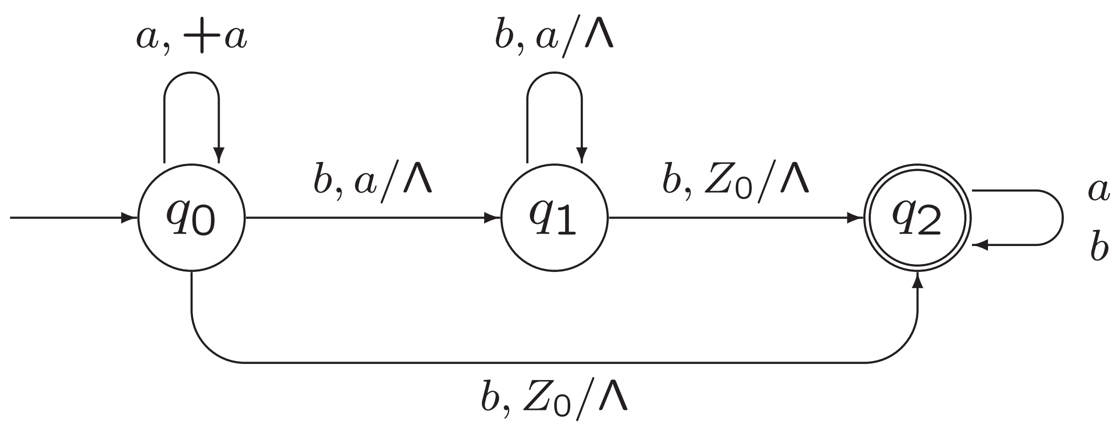
Exercise.

What language is accepted by the following pushdown automaton:



From exercise class 8:

Exercise 5.16.

Show that if L is accepted by a PDA, then L is accepted by a PDA that never crashes (i.e., for which the stack never empties and no configuration is reached from which there is no move defined).

From lecture 8:

Stack in PDA contains symbols from certain alphabet.

Usual stack operations: pop, top, push

Extra possiblity: replace top element X by string α

$\alpha = \Lambda$	pop
$\alpha = X$	top
$\alpha = YX$	push
$\alpha = \beta X$	push*
$\alpha = \ldots$	

Top element X is required to do a move!

From exercise class 8:

Exercise 5.17.

Show that if L is accepted by a PDA, then L is accepted by a PDA in which every move
$*$ either pops something from the stack (i.e., removes a stack symbol without putting anything else on the stack);
$*$ or pushes a single symbol onto the stack on top of the symbol that was previously on top;
$*$ or leaves the stack unchanged.

Hence, each action on the stack due to a move in the PDA has one of the following forms:
$*$ either X/Λ (with $X \in \Gamma$),
$*$ or X/YX (with $X, Y \in \Gamma$),
$*$ or X/X (with $X \in \Gamma$).

From lecture 6:

Theorem 4.9.

If L_1 and L_2 are context-free languages over an alphabet Σ, then

$$L_1 \cup L_2, \quad L_1 L_2 \quad \text{and } L_1^*$$

are also CFLs.

Exercise 5.19. ♣ ♠

Suppose M_1 and M_2 are PDAs accepting L_1 and L_2, respectively. For both the languages L_1L_2 and L_1^*, describe a procedure for constructing a PDA accepting the language.

In each case, nondeterminism will be necessary. Be sure to say precisely how the stack of the new machine works; no relationship is assumed between the stack alphabets of M_1 and M_2.

Answer begins with:
Let $M_1 = (Q_1, \Sigma, \Gamma_1, q_{01}, Z_{01}, A_1, \delta_1)$
and let $M_2 = (Q_2, \Sigma, \Gamma_2, q_{02}, Z_{02}, A_2, \delta_2)$.

Exercise 5.25.

A *counter automaton* is a PDA with just two stack symbols, A and Z_0, for which the string on the stack is always of the form $A^n Z_0$ for some $n \geq 0$.
(In other words, the only possible change in the stack contents is a change in the number of A's on the stack.)
For some context-free languages, such as *AnBn*, the obvious PDA to accept the language is in fact a counter automaton.
Construct a counter automaton to accept the given language in each case below.

a. $\{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$

b. ♣ ♠ $\{x \in \{a, b\}^* \mid n_a(x) = 2n_b(x)\}$

Exercise 5.28.

In each case below, you are given a CFG G and a string x that it generates.
Draw the nondeterministic top-down PDA $NT(G)$.

Trace a sequence of moves in $NT(G)$ by which x is accepted, showing at each step the state, the unread input, and the stack contents.
Show at the same time the corresponding leftmost derivation of x in the grammar. See Example 5.19 for a guide.

b. ♣ The grammar has productions $S \to S{+}S \quad | \quad S{*}S \quad | \quad [S] \quad | \quad a$, and $x = [a * a + a]$.

Exercise 5.34.

In each case below, you are given a CFG G and a string x that it generates.
Draw the nondeterministic bottom-up PDA *NB*(G).

Trace a sequence of moves in *NB*(G) by which x is accepted, showing at each step the state, the (reversed) stack contents and the unread input.
Show at the same time the corresponding rightmost derivation of x (in reverse order) in the grammar. See Example 5.24 for a guide.

a. ♣ The grammar has productions $S \to S[S] \mid \Lambda$ and $x = [][[]]$.

Exercise 5.30.

For a certain CFG G, the moves shown below are those by which the nondeterministic bottom-up PDA *NB*(G) accepts the input aabbab. Each occurrence of $\vdash^*$ indicates a sequence of moves constituting a reduction. Draw the derivation tree for $aabbab$ that corresponds to this sequence of moves.

$$
\begin{array}{rcl}
(q_0, aabbab, Z_0) & \vdash & (q_0, abbab, aZ_0) \vdash (q_0, bbab, aaZ_0) \\
& \vdash & (q_0, bab, baaZ_0) \vdash^* (q_0, bab, SaZ_0) \\
& \vdash & (q_0, ab, bSaZ_0) \vdash^* (q_0, ab, SZ_0) \\
& \vdash & (q_0, b, aSZ_0) \vdash (q_0, \Lambda, baSZ_0) \\
& \vdash^* & (q_0, \Lambda, SSZ_0) \vdash^* (q_0, \Lambda, SZ_0) \\
& \vdash & (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)
\end{array}
$$

From exam Automata Theory, 23 December, 2021

Let

$$L = \{a^i b^j a^k \mid i, j, k \geq 0 \text{ and } i + k \leq 2j\}$$

a. Give the first six elements in the canonical (shortlex) order of L.

b. Draw a pushdown automaton M, such that $L(M) = L$.
This pushdown automaton must be directly based on the properties of the language. So it must not be the result of a standard construction to, e.g., transform a context-free grammar into a pushdown automaton.
Try to make M deterministic, and not containing any Λ-transitions.