Theorie van Concurrency

najaar 2011

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laatste college op dinsdag

6. Processes

6.3 Processes

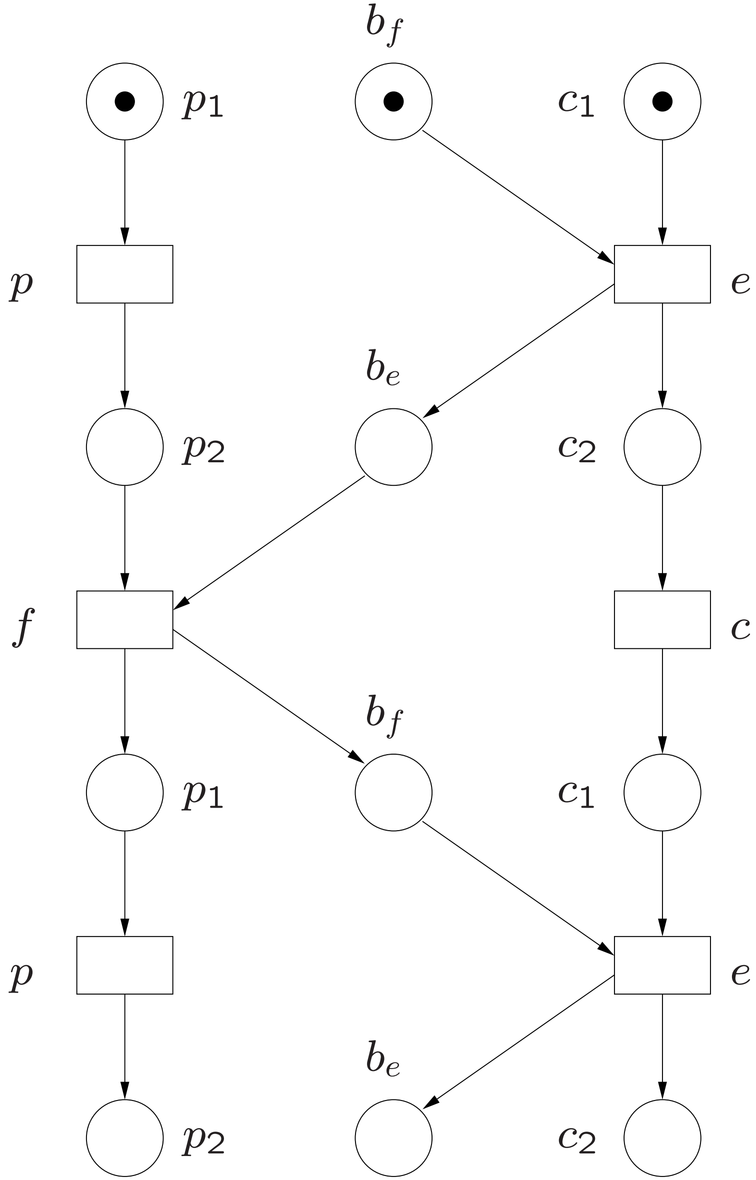
6.4 Pruned Contracted Processes

Theorem 92. Let $M = (P, T, F, C_{in})$ be a contact-free EN system, let $t_1, \ldots, t_n$ be transitions in T, and let $C \subseteq P$. Then

$C_{in}[t_1 \cdots t_n\rangle_M C$ iff

there exists a process $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ of M and there exist transitions $s_1, \ldots, s_n$ in T_N such that

(1) $\phi(s_i) = t_i$ for $1 \leq i \leq n$,
(2) $\phi(N^\circ) = C$, and
(3) ${}^\circ N[s_1 \cdots s_n\rangle_N N^\circ$.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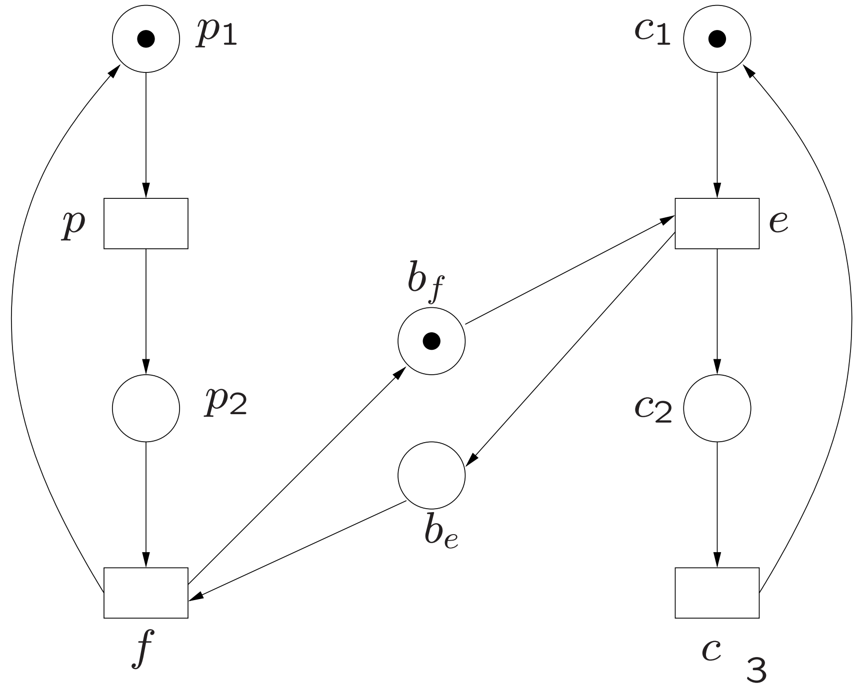
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Theorem 93. Let M be a contact-free EN system. Let $C, D \in \mathbb{C}_M$ and $t \in T_M$.

If $C[t\rangle_M D$, then

there exists a process N of M and
there exist $C', D' \in \mathbb{C}_N$ and $s \in T_N$, such that

$C'[s\rangle_N D'$,
$\phi_N(C') = C$, $\phi_N(s) = t$, and $\phi_N(D') = D$.

Theorem 94. Let M be a contact-free EN system. Let $C, D \in \mathbb{C}_M$ and let $U \subseteq T_M$.

If $C[U\rangle_M D$, then

(1) there exists a process N of M and there exist $C', D' \in \mathbb{C}_N$ and $V \subseteq T_N$, such that
$C'[V\rangle_N D'$, $\phi_N(C') = C$, $\phi_N(V) = U$, and $\phi_N(D') = D$,

(2) there exist a process N of M and a $\mathbf{co}$-clique $V \subseteq T_N$ such that $\phi_N(V) = U$.

Definition 86. $N = (P, T, F, \phi_1, \phi_2)$ and $N' = (P', T', F', \phi'_1, \phi'_2)$.

N and N' are *isomorphic*, $N \equiv N'$, if there exist bijections
$\alpha : \Sigma_1 \to \Sigma'_1$ and $\beta : \Sigma_2 \to \Sigma'_2$,
$\gamma : P \to P'$ and $\delta : T \to T'$,
such that

(1) $\mathbf{und}(N) \equiv^{\gamma}_{\delta} \mathbf{und}(N')$,

(2) for all $p \in P$, $\phi'_1(\gamma(p)) = \alpha(\phi_1(p))$, and

(3) for all $t \in T$, $\phi'_2(\delta(t)) = \beta(\phi_2(t))$.

Notation: $N \equiv^{\alpha}_{\beta} N'$.

Definition 87. Let $\mathcal{P}$ and $\mathcal{P}'$ be two sets of (Σ_1, Σ_2)-, respectively (Σ_1', Σ_2')-labelled nets.

Then $\mathcal{P}$ and $\mathcal{P}'$ are *isomorphic*, $\mathcal{P} \equiv \mathcal{P}'$, if there exist bijections
$\alpha : \Sigma_1 \to \Sigma_1'$ and
$\beta : \Sigma_2 \to \Sigma_2'$, such that

(1) for every $N \in \mathcal{P}$ there exists $N' \in \mathcal{P}'$ such that $N \equiv_\beta^\alpha N'$, and

(2) for every $N' \in \mathcal{P}'$ there exists $N \in \mathcal{P}$ such that $N \equiv_\beta^\alpha N'$.

Theorem 95. Let M and M' be two contact-free reduced EN systems.

Then PROC(M) $\equiv$ PROC(M') iff $M \equiv M'$.

6.4. Pruned contracted processes

Definition 96. Let Σ be an alphabet.
A (Σ-)*labelled graph* is a quadruple $G = (V, \Gamma, \Sigma, \phi)$, where

V is a finite set of *nodes*,
$\Gamma \subseteq V \times V$ is a set of *edges*, and
ϕ is a function from V to Σ, the *labelling of* G.

G is *acyclic* if Γ^+ is irreflexive.

Definition 97. Let $G = (V, \Gamma, \Sigma, \phi)$ and $G' = (V', \Gamma', \Sigma', \phi')$ be two Σ-, respectively Σ'-labelled graphs.

G and G' are *isomorphic*, $G \equiv G'$, if there exist bijections

$\beta : \Sigma \to \Sigma'$ and
$\delta : V \to V'$, such that

(1) for all $v, w \in V$, $(v, w) \in \Gamma$ iff $(\delta(v), \delta(w)) \in \Gamma'$, and

(2) for all $v \in V$, $\phi'(\delta(v)) = \beta(\phi(v))$.

Notation: $G \equiv_\beta G'$.

Definition 98. Let $\mathcal{P}$ and $\mathcal{P}'$ be two sets of Σ-, respectively Σ'-labelled graphs.

$\mathcal{P}$ and $\mathcal{P}'$ are *isomorphic*, $\mathcal{P} \equiv \mathcal{P}'$, if there exists a bijection

$\beta : \Sigma \to \Sigma'$, such that

(1) for every $G \in \mathcal{P}$ there exists $G' \in \mathcal{P}'$ such that $G \equiv_\beta G'$, and

(2) for every $G' \in \mathcal{P}'$ there exists $G \in \mathcal{P}$ such that $G \equiv_\beta G'$.

Definition 99. Transitive closure.

Definition 100. Pruned version.

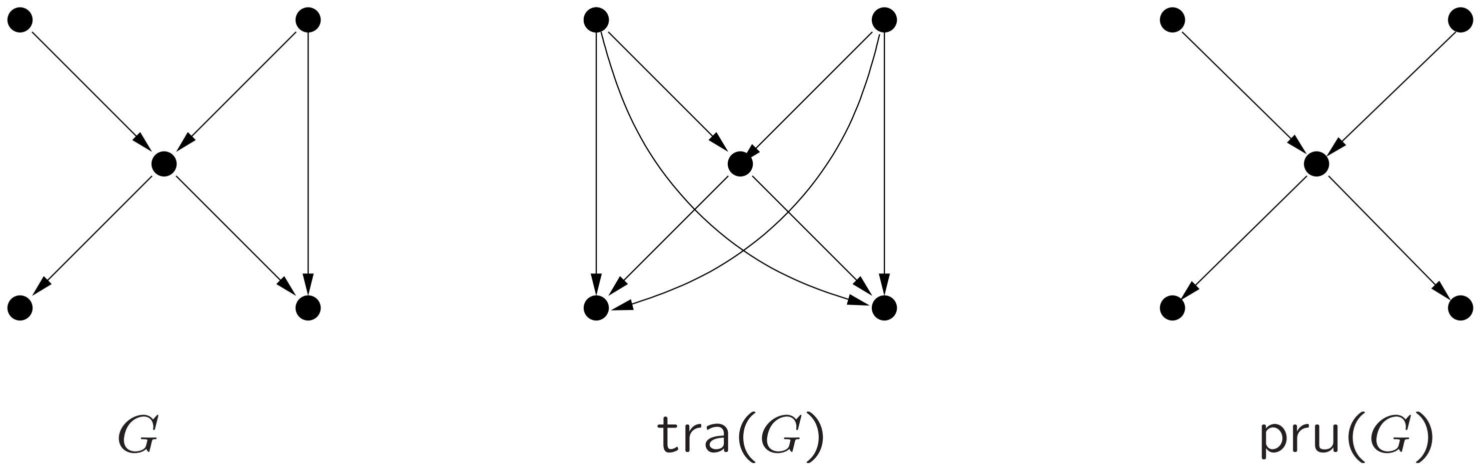


Fig. 61. A graph with
its transitive closure and its pruned version.

Theorem 101. For every acyclic labelled graph G,

$\mathbf{tra}(\mathbf{pru}(G)) = \mathbf{tra}(G)$ and $\mathbf{pru}(\mathbf{tra}(G)) = \mathbf{pru}(G)$.

Theorem 102. Let Let G and G' be two acyclic Σ-, respectively Σ'-labelled graphs and let $\beta : \Sigma \to \Sigma'$ be a bijection.

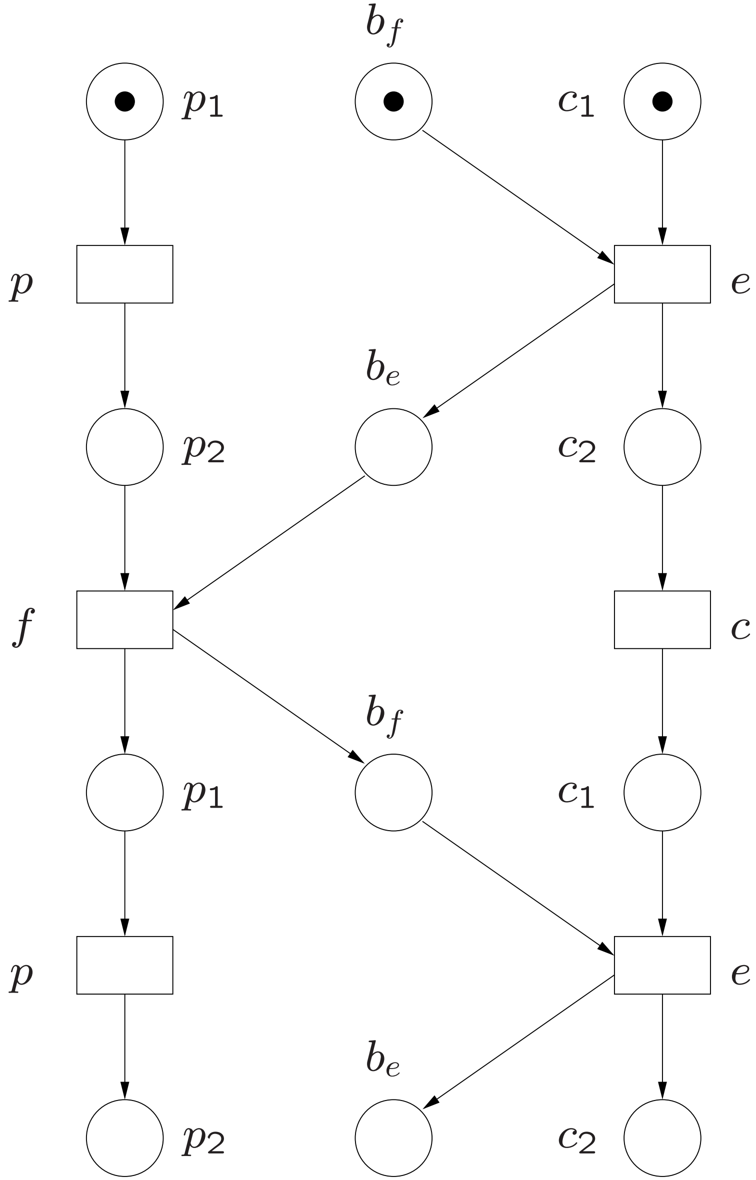
$\mathbf{tra}(G) \equiv_\beta \mathbf{tra}(G')$ iff $\mathbf{pru}(G) \equiv_\beta \mathbf{pru}(G')$.

Definition 103. Let $N = (P, T, F, \phi_1, \phi_2)$ be an acyclic (Σ_1, Σ_2)-labelled net.

(1) The *contracted version of* N, $\mathbf{ctr}(N)$, is the labelled graph $(T, \Gamma, \Sigma_2, \phi_2)$ such that,
for all $s, t \in T$,
$(s, t) \in \Gamma$ iff $s^\bullet \cap {}^\bullet t \neq \varnothing$.

(2) The *pruned contracted version of* N is the labelled graph $\mathbf{pru}(\mathbf{ctr}(N))$.

Lemma 104. Let $N = (P, T, F, \phi_1, \phi_2)$ be an acyclic (Σ_1, Σ_2)-labelled net and let $\mathbf{pru}(\mathbf{ctr}(N)) = (T, \Gamma, \Sigma_2, \phi_2)$. Then,
for all $s, t \in T$,
$(s, t) \in \Gamma^+$ iff $(s, t) \in F^+$.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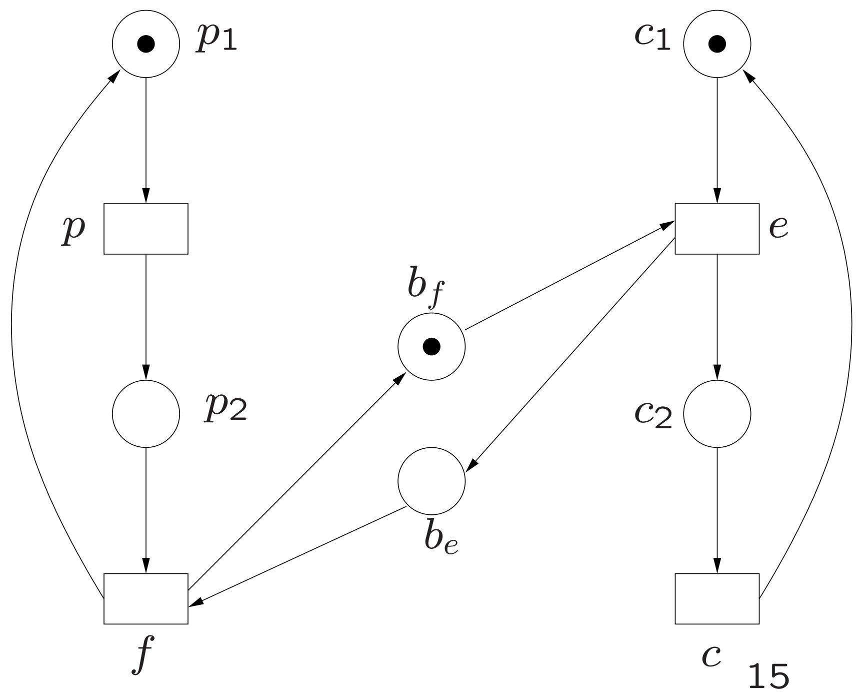
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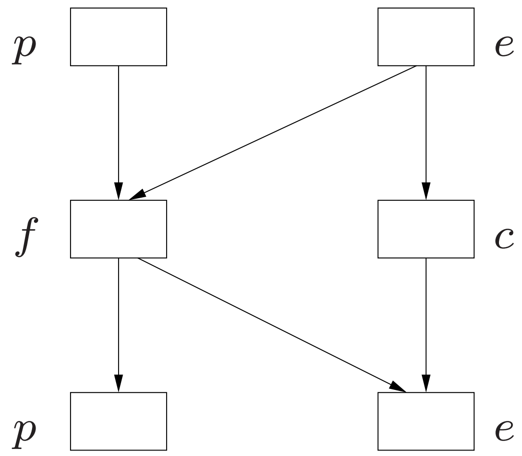


or

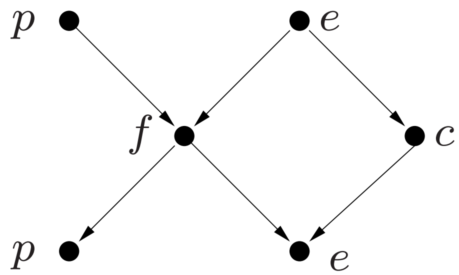


Fig. 62. Pruned/contracted version.

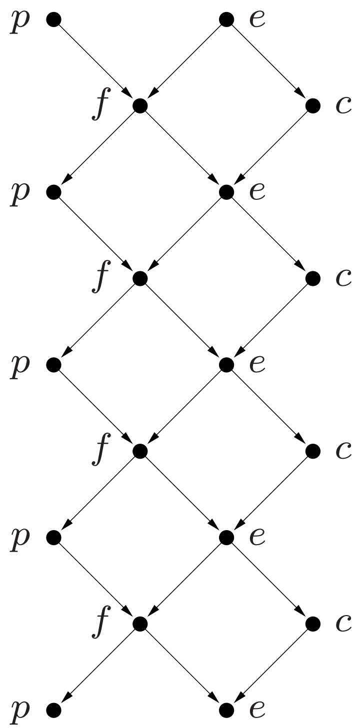


Fig. 63. Another pruned contracted process.

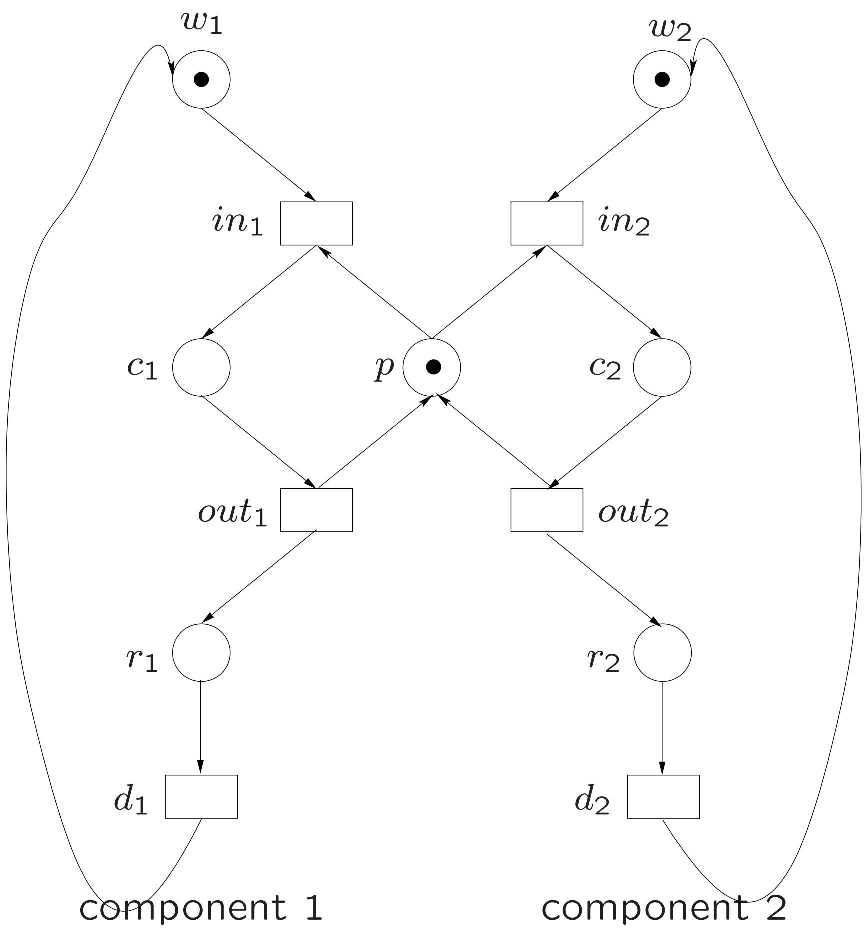
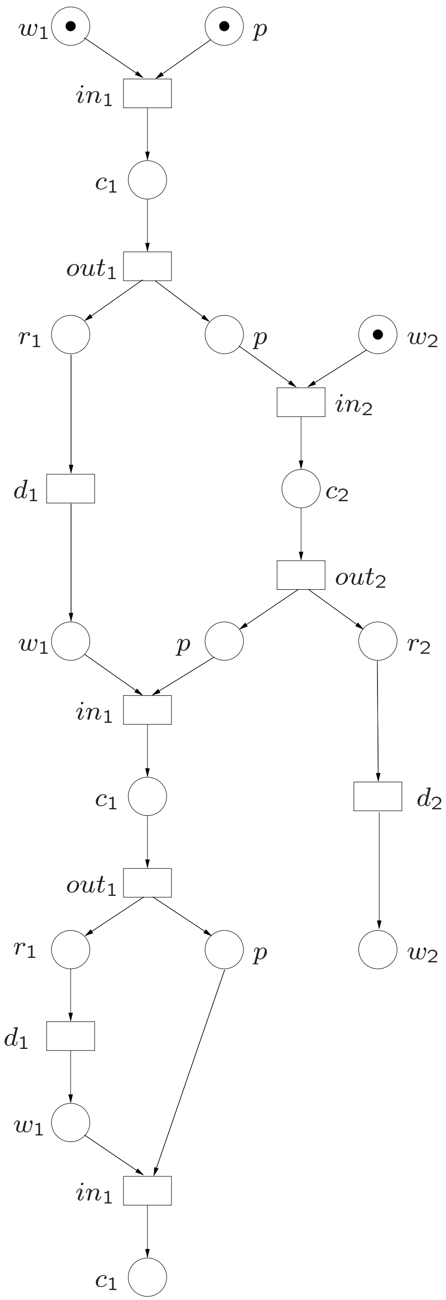


Fig. 5. The mutual exclusion problem.



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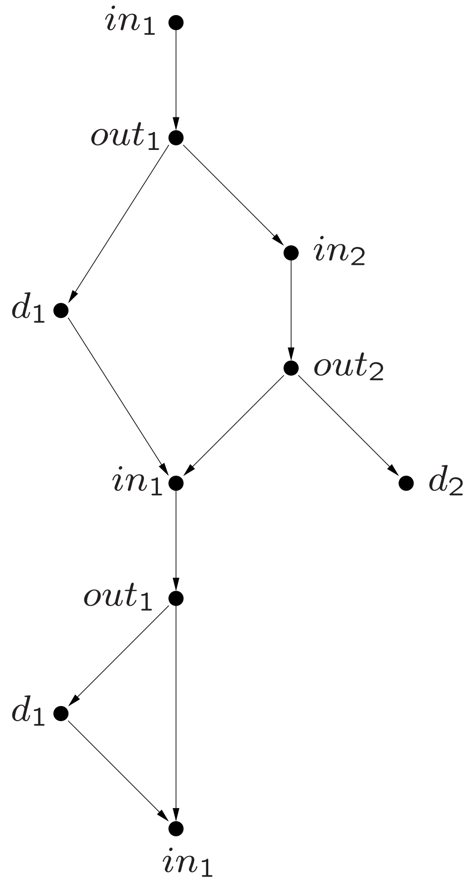
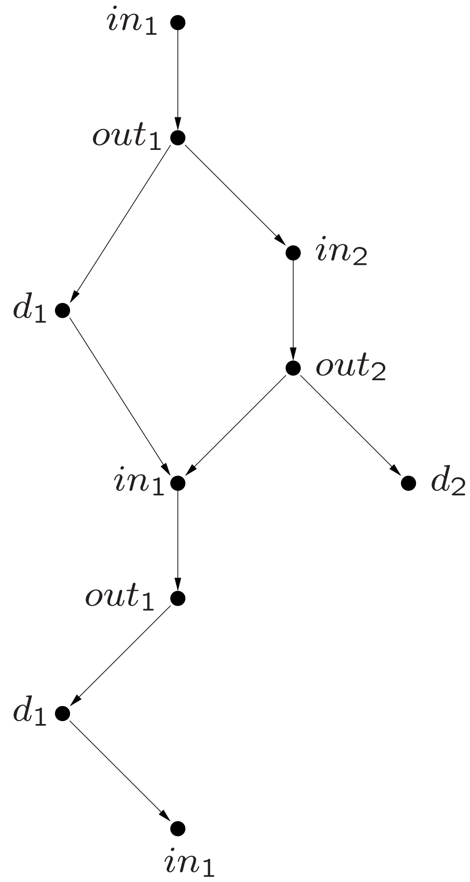


ctr(N)



pru(ctr(N))

For a contact-free EN system M
LPO(M) is the set of all pruned contracted processes of M:
LPO(M) $= \{\mathbf{pru}(\mathbf{ctr}(N)) \mid N \in$ PROC(M)$\}$

Definition 105. Two contact-free EN systems M and M' are *lpo-equivalent* if LPO(M) $\equiv$ LPO(M').

Theorem 106. There exist reduced contact-free EN systems M and M' such that LPO(M) $\equiv$ LPO(M') but $M \equiv M'$ does not hold.

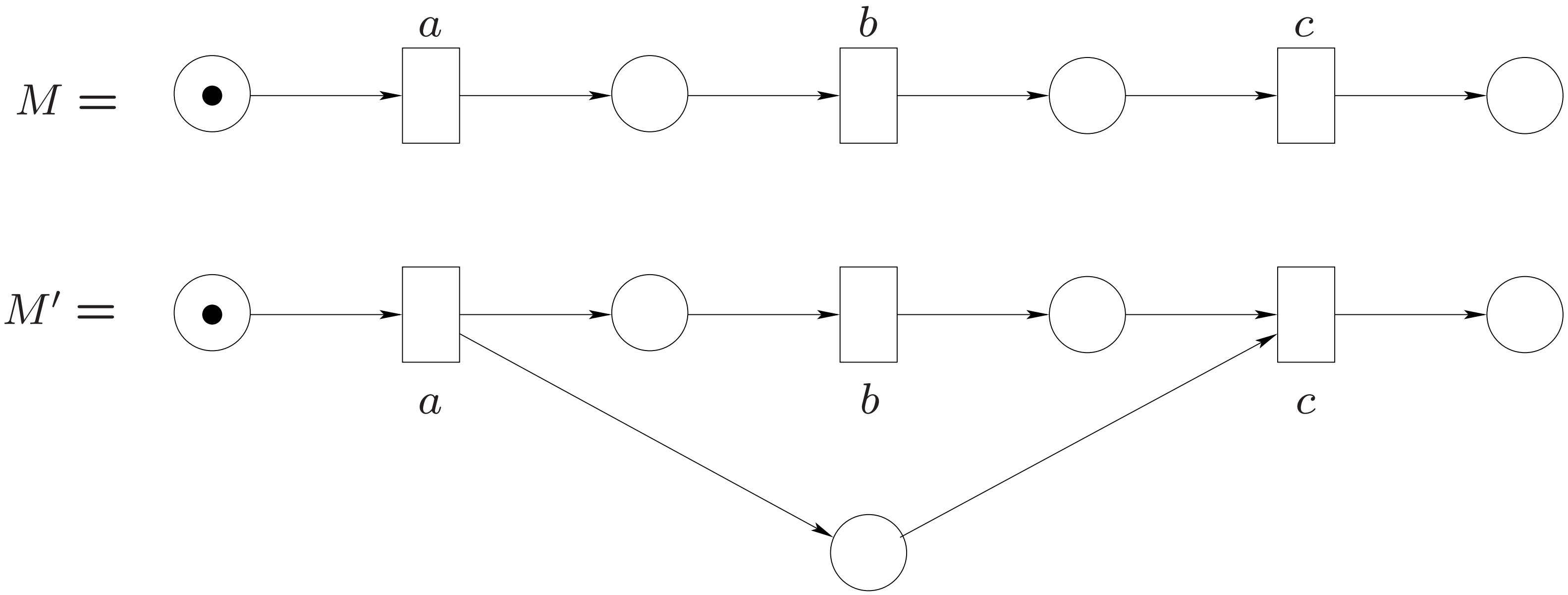


Fig. 65. Two non-isomorphic lpo-equivalent EN systems.

Theorem 107. Let M be a contact-free EN system and let N, N' be two processes of M.
Let α be the identity on P_M and β the identity on $\mathbf{use}(T_M)$.

$N \equiv_{\beta}^{\alpha} N'$ iff $\mathbf{pru}(\mathbf{ctr}(N)) \equiv_{\beta} \mathbf{pru}(\mathbf{ctr}(N'))$.

Proof

Only-if direction: both $\mathbf{ctr}$ and $\mathbf{pru}$ preserve isomorphism.
If-direction:
Reconstruct the process N directly from the pruned contracted process $\mathbf{pru}(\mathbf{ctr}(N))$, i.e., from $(T_N, F_N^+ \cap (T_N \times T_N), T_M, \phi_{2N})$. *
This (re)construction must be known for the exam, but the rest of the proof does not have to be known for the exam.

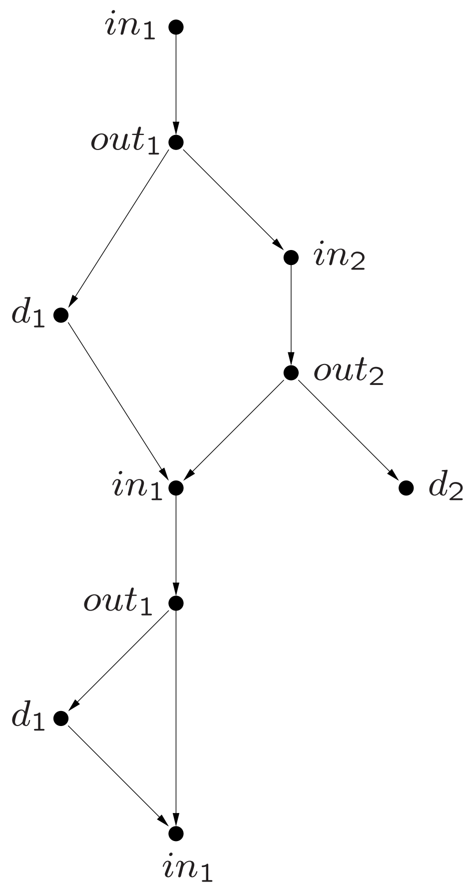
The following two lemmas do not have to be known for the exam

* **Lemma 108.** Let $N = (P, T, F, \phi_1, \phi_2)$ be a process of a contact-free EN system M and let $q_1, q_2 \in P$.

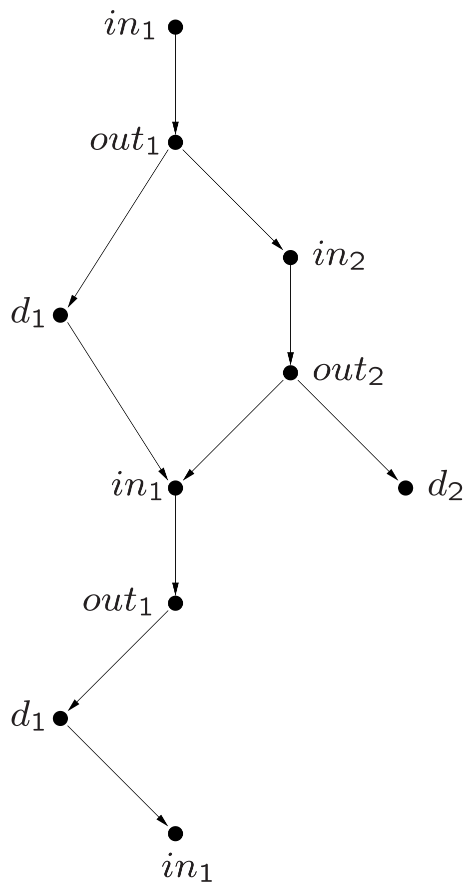
(1) For all $s_1, s_2 \in T$,
if $s_1 \in {}^\bullet q_1$, $s_2 \in q_2{}^\bullet$, $(s_1, s_2) \in F^+$, and $\phi(q_1) = \phi(q_2)$,
then $(q_1, q_2) \in F^*$.

(2) If ${}^\bullet q_1 = \varnothing$ and $\phi(q_1) = \phi(q_2)$,
then $(q_1, q_2) \in F^*$.

* **Lemma 109.** The position of the places of $N = (P, T, F, \phi_1, \phi_2)$ can be determined from $(T_N, F_N^+ \cap (T_N \times T_N), T_M, \phi_{2N})$.



ctr(N)



pru(ctr(N))

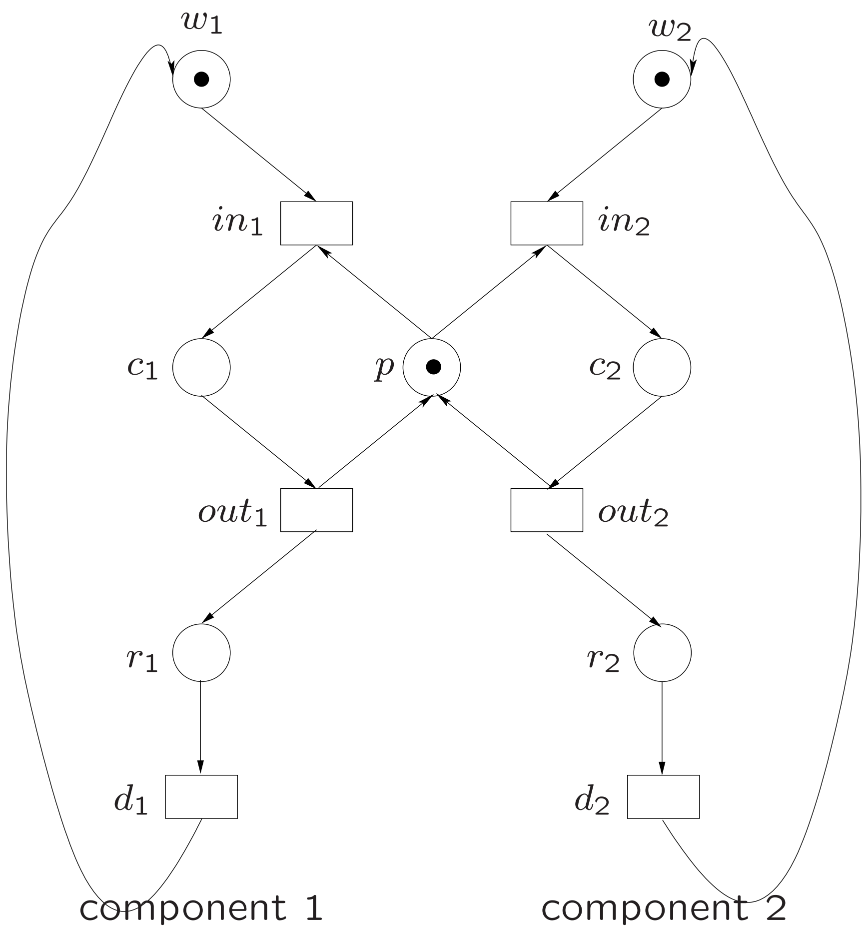
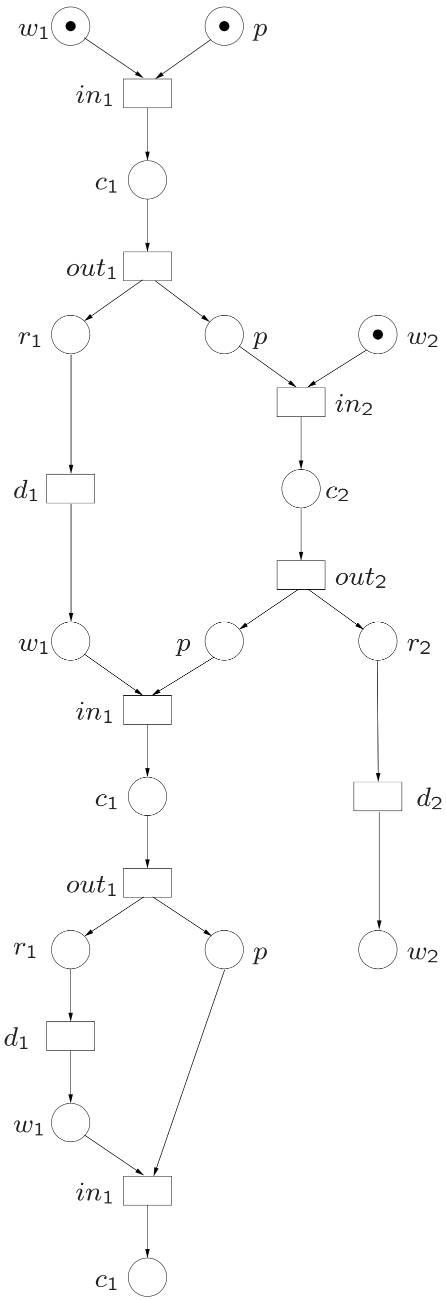


Fig. 5. The mutual exclusion problem.



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