $t_1, \ldots, t_n \in T$.

Then ${}^\circ N[t_1 \cdots t_n\rangle N^\circ$
iff

(1) all t_i are distinct,
(2) $T = \{t_1, \ldots, t_n\}$, and

(3) for all $1 \leq i, j \leq n$,
if $(t_i, t_j) \in F^+$, then $i < j$.

Definition 131. Let I be an independency relation over Σ.

The relation $\doteq_I \subseteq \Sigma^* \times \Sigma^*$ is defined as follows:

for $x, y \in \Sigma^*$,
$x \doteq_I y$ iff there exist $a, b \in \Sigma$ and $x_1, x_2 \in \Sigma^*$, such that
$x = x_1abx_2$, $y = x_1bax_2$, and $(a, b) \in I$.

The relation $\approx_I \subseteq \Sigma^* \times \Sigma^*$ is then defined as the smallest equivalence relation that contains $\doteq_I$.
If $x \approx_I y$ then x and y are *trace equivalent* (*over* I).

$[x]_I$ denotes the equivalence class of $\approx_I$ (*trace*) that contains x.

Lemma 132. Let I be an independency relation over Σ, and let $x, y \in \Sigma^*$.

(1) $x \approx_I y$ iff there exist $n \geq 0$ and $x_0, \ldots, x_n \in \Sigma^*$ such that $x_0 = x$, $x_n = y$, and $x_{i-1} \doteq_I x_i$ for all $1 \leq i \leq n$.

(2) $x \approx_I y$ implies $|x| = |y|$.

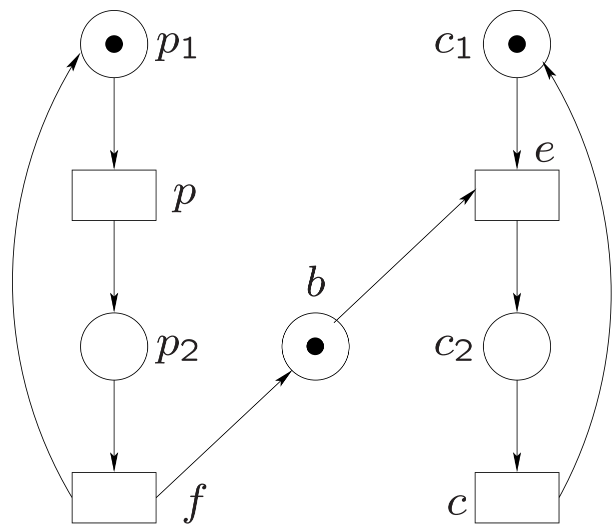


Fig. 12. The producer/consumer problem
(not contact-free!)

Lemma 133. Let $G = (V, \Gamma, \Sigma, \phi)$ be an acyclic graph. Let $J \subseteq V \times V$ be the independency relation (of G) defined by:
for all $u, w \in V$, $(u, w) \in J$ iff $u \neq w$, $(u, w) \notin \Gamma$, and $(w, u) \notin \Gamma$.

Then, for every topological order $u_1 \cdots u_n$ of G,
$\mathbf{top}(G) = \{w_1 \cdots w_n \in V^* \mid u_1 \cdots u_n \approx_J w_1 \cdots w_n\}$.

The proof of Lemma 133 does not have to be known for the exam

Theorem 134. Let I be an independency relation over Σ, and $x \in \Sigma^*$.
Then $[x]_I = \mathbf{words}(\mathbf{dep}_I(x))$.

The proof of Theorem 134 does not have to be known for the exam

Theorem 135. Let I be an independency relation over Σ, and let $x, y \in \Sigma^*$. Then the following four statements are equivalent.

(1) $x \approx_I y$,

(2) $[x]_I = [y]_I$,

(3) $\mathbf{dep}_I(x) \equiv_\beta \mathbf{dep}_I(y)$,

(4) $\mathbf{pru}(\mathbf{dep}_I(x)) \equiv_\beta \mathbf{pru}(\mathbf{dep}_I(y))$, where β is the identity on Σ.

Theorem 136. Let M be a contact-free EN system and $x, x' \in \mathsf{FS}(M)$. Then

$x \approx_{lpo} x'$ iff $x \approx_{\mathbf{ind}(M)} x'$.

A *trace language* (*over* an independency relation I) is a set of equivalence classes of $\approx_I$.

$\mathsf{TR}(M) = \{[x]_{\mathbf{ind}(M)} \mid x \in \mathsf{FS}(M)\}$, i.e., the language $\mathsf{FS}(M)$ in which trace equivalent words are grouped together.

Trace equivalence coincides with lpo-equivalence.