Theorie van Concurrency

najaar 2011

http://www.liacs.nl/home/rvvliet/tvc/

tiende college: 18 oktober 2011

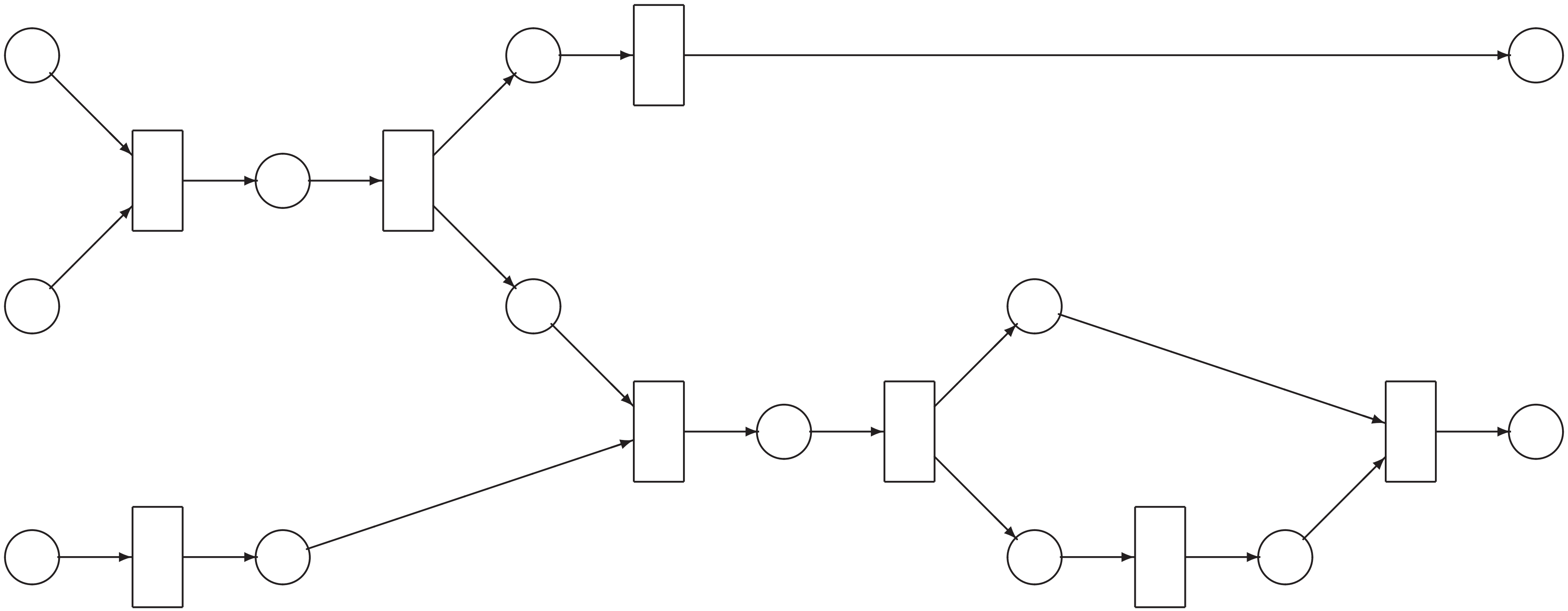
6. Processes

6.2 Process Nets

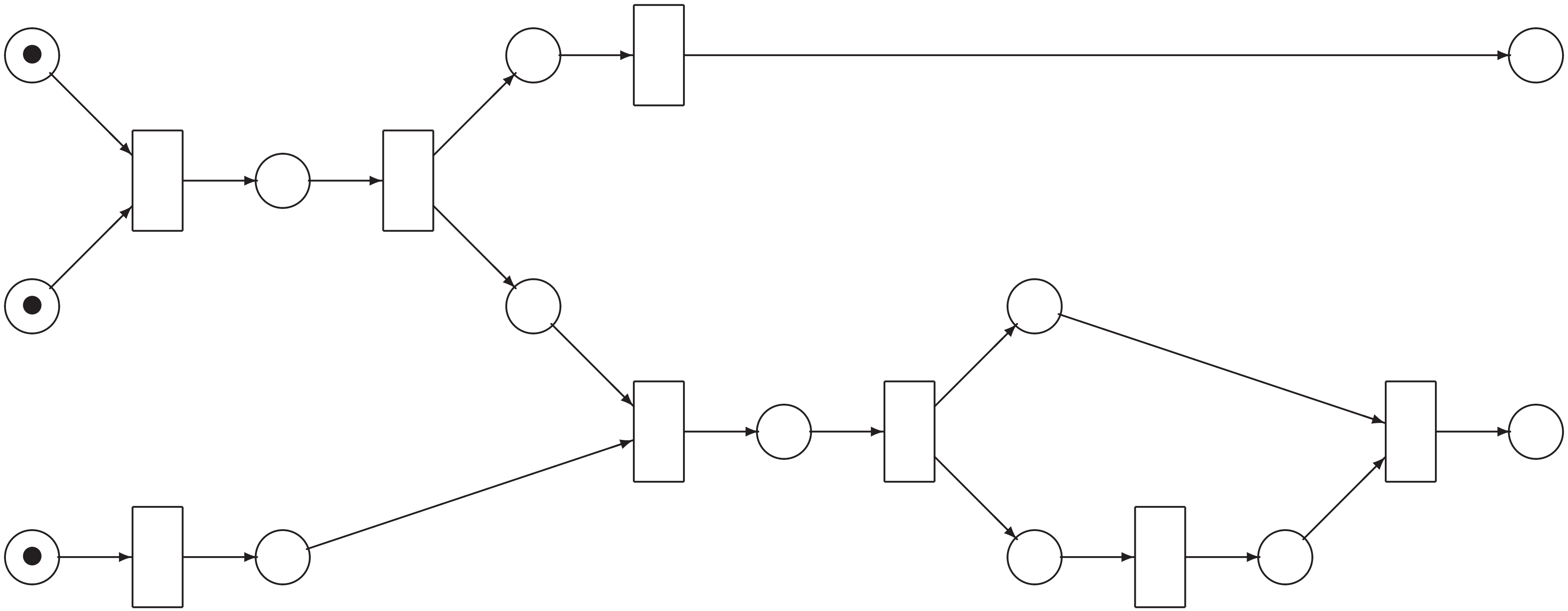
6.3 Processes

6.4 Pruned Contracted Processes

Vierde werkcollege: 20 oktober 2011



A process net.



A process net as an EN system.

Theorem 79. Let $N = (P, T, F, {}^\circ N)$ be a process net and let $C \subseteq P$.

$C \in \mathbb{C}_N$ iff C is a slice of N.

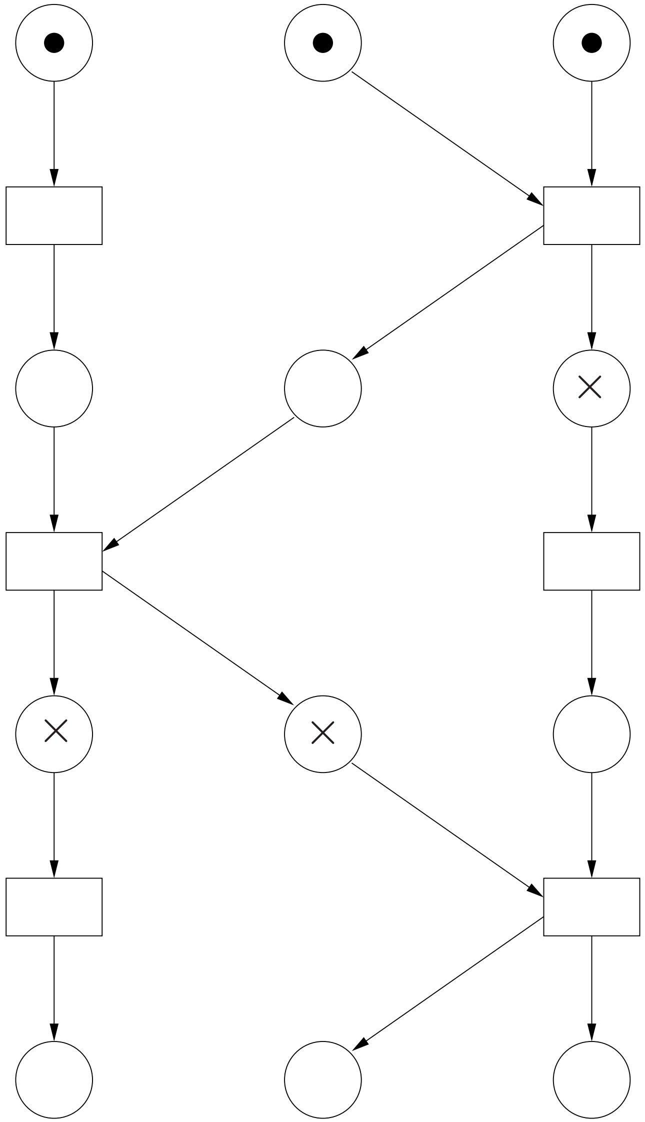


Fig. 52. A process net, reachable configurations.

Lemma 82. NEW VERSION
Let $N = (P, T, F, {}^\circ N)$ be a process net with $P \neq \varnothing$,
$L \subseteq P \cup T$.

L is a line of N iff

there exist $x_1, \ldots, x_k$ such that
$L = \{x_1, \ldots, x_k\}$
$x_1 \in {}^\circ N$ and $x_k \in N^\circ$
$x_i F x_{i+1}$ for every $1 \leq i \leq k-1$

Theorem 83. Let $N = (P, T, F, {}^\circ N)$ be a process net with $P \neq \varnothing$.

(1) If M is a sequential component of N,
then $P_M \cup T_M$ is a line of N.

(2) If L is a line of N,
then $(L \cap P, L \cap T, (L \times L) \cap F, L \cap {}^\circ N)$
is a sequential component of N.

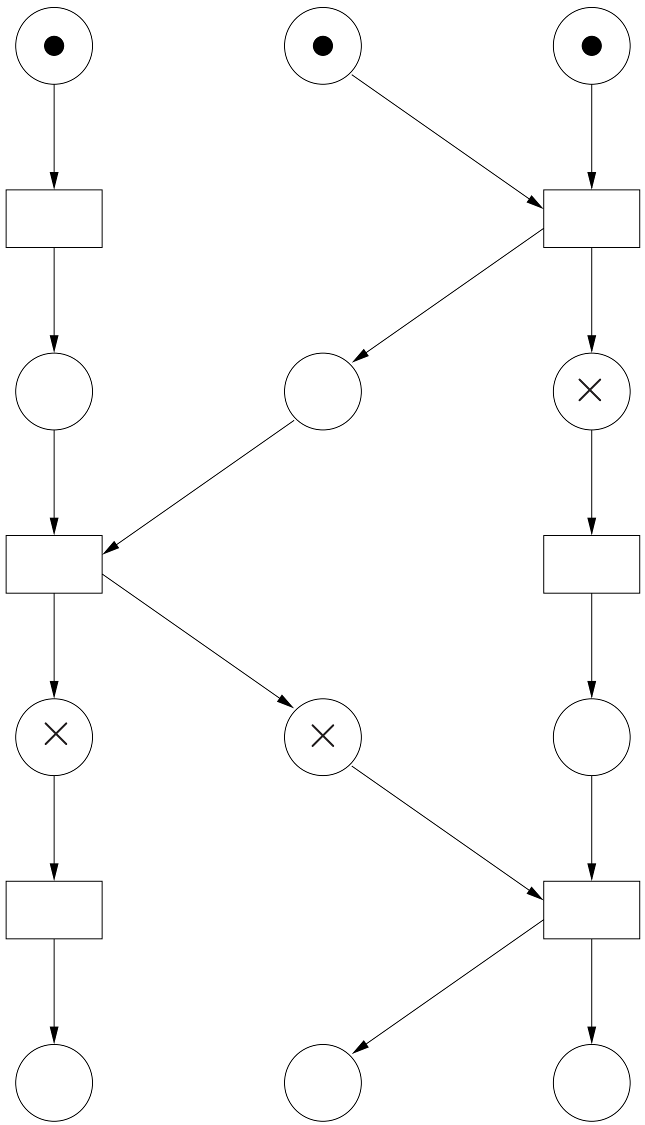
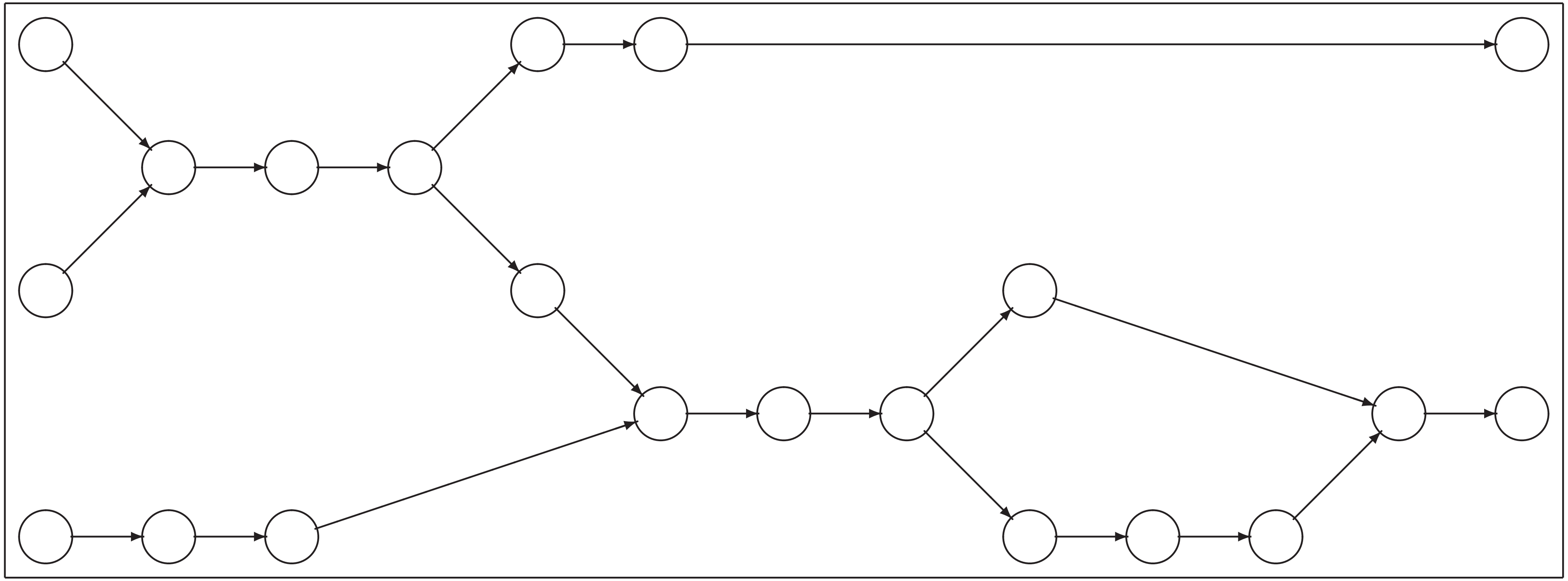


Fig. 52. A process net, covered by sequential components.



Definition 68. Let (A, ρ) be a partially ordered set.

The ordering ρ is *dense* if every line and every cut of ρ have a nonempty intersection.

For every line L and every slice C, $L \cap U \neq \varnothing$ (almost dense).

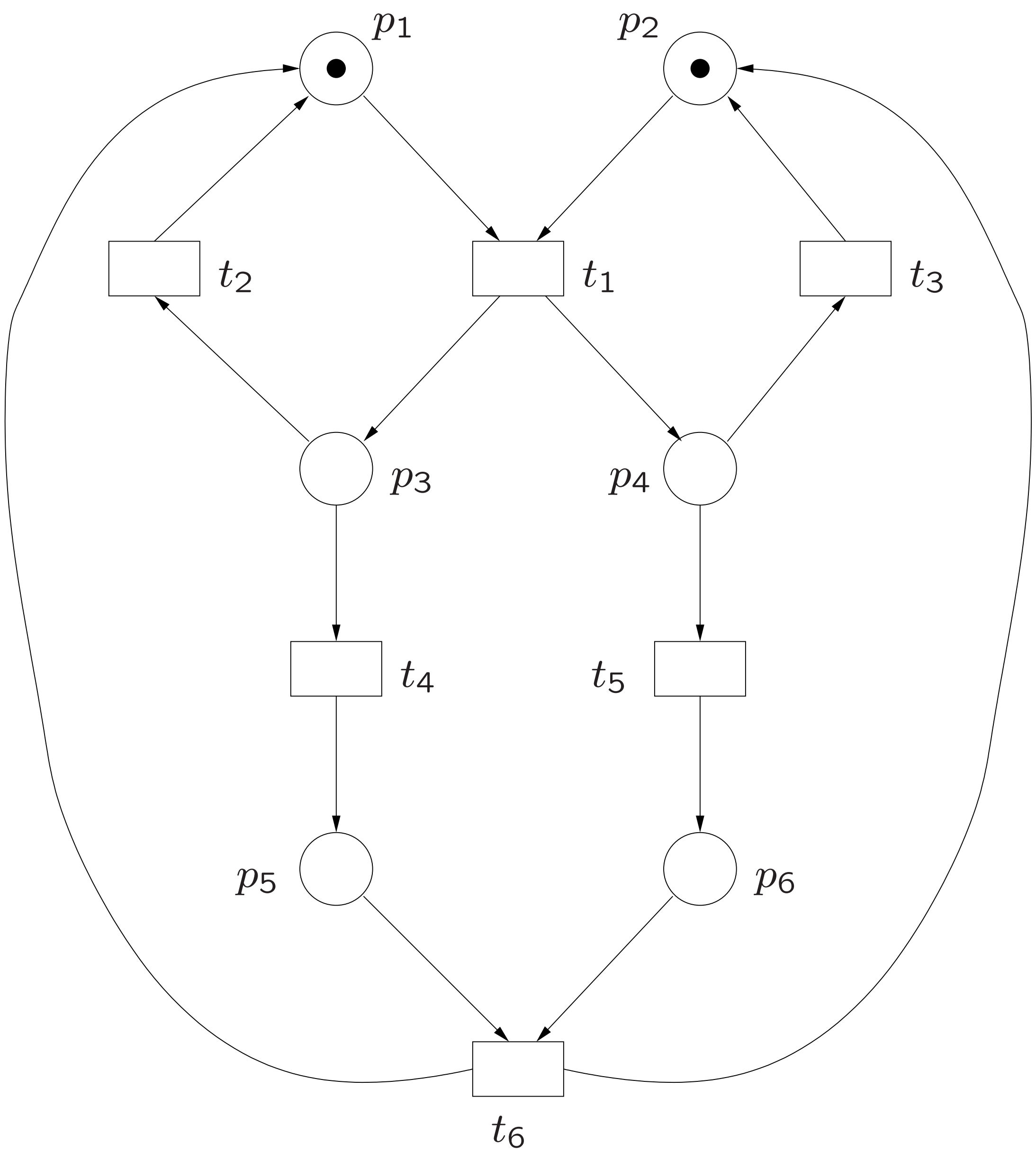
Theorem 49. Let $M = (P, T, F, C_{in})$ be a reduced EN system and let $S \subseteq P$.
Then the following statements are equivalent.

(1) There is a sequential component M' of M with $P_{M'} = S$,

(2) $\#(C \cap S) = 1$ for all $C \in \mathbb{C}_M$,

(3) (i) $\#(C_{in} \cap S) = 1$, and
(ii) $\forall t \in T$:
$\#(^\bullet t \cap S) = \#(t^\bullet \cap S) = 1$ or $\#(^\bullet t \cap S) = \#(t^\bullet \cap S) = 0$.

Theorem 84. Every process net $N = (P, T, F, {}^\circ N)$, with $P \neq \varnothing$, is dense.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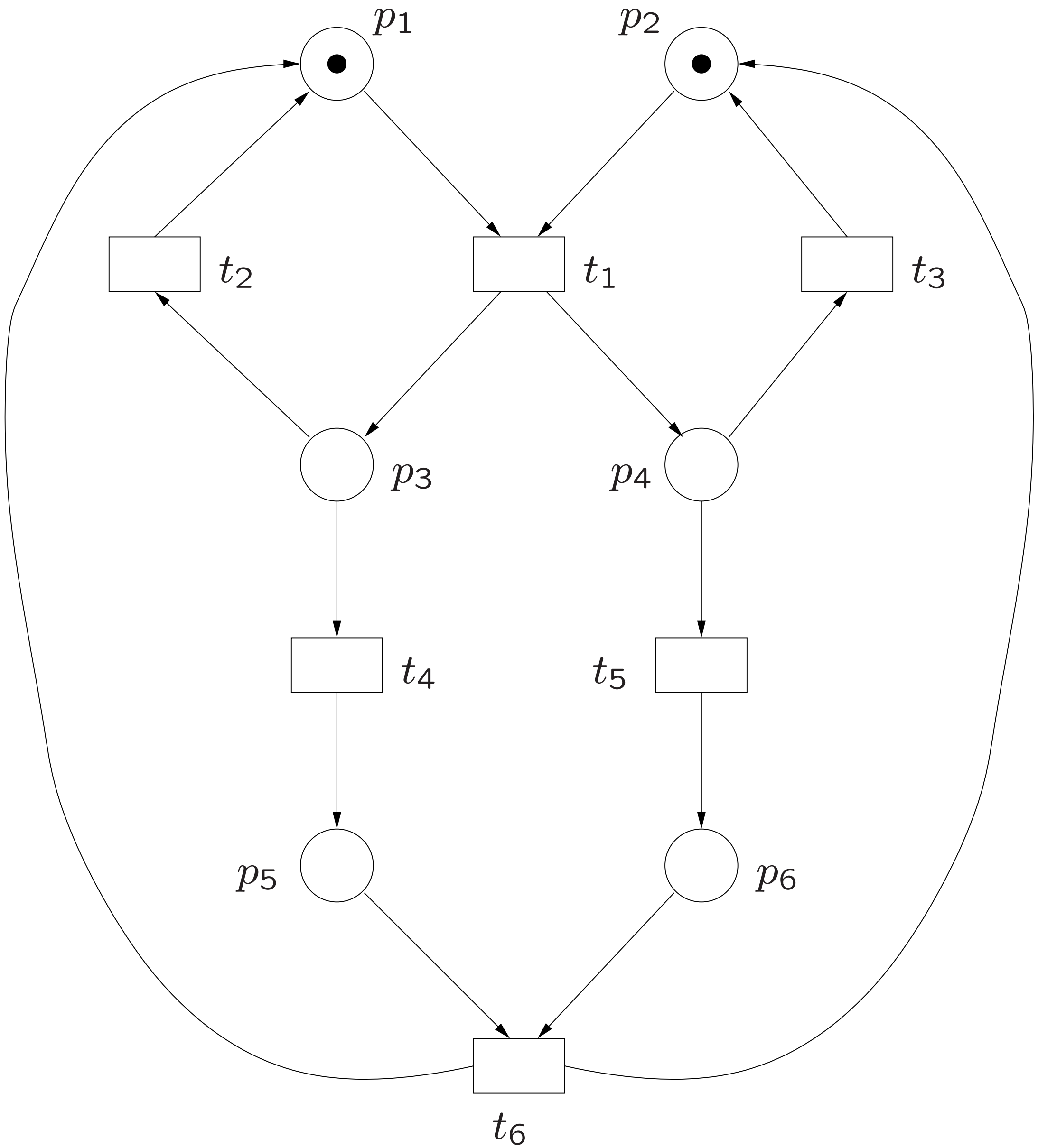

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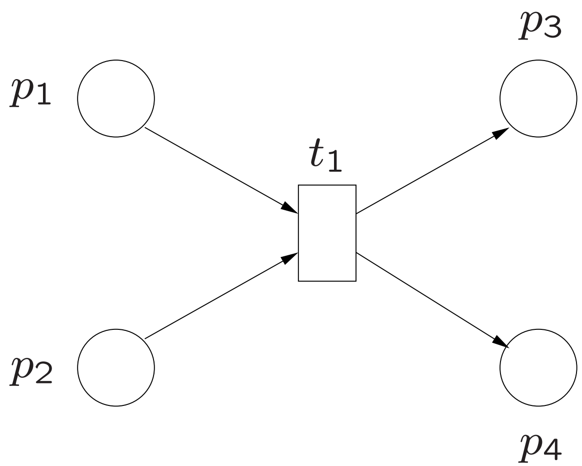

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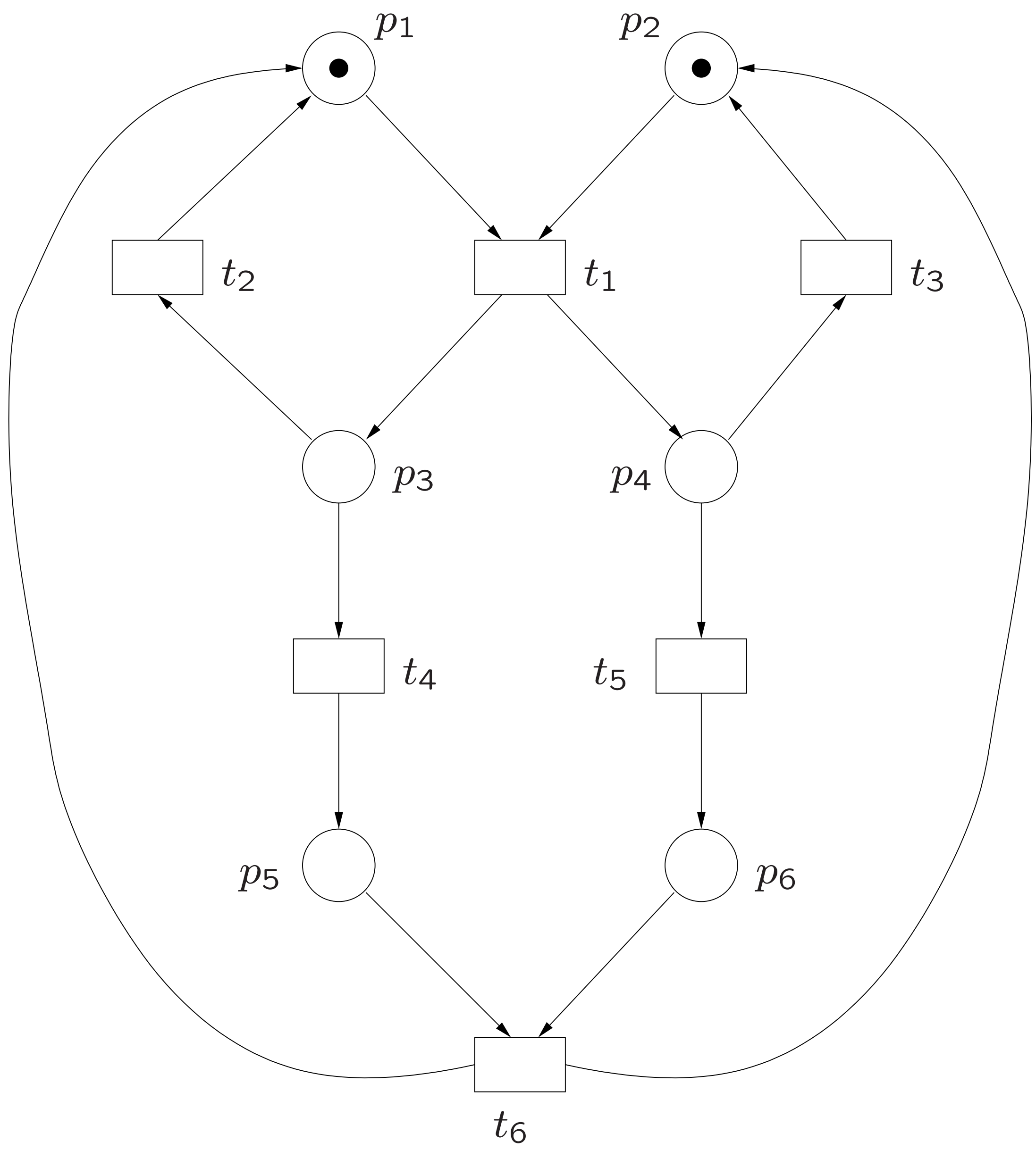
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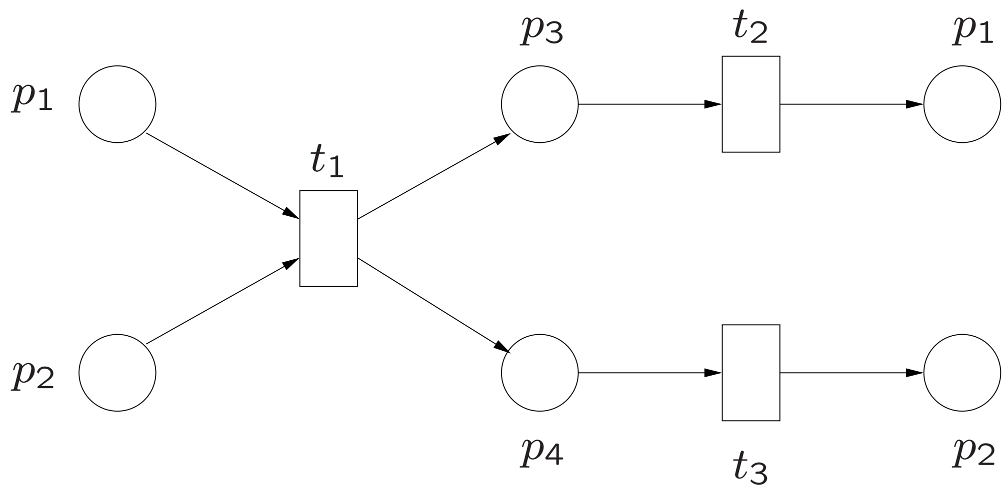
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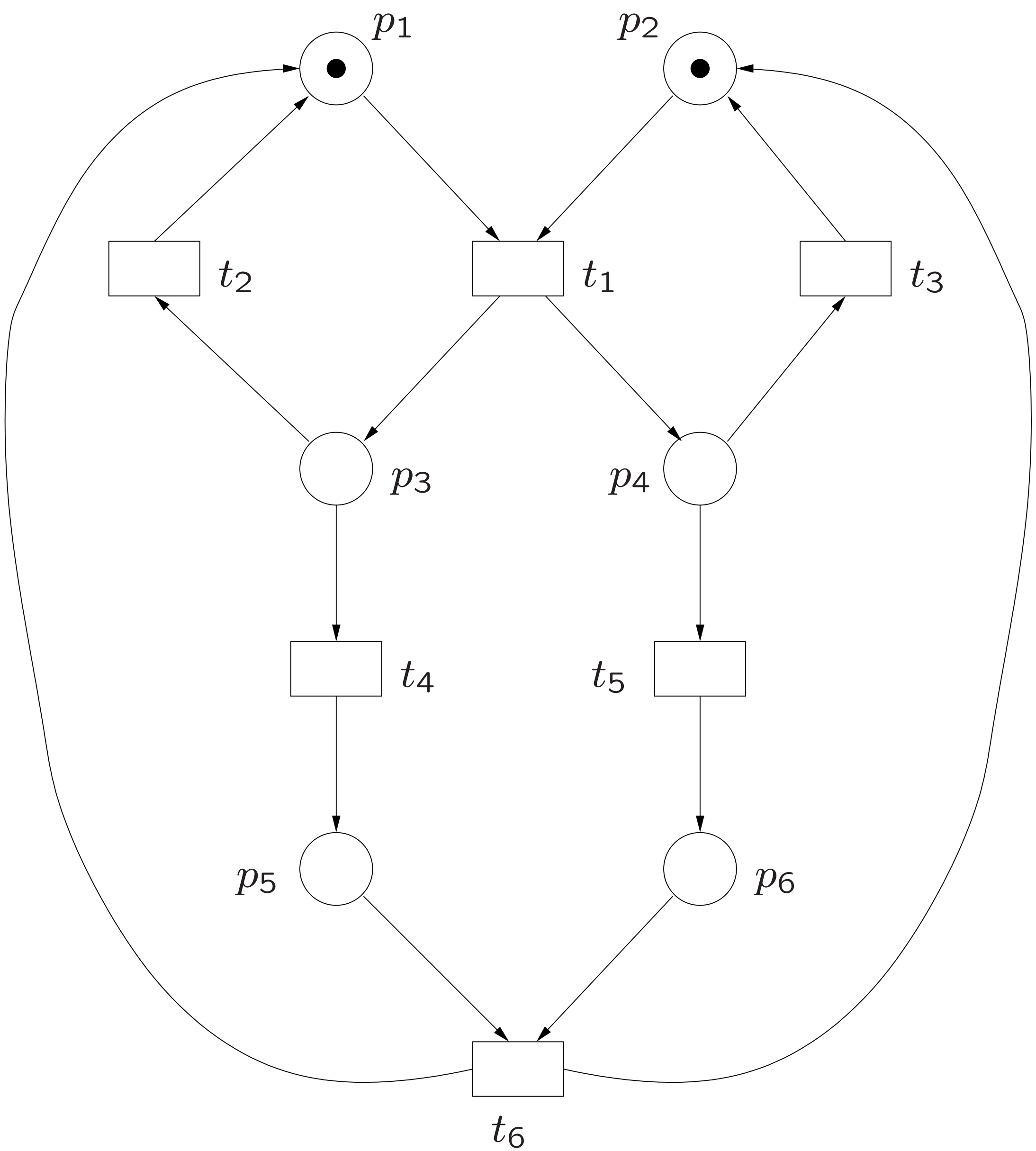
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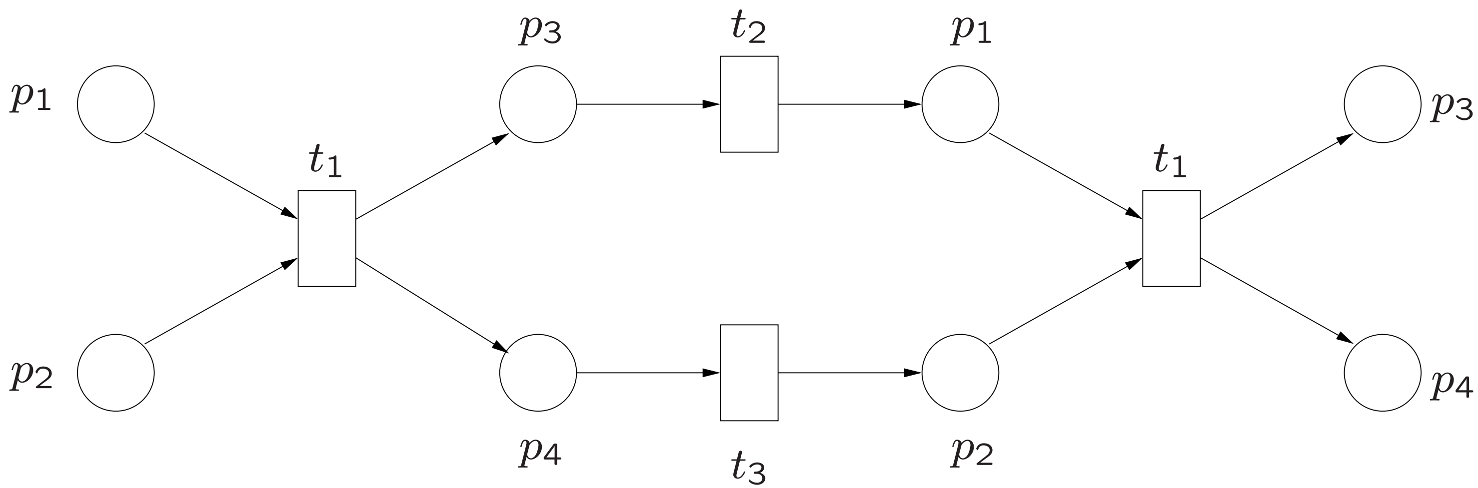

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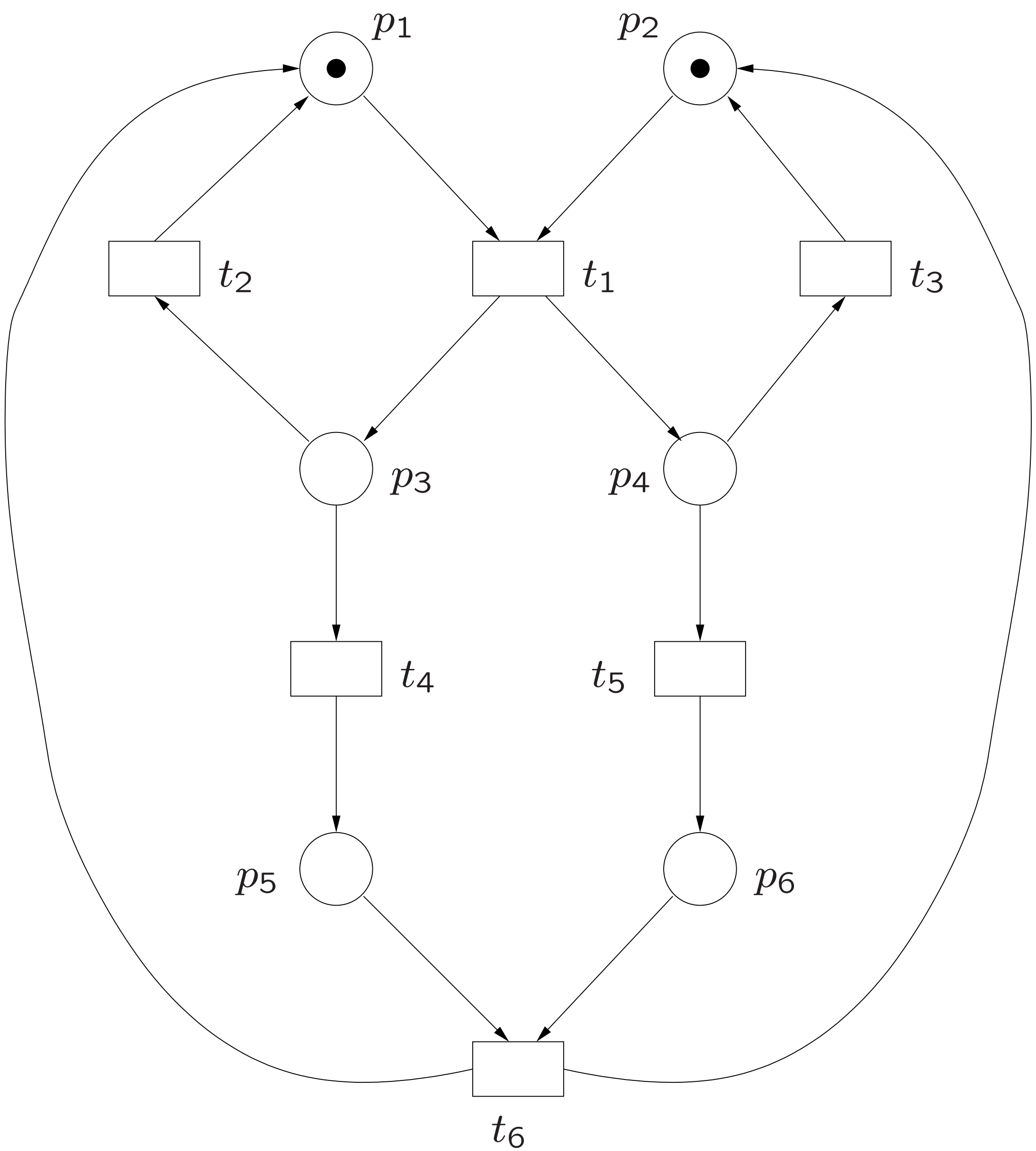
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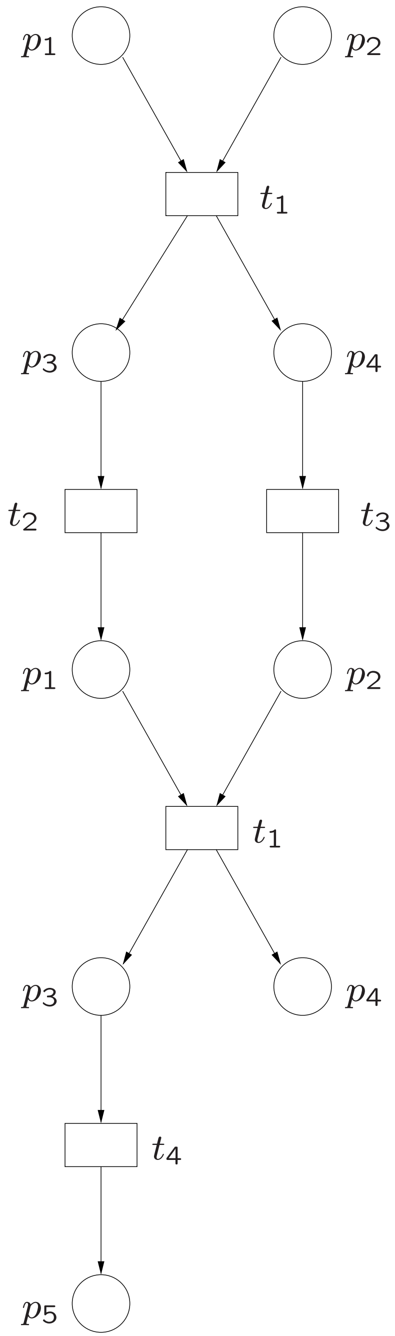
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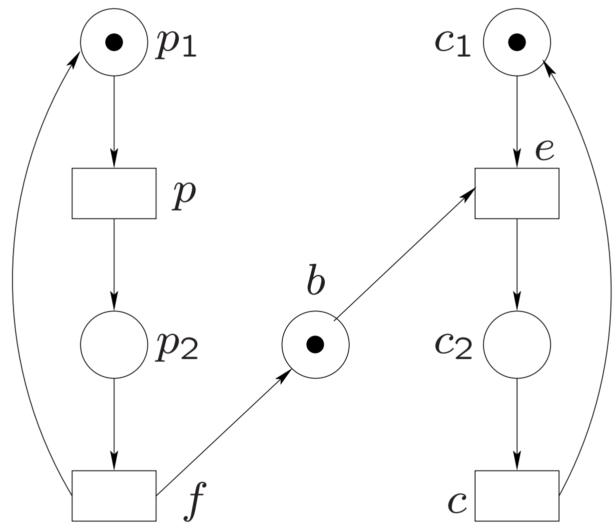


Fig. 12. The producer/consumer system (not contact free).

Definition 85. Let Σ_1 and Σ_2 be disjoint alphabets.
A (Σ_1, Σ_2)-*labelled net* is a 5-tuple $N = (P, T, F, \phi_1, \phi_2)$, where

(P, T, F) is a net
the *underlying net of* $\mathbf{und}(N)$ of N,

ϕ_1 is a function from P to Σ_1
the *place labelling of* N, and

ϕ_2 is a function from T to Σ_2
the *transition labelling of* N.

ϕ_1, ϕ_2 are not necessarily injective.
Different places / transitions may have same label.

Definition 86. $N = (P, T, F, \phi_1, \phi_2)$ and $N' = (P', T', F', \phi'_1, \phi'_2)$.

N and N' are *isomorphic*, $N \equiv N'$, if there exist bijections
$\alpha : \Sigma_1 \to \Sigma'_1$ and $\beta : \Sigma_2 \to \Sigma'_2$,
$\gamma : P \to P'$ and $\delta : T \to T'$,
such that

(1) $\mathbf{und}(N) \equiv^{\gamma}_{\delta} \mathbf{und}(N')$,

(2) for all $p \in P$, $\phi'_1(\gamma(p)) = \alpha(\phi_1(p))$, and

(3) for all $t \in T$, $\phi'_2(\delta(t)) = \beta(\phi_2(t))$.

Notation: $N \equiv^{\alpha}_{\beta} N'$.

Definition 87. Let $\mathcal{P}$ and $\mathcal{P}'$ be two sets of (Σ_1, Σ_2)-, respectively (Σ_1', Σ_2')-labelled nets.

Then $\mathcal{P}$ and $\mathcal{P}'$ are *isomorphic*, $\mathcal{P} \equiv \mathcal{P}'$, if there exist bijections
$\alpha : \Sigma_1 \to \Sigma_1'$ and
$\beta : \Sigma_2 \to \Sigma_2'$, such that

(1) for every $N \in \mathcal{P}$ there exists $N' \in \mathcal{P}'$ such that $N \equiv_\beta^\alpha N'$, and

(2) for every $N' \in \mathcal{P}'$ there exists $N \in \mathcal{P}$ such that $N \equiv_\beta^\alpha N'$.

Definition 88. Let $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ be a labelled process net and let $M = (P, T, F, C_{in})$ be a contact-free EN system.

Then N is a *process of* M if

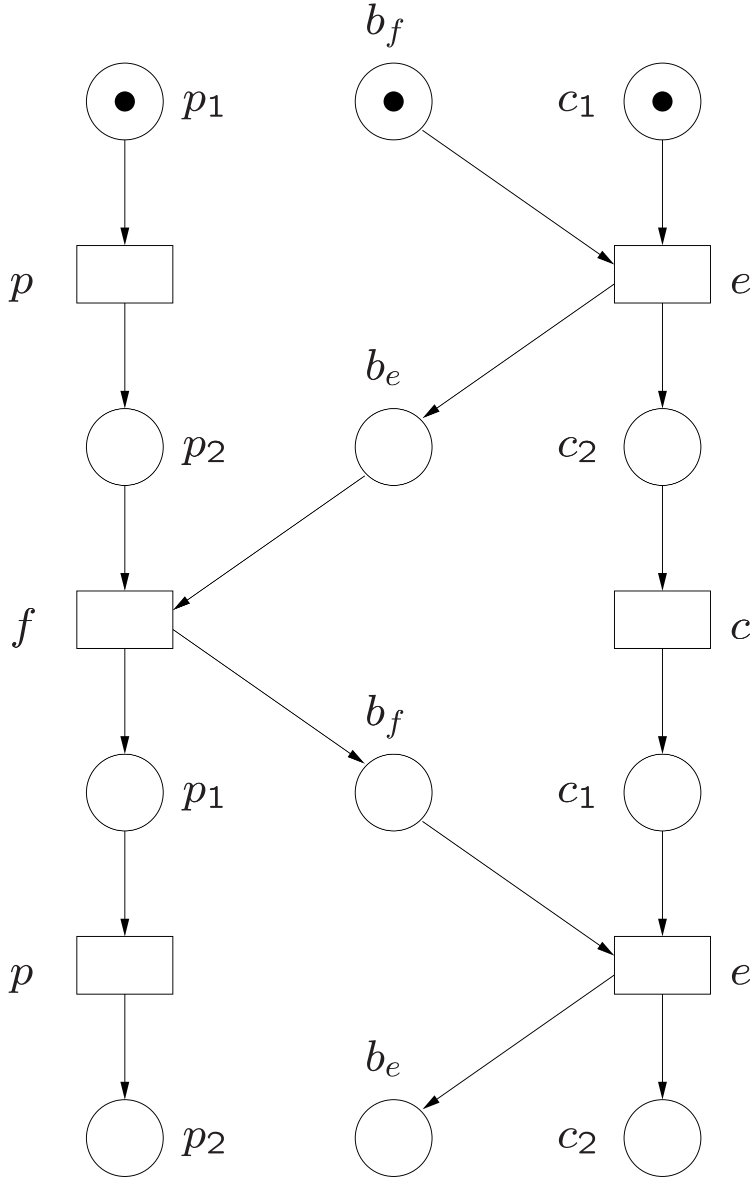
(1) $\Sigma_1 = P$ and $\Sigma_2 = \mathbf{use}(T)$,

(2) $\phi_1 \restriction {}^{\circ}N$ is injective,
(3) $\phi_1({}^{\circ}N) = C_{in}$,

(4) for every $t \in T_N$, $\phi_1 \restriction {}^{\bullet}t$ is injective and $\phi_1 \restriction t^{\bullet}$ is injective,
(5) for every $t \in T_N$, $\phi_1({}^{\bullet}t) = {}^{\bullet}(\phi_2(t))$ and $\phi_1(t^{\bullet}) = (\phi_2(t))^{\bullet}$.

For a contact-free EN system M, PROC(M) denotes the set of all processes of M.

We often write ϕ for both ϕ_1 and ϕ_2.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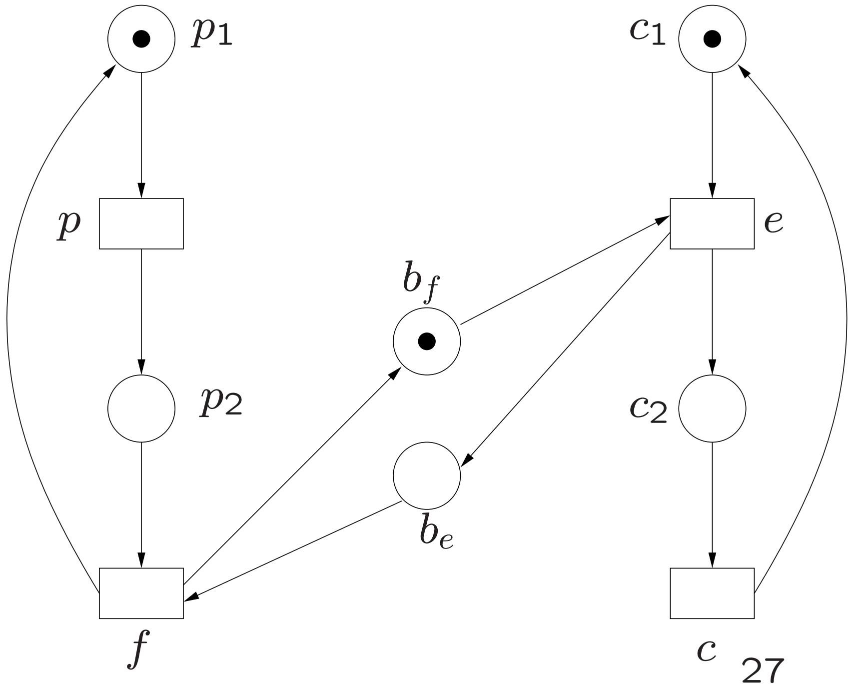
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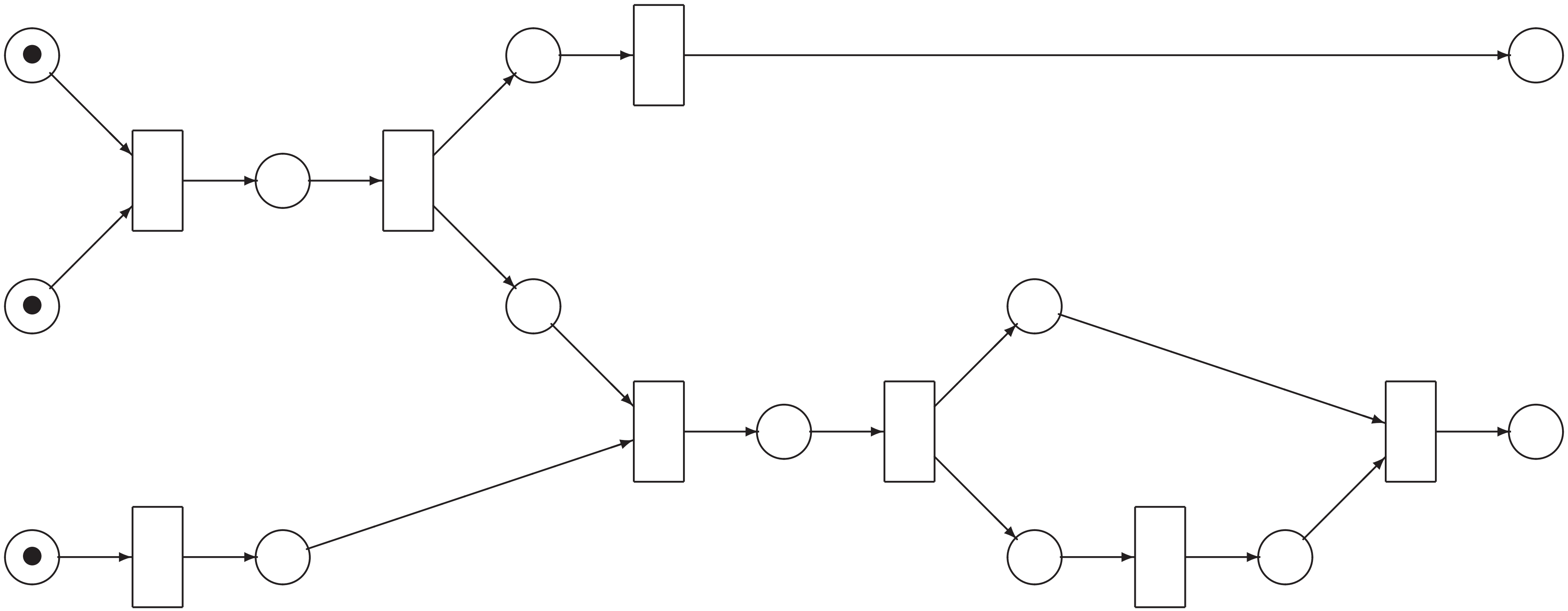


A process net as an EN system.

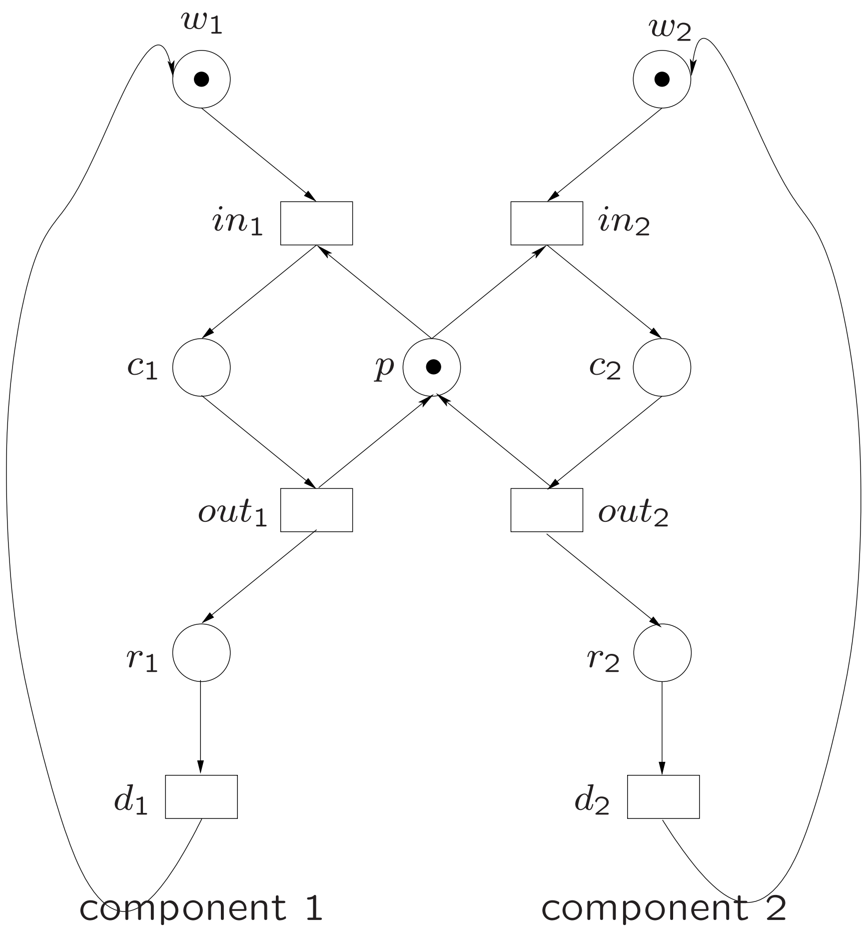
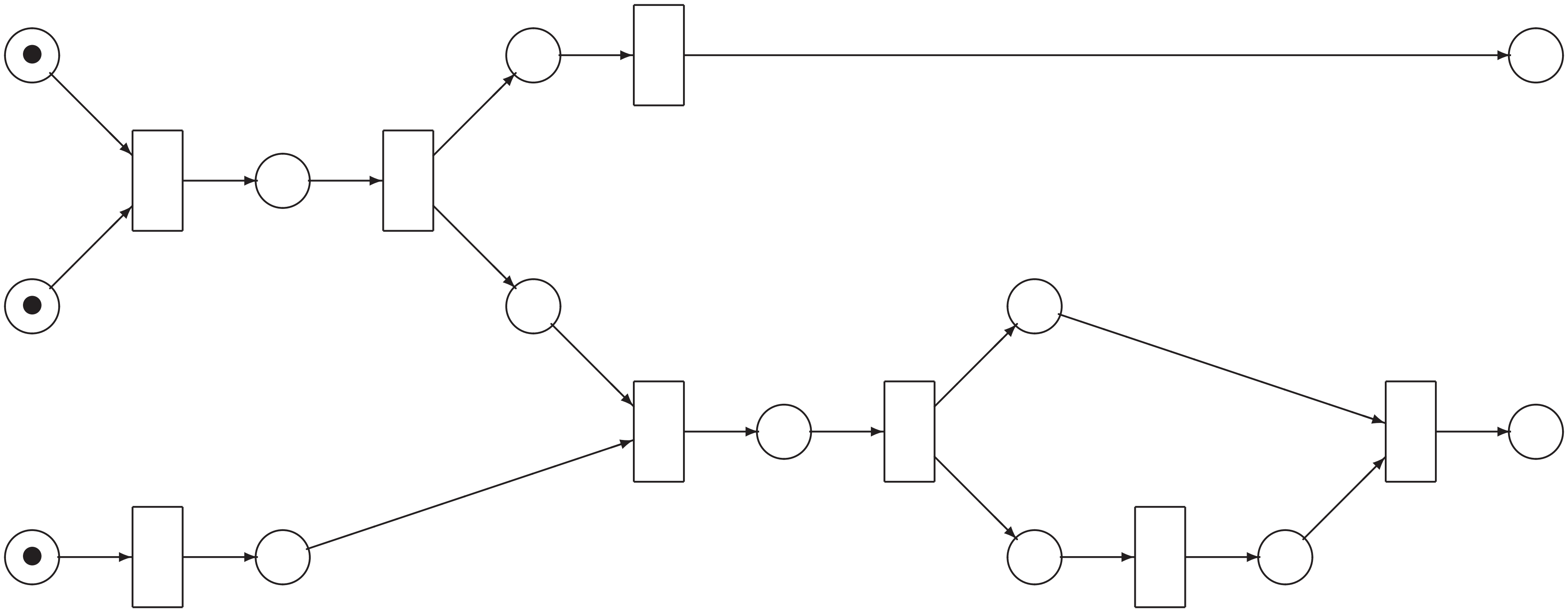
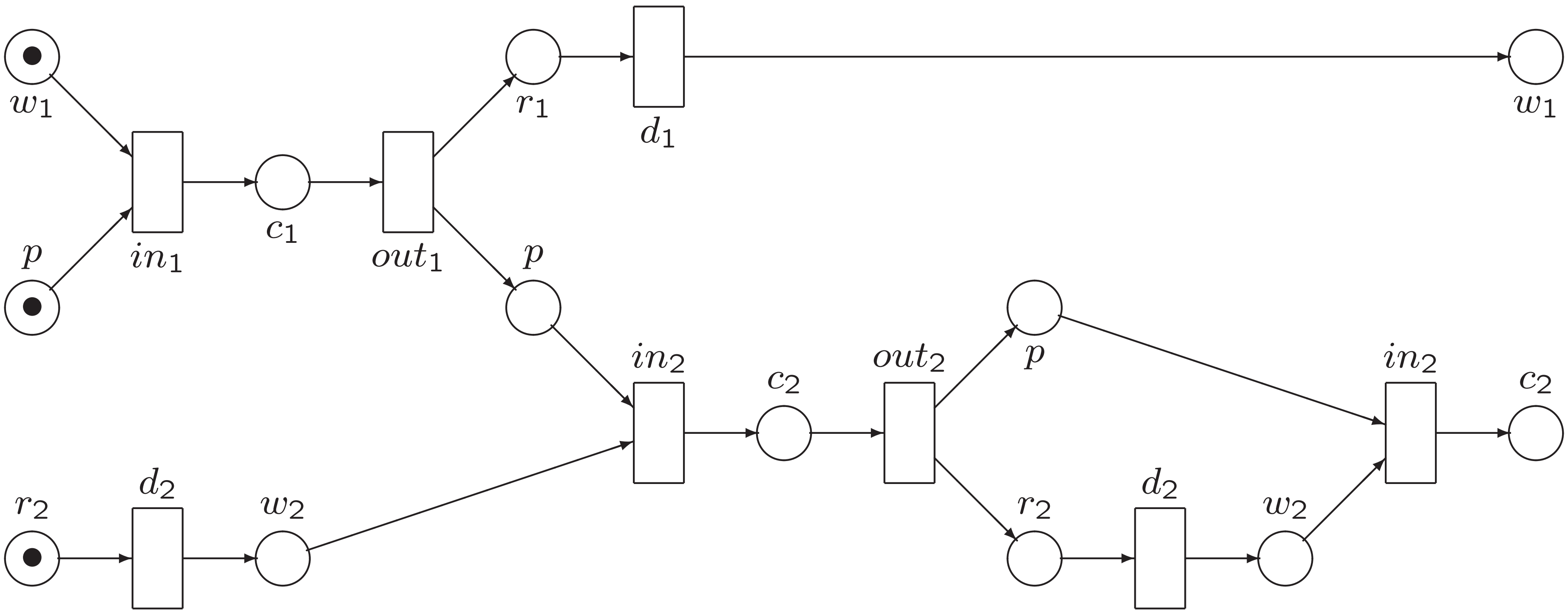


Fig. 5. The mutual exclusion problem.



A process net as an EN system.



A process of the mutual exclusion system with different initial configuration.

Lemma 89. Let $M = (P, T, F, C_{in})$ be a contact-free EN system and let $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ be a process of M.

Let $C, D \in \mathbb{C}_N$ and $t \in T_N$.

If $\phi \restriction C$ is injective, $\phi(C) \in \mathbb{C}_M$, and $C[t\rangle_N D$,

then $\phi \restriction D$ is injective and $\phi(C)[\phi(t)\rangle_M \phi(D)$.

Theorem 90. Let $M = (P, T, F, C_{in})$ be a contact-free EN system and let $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ be a process of M.

(1) For every $C \in \mathbb{C}_N$, $\phi \upharpoonright C$ is injective and $\phi(C) \in \mathbb{C}_M$.

(2) For every $C, D \in \mathbb{C}_N$ and $t \in T_N$,
if $C[t\rangle_N D$ then $\phi(C)[\phi(t)\rangle_M \phi(D)$.

Theorem 91. Let $M = (P, T, F, C_{in})$ be a contact-free EN system and let $N = (P_N, T_N, F_N, \phi_1, \phi_2)$ be a process of M.

(1) For every $\mathbf{co}$-clique D of X_N, $\phi \restriction D$ is injective.

(2) For every $C, D \in \mathbb{C}_N$ and $U \subseteq T_N$,
if $C[U\rangle_N D$ then $\phi(C)[\phi(U)\rangle_M \phi(D)$.

(3) For every $\mathbf{co}$-clique $U \subseteq T_N$ there exist $C, D \in \mathbb{C}_M$ such that $C[\phi(U)\rangle_M D$.

Definition 96. Let Σ be an alphabet.
A (Σ-)*labelled graph* is a quadruple $G = (V, \Gamma, \Sigma, \phi)$, where

V is a finite set of *nodes*,
$\Gamma \subseteq V \times V$ is a set of *edges*, and
ϕ is a function from V to Σ, the *labelling of* G.

G is *acyclic* if Γ^+ is irreflexive.

Definition 99. Let $G = (V, \Gamma, \Sigma, \phi)$ be an acyclic labelled graph.

The *transitive closure* of G, $\mathbf{tra}(G)$, is the labelled graph

$(V, \Gamma^+, \Sigma, \phi)$.

G *represents* $\mathbf{tra}(G)$.

Definition 100. Let $G = (V, \Gamma, \Sigma, \phi)$ be an acyclic labelled graph.

The *pruned version of* G, $\mathbf{pru}(G)$, is the labelled graph

$(V, \Gamma', \Sigma, \phi)$ with
$\Gamma' = \{(v, w) \in \Gamma \mid \neg\ \exists u \in V : (v, u) \in \Gamma^+$ and $(u, w) \in \Gamma^+\}$.

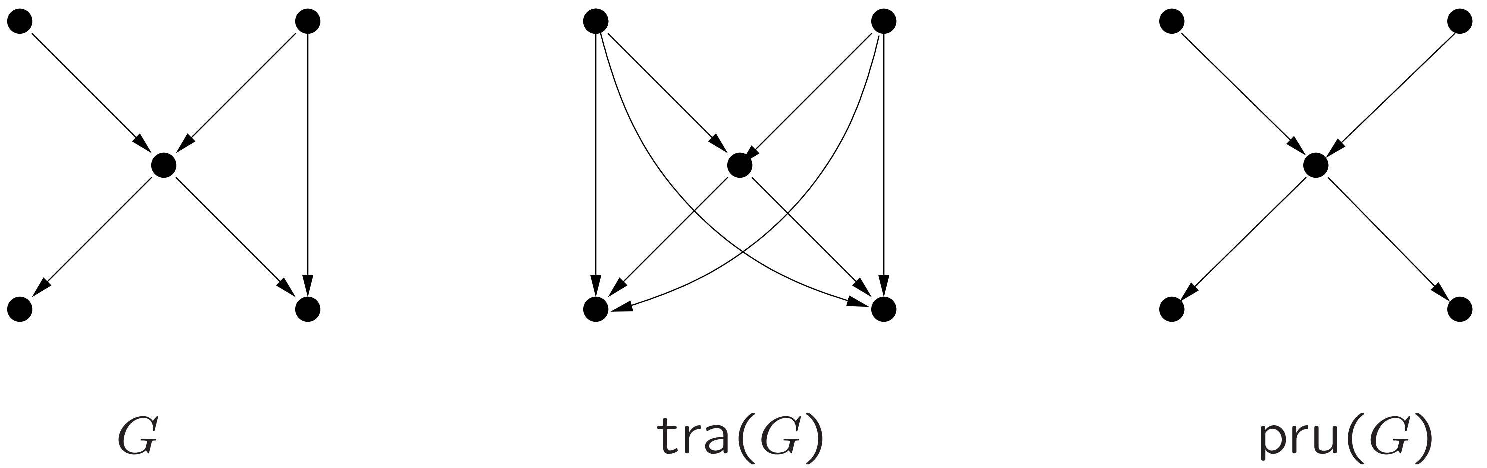


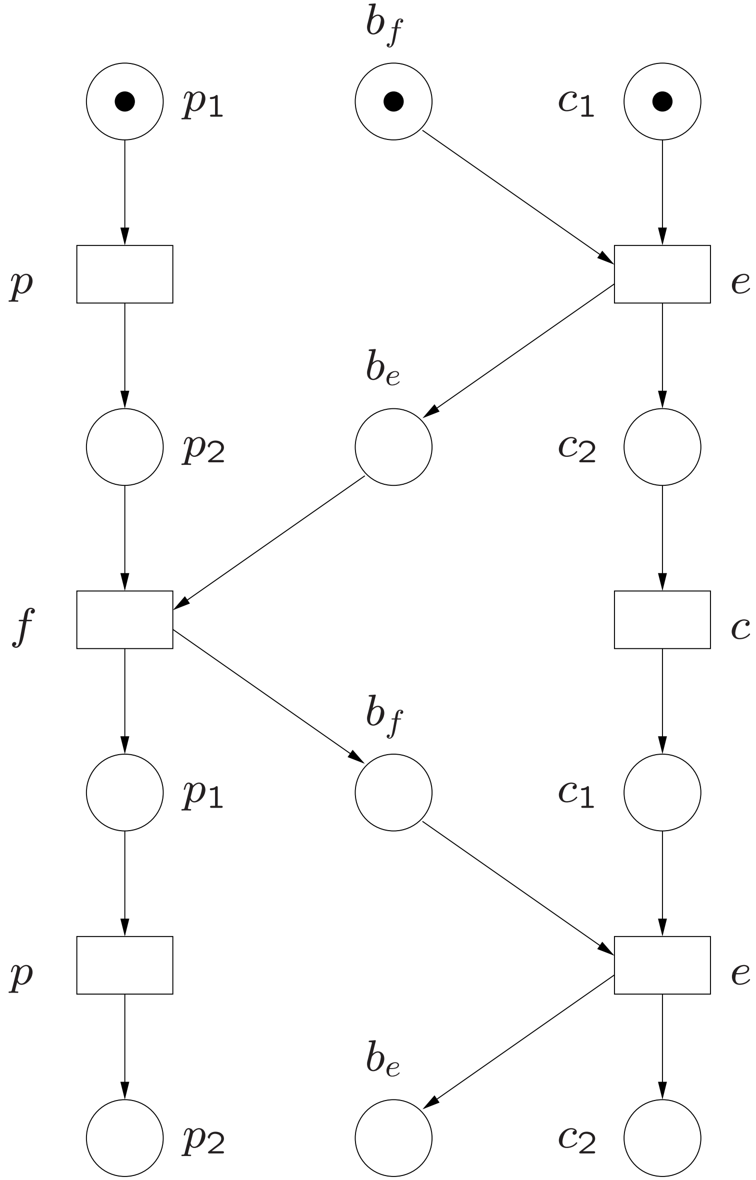
Fig. 61. A graph with
its transitive closure and its pruned version.

Definition 103. Let $N = (P, T, F, \phi_1, \phi_2)$ be an acyclic (Σ_1, Σ_2)-labelled net.

(1) The *contracted version of* N, $\mathbf{ctr}(N)$, is the labelled graph $(T, \Gamma, \Sigma_2, \phi_2)$ such that,

for all $s, t \in T$,
$(s, t) \in \Gamma$ iff $s^\bullet \cap {}^\bullet t \neq \varnothing$.

(2) The *pruned contracted version of* N is the labelled graph $\mathbf{pru}(\mathbf{ctr}(N))$.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c_1
p
e
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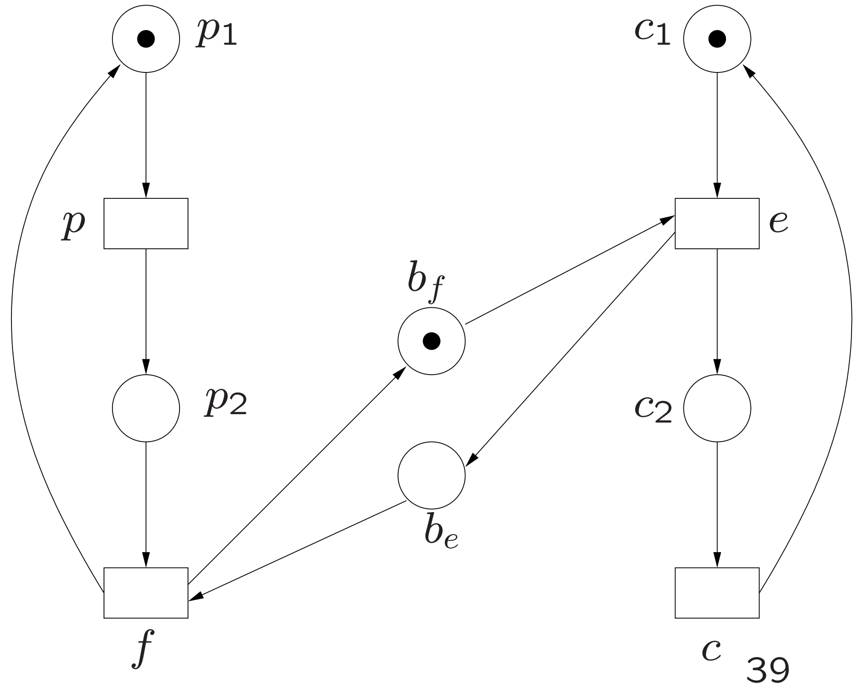
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b_f
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f
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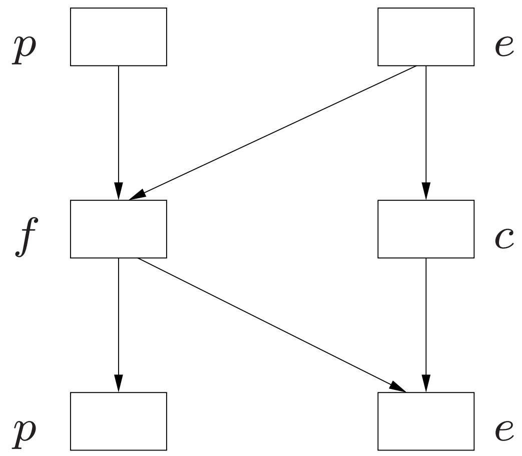

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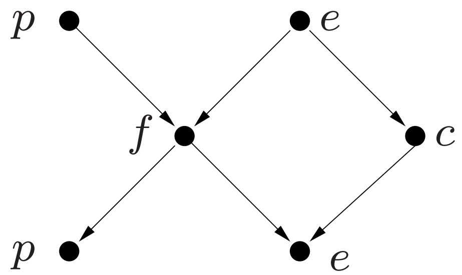

Fig. 62. Pruned/contracted version.