Theorie van Concurrency

najaar 2011

`http://www.liacs.nl/home/rvvliet/tvc/`

twaalfde college: 27 oktober 2011

vanaf volgende week begint college op donderdag om 10:00 uur

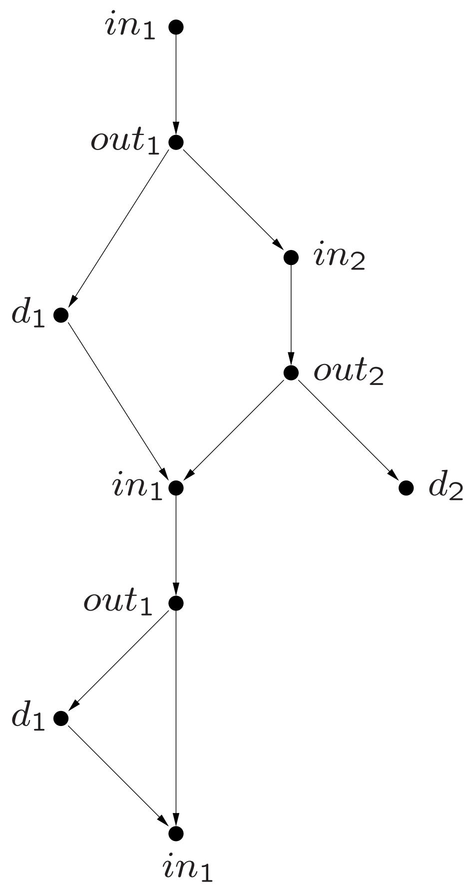
7. Comparison of Partial and Linear Order

For a contact-free EN system M
LPO(M) is the set of all pruned contracted processes of M:
LPO(M) $= \{\mathbf{pru}(\mathbf{ctr}(N)) \mid N \in \mathsf{PROC}(M)\}$

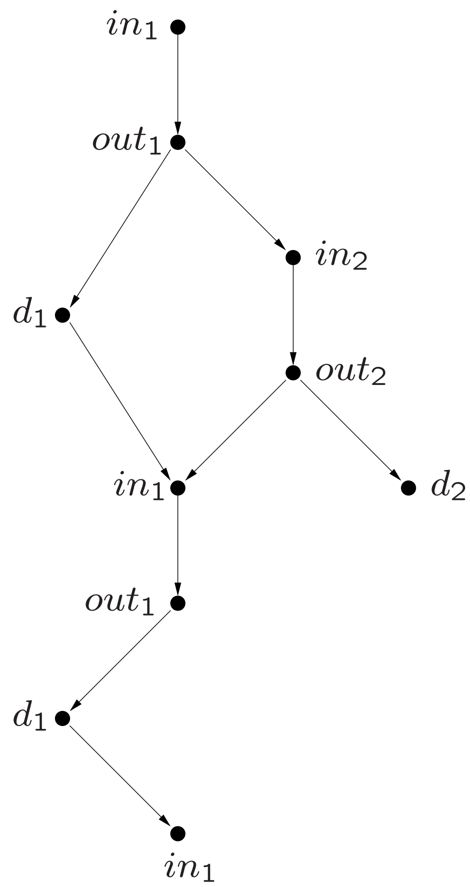
Definition 105. Two contact-free EN systems M and M' are *lpo-equivalent* if LPO(M) $\equiv$ LPO(M').

$M \equiv M'$	$\Longrightarrow$	$M \approx M'$	$\Longrightarrow$	$M \approx_w M'$	$\Longleftrightarrow$	$M \approx_{fs} M'$
isomorphic		config. equivalent		weakly config. equivalent		firing seq. equivalent

$$\text{SCG}(M) \equiv \text{SCG}(M')$$
$$\text{CG}(M) \equiv \text{CG}(M')$$



ctr(N)



pru(ctr(N))

Definition 110. A *topological order* of an acyclic labelled graph $G = (V, \Gamma, \Sigma, \phi)$ is a sequence $u_1 \cdots u_n \in V^*$,

with $u_i \in V$ for $1 \leq i \leq n$, and
(1) all u_i are distinct,
(2) $V = \{u_1, \ldots, u_n\}$, and

(3) for all $1 \leq i, j \leq n$, if $(u_i, u_j) \in \Gamma$, then $i < j$.

$\mathbf{top}(G)$ is the set of all topological orders of G.

$\phi(u_1) \cdots \phi(u_n) \in \Sigma^*$ with $u_1 \cdots u_n \in \mathbf{top}(G)$ is a *word of* G.
$\mathbf{words}(G) = \{\phi(u_1) \cdots \phi(u_n) \mid u_1 \cdots u_n \in \mathbf{top}(G)\}$.

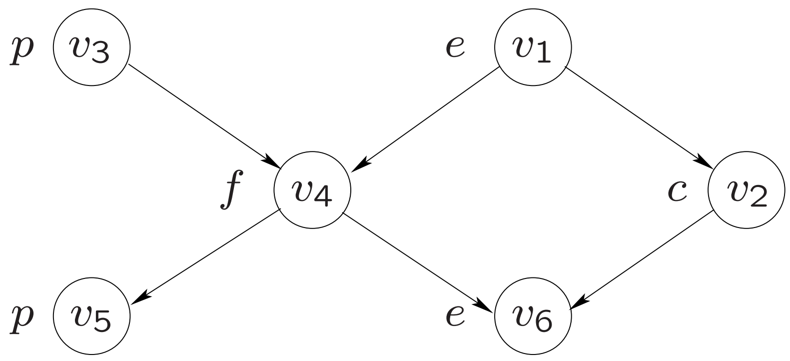


Fig. 66. An acyclic labelled graph.

Examples of topological orders:

$v_3v_1v_4v_2v_6v_5$ gives word $pefcep$

$v_1v_2v_3v_4v_5v_6$ gives word $ecpfpe$

Not a topological order:

$v_3v_4v_6v_4v_1$

$v_3v_4v_1v_2v_6v_5$

Lemma 111. Let G and G' be acyclic labelled graphs.

If $\mathbf{tra}(G) = \mathbf{tra}(G')$, then

$\mathbf{top}(G) = \mathbf{top}(G')$ and $\mathbf{words}(G) = \mathbf{words}(G')$.

Exercise:

Show that for every acyclic graph G,

$$\mathbf{top}(G) = \mathbf{top}(\mathbf{tra}(G))$$

Theorem 92. Let $M = (P, T, F, C_{in})$ be a contact-free EN system, let $t_1, \ldots, t_n$ be transitions in T, and let $C \subseteq P$. Then

$C_{in}[t_1 \cdots t_n\rangle_M C$ iff

there exists a process $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ of M and there exist transitions $s_1, \ldots, s_n$ in T_N such that

(1) $\phi(s_i) = t_i$ for $1 \leq i \leq n$,
(2) $\phi(N^\circ) = C$, and
(3) ${}^\circ N[s_1 \cdots s_n\rangle_N N^\circ$.

Lemma 113. Let $N = (P, T, F, {}^\circ N)$ be a process net, $C \subseteq P$, and $t_1, \ldots, t_n \in T$.

Then ${}^\circ N[t_1 \cdots t_n\rangle C$
iff

(1) all t_i are distinct,
(2) ${}^{\rightarrow}C \cap T = \{t_1, \ldots, t_n\}$,

(3) for all $1 \leq i, j \leq n$,
if $(t_i, t_j) \in F^+$, then $i < j$, and

(4) C is a slice.

Definition 110. A *topological order* of an acyclic labelled graph $G = (V, \Gamma, \Sigma, \phi)$ is a sequence $u_1 \cdots u_n \in V^*$,

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Lemma 113. Let $N = (P, T, F, {}^\circ N)$ be a process net, $C \subseteq P$, and $t_1, \ldots, t_n \in T$.

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(3) for all $1 \leq i, j \leq n$,
if $(t_i, t_j) \in F^+$, then $i < j$, and

(4) C is a slice.

Lemma 78 Ctd. Let $N = (P, T, F)$ be a process net.

(3b) For every slice C and every $t \in T$,
if $t^\bullet \subseteq C$, then
${}^\bullet t \cap C = \varnothing$ and
$D = (C - t^\bullet) \cup {}^\bullet t$ is a slice such that ${}^\rightarrow D = {}^\rightarrow C - t^\bullet - \{t\}$.

(4) For every slice C and every transition t,
if $t \in {}^\rightarrow C$, then
$\mathbf{nbh}(t) \subseteq {}^\rightarrow C$.

(5) For every slice $C \neq {}^\circ N$
there exists $t \in T$ such that $t^\bullet \subseteq C$.

Lemma 113. Let $N = (P, T, F, {}^\circ N)$ be a process net, $C \subseteq P$, and $t_1, \ldots, t_n \in T$.

Then ${}^\circ N[t_1 \cdots t_n\rangle C$
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(1) all t_i are distinct,
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if $(t_i, t_j) \in F^+$, then $i < j$, and

(4) C is a slice.

Theorem 112. Let $N = (P, T, F)$ be a process net and let $t_1, \ldots, t_n \in T$.

Then ${}^\circ N[t_1 \cdots t_n\rangle N^\circ$
iff

(1) all t_i are distinct,
(2) $T = \{t_1, \ldots, t_n\}$, and

(3) for all $1 \leq i, j \leq n$,
if $(t_i, t_j) \in F^+$, then $i < j$.

Theorem 112. Let $N = (P, T, F)$ be a process net and let $t_1, \ldots, t_n \in T$.

Then $^\circ N[t_1 \cdots t_n\rangle N^\circ$
iff
(1) all t_i are distinct,
(2) $T = \{t_1, \ldots, t_n\}$, and
(3) for all $1 \leq i, j \leq n$,
if $(t_i, t_j) \in F^+$, then $i < j$.

Lemma 104. Let $N = (P, T, F, \phi_1, \phi_2)$ be an acyclic (Σ_1, Σ_2)-labelled net and let $\mathbf{pru}(\mathbf{ctr}(N)) = (T, \Gamma, \Sigma_2, \phi_2)$. Then,
for all $s, t \in T$,
$(s, t) \in \Gamma^+$ iff $(s, t) \in F^+$.

Theorem 114. Let M be a contact-free EN system. Then

$$\mathsf{FS}(M) = \bigcup\{\mathbf{words}(G) \mid G \in \mathsf{LPO}(M)\}.$$

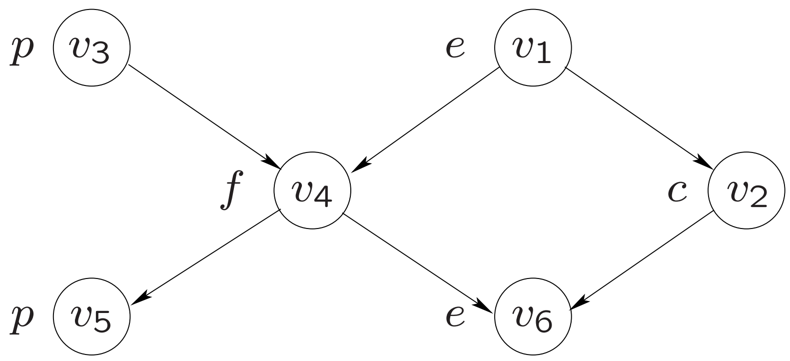

Fig. 66. An acyclic labelled graph.

Examples of topological orders:

$v_3v_1v_4v_2v_6v_5$ gives word $pefcep$

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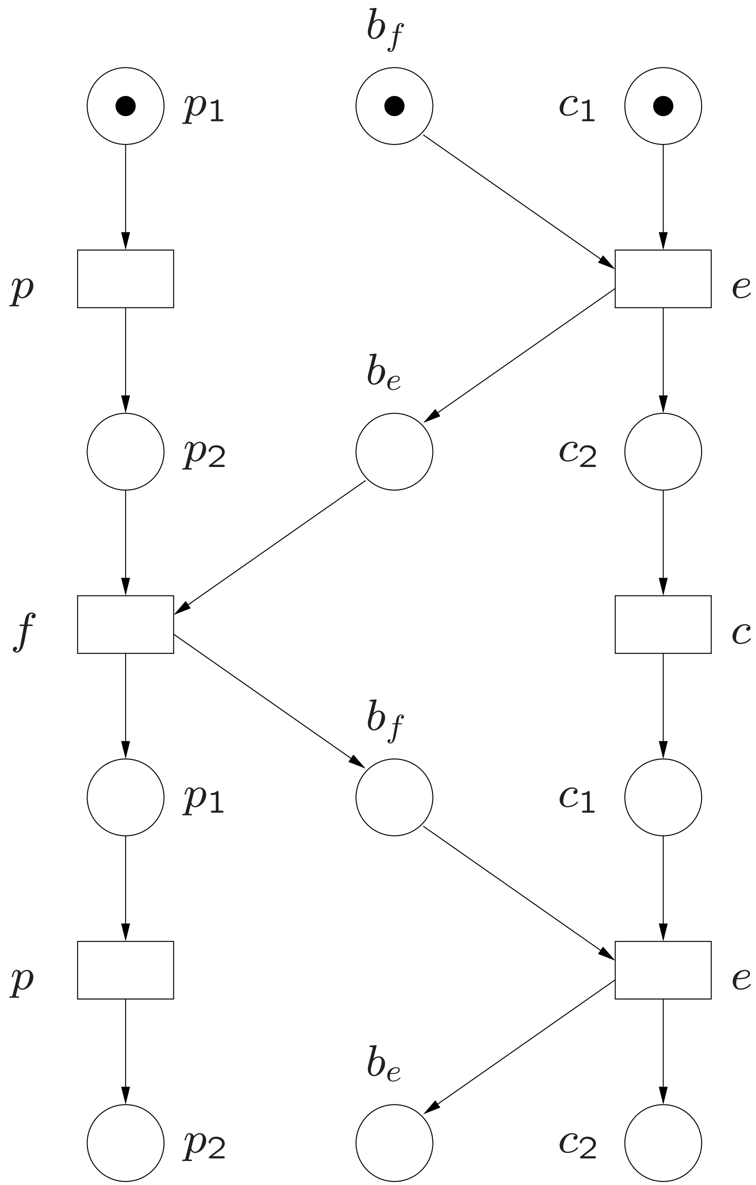
b_f
p_1
c_1
p
e
b_e
p_2
c_2
f
c
b_f
p_1
c_1
p
e
b_e
p_2
c_2

Theorem 114. Let M be a contact-free EN system. Then

$$\mathsf{FS}(M) = \bigcup\{\mathbf{words}(G) \mid G \in \mathsf{LPO}(M)\}.$$

Theorem 115. Let M and M' be two contact-free EN systems and let β be a bijection from $\mathbf{use}(T_M)$ to $\mathbf{use}(T_{M'})$.

If $\mathsf{LPO}(M) \equiv_\beta \mathsf{LPO}(M')$ then $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$.

Theorem 107. Let M be a contact-free EN system and let N, N' be two processes of M.
Let α be the identity on P_M and β the identity on $\mathbf{use}(T_M)$.

$N \equiv_\beta^\alpha N'$ iff $\mathbf{pru}(\mathbf{ctr}(N)) \equiv_\beta \mathbf{pru}(\mathbf{ctr}(N'))$.

Proof

Only-if direction: both $\mathbf{ctr}$ and $\mathbf{pru}$ preserve isomorphism.
If-direction:
Reconstruct the process N directly from the pruned contracted process $\mathbf{pru}(\mathbf{ctr}(N))$, i.e., from $(T_N, F_N^+ \cap (T_N \times T_N), T_M, \phi_{2N})$. *

Next (**Thm 125**):

M and M' contact-free EN systems.

If $\beta(\mathsf{FS}(M)) = \mathsf{FS}(M')$ then $\mathsf{LPO}(M) \equiv_\beta \mathsf{LPO}(M')$.

Definition 117. Let $M = (P, T, F, C_{in})$ be an EN system.

(1) The *independency relation of* M is the independency relation $\mathbf{ind}(M)$ over $\mathbf{use}(T)$ defined by
$\mathbf{ind}(M) = \{(s,t) \in T \times T \mid s \neq t$ and $\exists C \in \mathbb{C}_M : \{s,t\}\ \mathbf{con}\ C\}$.

(2) The *dependency relation of* M is the relation $\mathbf{dep}(M)$ defined by $\mathbf{dep}(M) = (\mathbf{use}(T) \times \mathbf{use}(T)) - \mathbf{ind}(M)$.

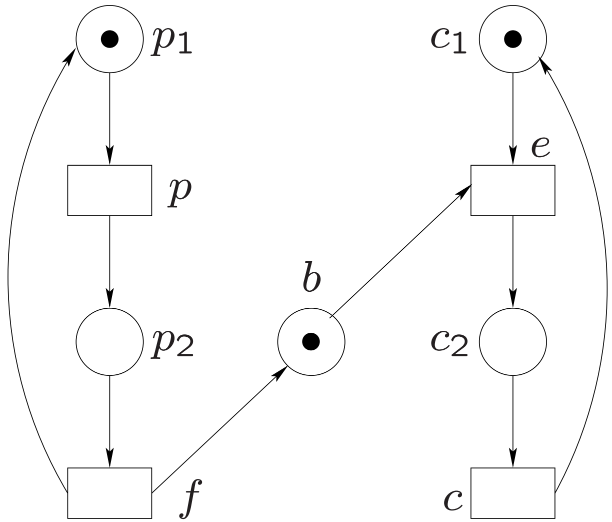

Fig. 12. The producer/consumer problem
(not contact-free!)

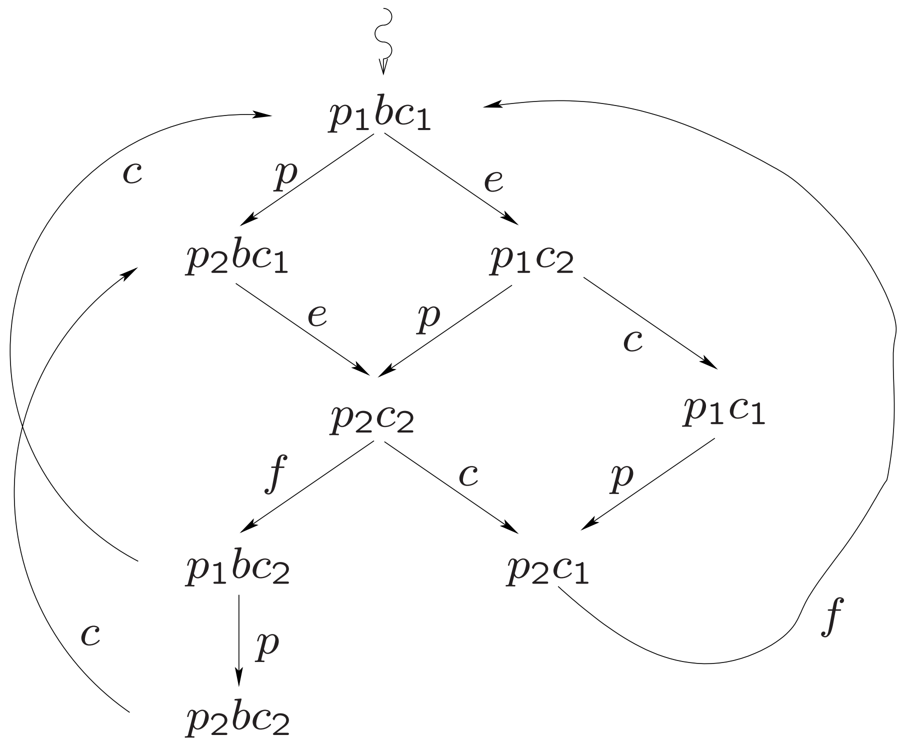

Fig. 16. A sequential configuration graph.

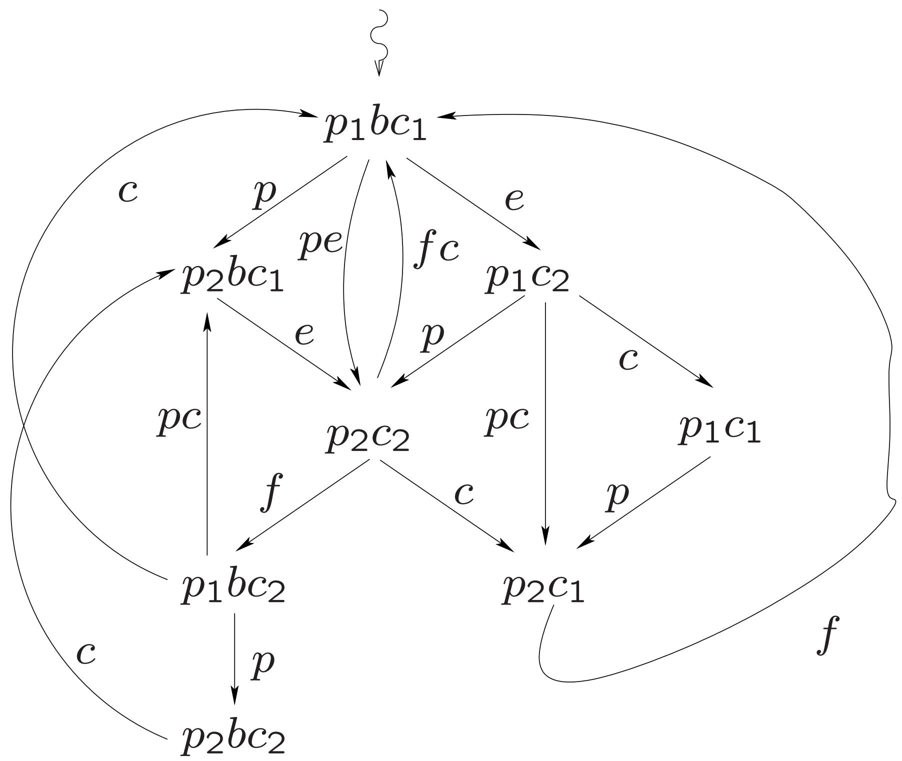

Fig. 18. A configuration graph.

Definition 116. Let Σ be an alphabet.
A relation $I \subseteq \Sigma \times \Sigma$ is an *independency relation* (*over* Σ) if I is irreflexive and symmetric.

Definition 119. Let Σ be an alphabet and I an independency relation over Σ.
Let $x = t_1 \cdots t_n \in \Sigma^*$, with $n \geq 0$ and $t_1, \ldots, t_n \in \Sigma$.

(1) The *dependency graph of* x (*over* I), denoted by $\mathbf{dep}_I(x)$, is the labelled graph $(V, \Gamma, \Sigma, \phi)$, where
$V = \{v_1, \ldots, v_n\}$,
$\phi(v_i) = t_i$ for all $1 \leq i \leq n$, and,

for all $1 \leq i, j \leq n$,
$(v_i, v_j) \in \Gamma$ iff $i < j$ and $(t_i, t_j) \notin I$.

(2) The *pruned dependency graph of* x (*over* I) is $\mathbf{pru}(\mathbf{dep}_I(x))$.

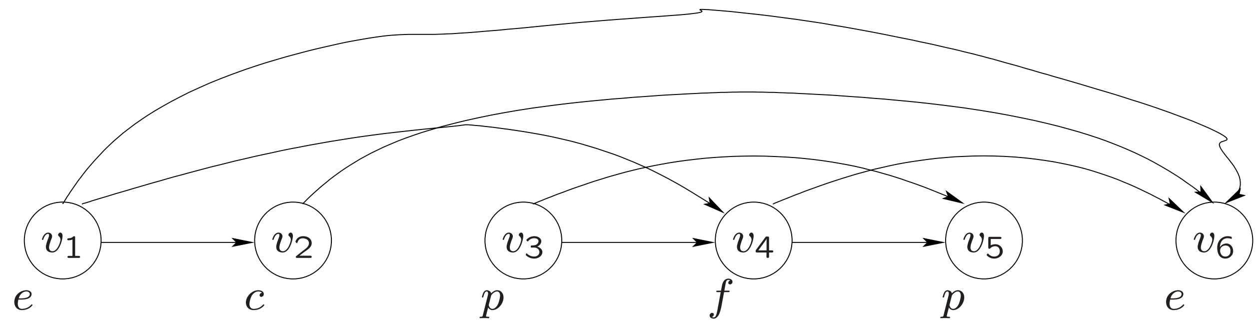

Fig. 67. A dependency graph.

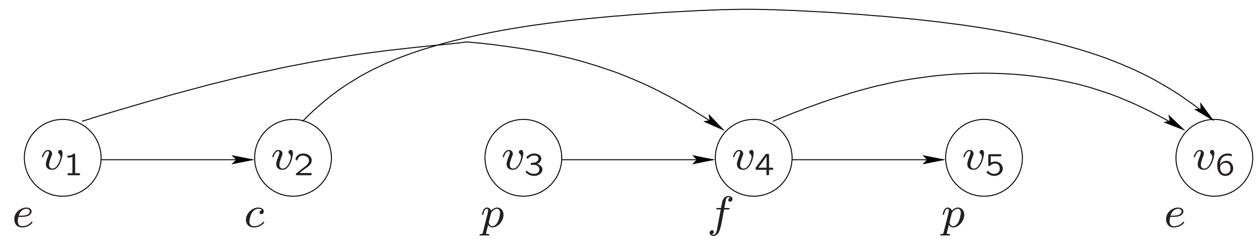

Fig. 68. A pruned dependency graph.

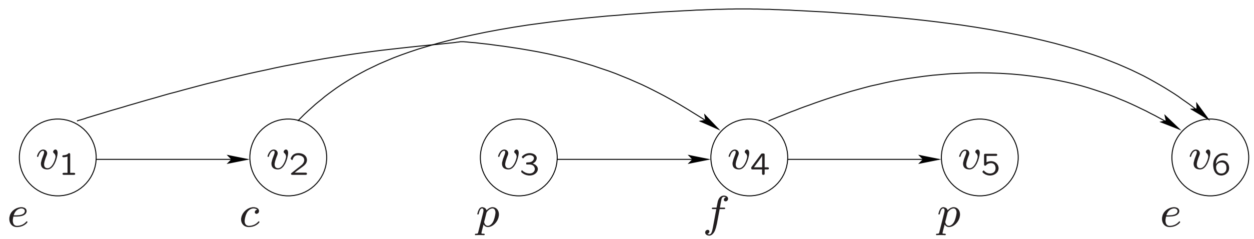

Fig. 68. A pruned dependency graph.

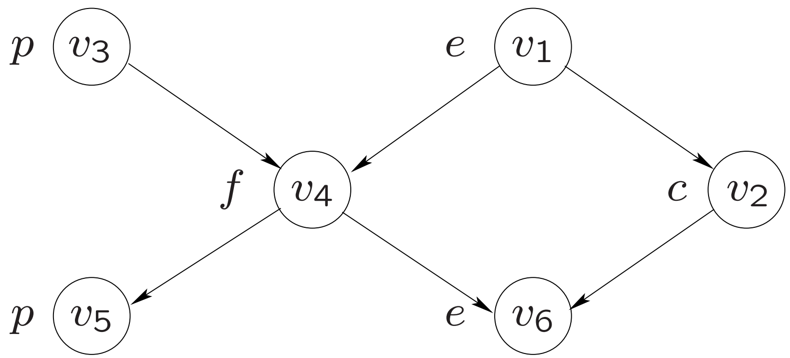

Fig. 66. An acyclic labelled graph.

Example of topological order:

$v_1 v_2 v_3 v_4 v_5 v_6$ gives word $ecpfpe$