

1. **[1 point]** Draw the parse tree of the formula $p \rightarrow ((q \wedge \neg\neg p) \vee \neg(q \rightarrow p))$ and list *all* its subformulas.
 2. **[2 points]** Give a proof by means of natural deduction of the following sequents:
 - a) $\vdash p \rightarrow ((p \rightarrow q) \rightarrow q)$.
 - b) $\neg p \vdash p \rightarrow (p \rightarrow q)$.
 - c) $(p \rightarrow q) \vee (r \rightarrow q) \vdash (p \wedge r) \rightarrow q$
 - d) $\neg p, (p \vee q) \vdash q$.
 3. **[1 point]** Use mathematical induction to prove that $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all $n \geq 1$.
 4. **[2 points]** Compute the conjunctive normal form of the following formulas and check which formulas are valid. Explain your answer.
 - a) $(p \wedge \neg q) \vee (p \wedge q)$.
 - b) $\neg(p \wedge \neg q) \wedge (q \vee \neg p)$.
 - c) $((p \rightarrow q) \vee p) \wedge (p \vee \neg(r \wedge \neg r \wedge q))$.
 - d) Construct a formula ϕ in conjunctive normal form from the truth table

p	q	ϕ
T	T	F
T	F	T
F	T	T
F	F	F
 5. **[1 point]** Apply the marking algorithm to check if the following Horn formulas are satisfiable:
 - a) $(\top \rightarrow p) \wedge ((p \wedge q) \rightarrow r) \wedge (p \rightarrow q) \wedge ((r \wedge p) \rightarrow q)$.
 - b) $(\top \rightarrow p) \wedge (p \rightarrow q) \wedge ((p \wedge q) \rightarrow r) \wedge (q \rightarrow \perp) \wedge (\top \rightarrow r)$.
 6. **[2 points]** Show the validity by means of natural deduction of the following sequents:
 - a) $\forall x P(x), \neg \exists x Q(x) \vdash P(a) \vee Q(a)$.
 - b) $P(a) \vdash \forall x (x = a \rightarrow P(x))$.
 - c) $\vdash \exists x (x = a \vee \neg(x = b))$.
 - d) $\vdash \neg \exists x \neg(x = x)$.
 7. **[1 point]** For each of the following sequents give a model showing that it is not valid:
 - a) $\vdash \forall x \forall y \forall z (P(x,y) \rightarrow P(y,z))$.
 - b) $\forall x (P(x) \vee Q(x)) \vdash \forall x P(x) \vee \forall x Q(x)$.
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The final score is given by the sum of the points obtained.