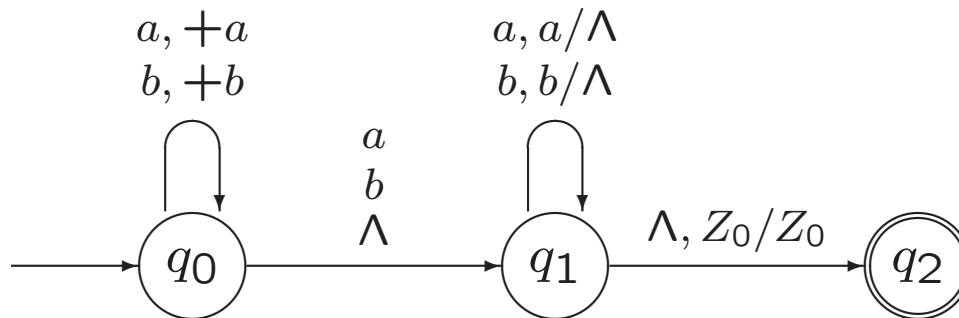


Exercise 5.2. ♣

For the PDA below, trace every possible sequence of moves for the two input strings aba and aab .

Example 5.7. A Pushdown Automaton Accepting Pal

$$Pal = \{y \in \{a, b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a, b\}^*\}$$



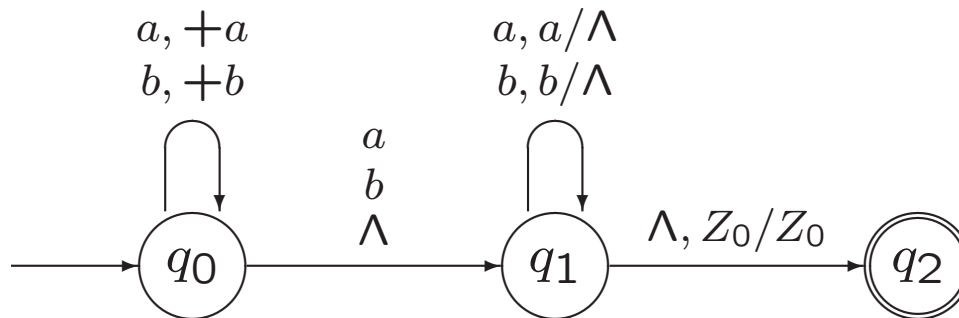
Exercise 5.4. ♣

For each of the following languages over $\{a, b\}^*$, modify the PDA below to obtain a PDA accepting the language.

- a. The language of even-length palindromes.
- b. The language of odd-length palindromes.

Example 5.7. A Pushdown Automaton Accepting *Pal*

$$Pal = \{y \in \{a, b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a, b\}^*\}$$



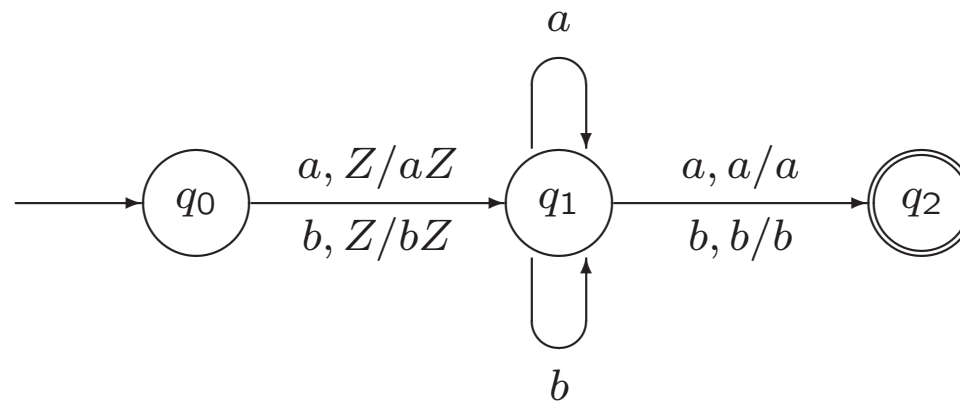
Exercise 5.5.

Give transition **diagrams** for PDAs accepting each of the following languages.

- a.** The language of all odd-length strings over $\{a, b\}$ with middle symbol a .
- b.** $\{a^n x \mid n \geq 0, x \in \{a, b\}^* \text{ and } |x| \leq n\}$.
- c.** $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } j = i \text{ or } j = k\}$.

Exercise 5.6. ♣

a. Below, a transition **diagram** is given for a PDA with initial state q_0 and accepting state q_2 . Describe the language that is accepted.



Exercise.

Let $L_1 = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ en } 2i > j\}$.

- a.** Give the first five elements of L_1 in the canonical order.
- b.** Give a PDA M_1 such that $L(M_1) = L_1$.

Exercise.

Let $L_1 = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ en } 2i > j\}$.

- a. Give the first five elements of L_1 in the canonical order.
- b. Give a **D**PPDA M_1 such that $L(M_1) = L_1$.

Exercise 5.10. ♣

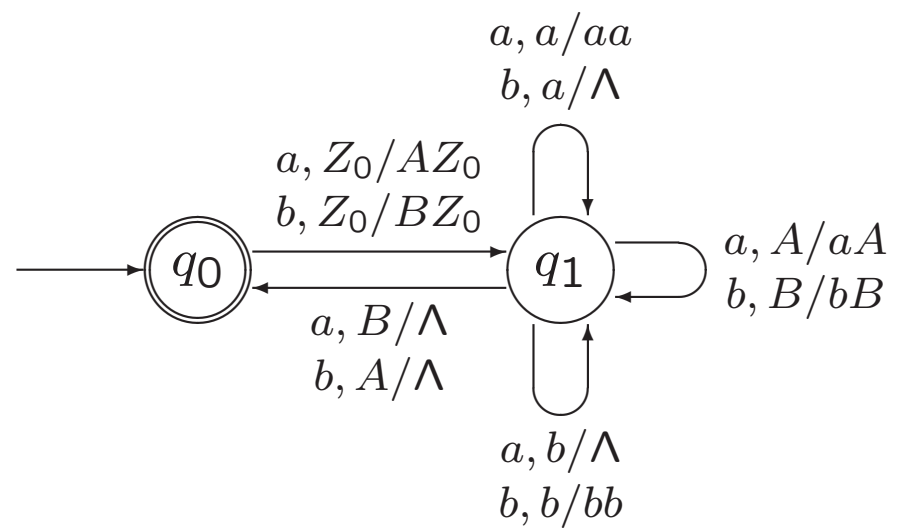
Show that every regular language can be accepted by a (deterministic) PDA M with only two states in which there are no Λ -transitions and no symbols are ever removed from the stack.

Exercise 5.12.

Show that if L is accepted by a PDA in which no symbols are ever removed from the stack, then L is regular.

From lecture 8:

A DPDA for $A \equiv B$:



Exercise 5.18.

For each of the following languages, give a transition **diagram** for a deterministic PDA that accepts that language.

- a. $\{x \in \{a, b\}^* \mid n_a(x) < n_b(x)\}$
- b. ♣ $\{x \in \{a, b\}^* \mid n_a(x) \neq n_b(x)\}$
- c. ♠ $\{x \in \{a, b\}^* \mid n_a(x) = 2n_b(x)\}$
- d. ♣ $\{a^n b^{n+m} a^m \mid n, m \geq 0\}$

Exercise 5.16.

Show that if L is accepted by a PDA, then L is accepted by a PDA that never crashes (i.e., for which the stack never empties and no configuration is reached from which there is no move defined).

From lecture 8:

Stack in PDA contains symbols from certain alphabet.

Usual stack operations: pop, top, push

Extra possibility: replace top element X by string α

$\alpha = \Lambda$ pop

$\alpha = X$ top

$\alpha = YX$ push

$\alpha = \beta X$ push*

$\alpha = \dots$

Top element X is required to do a move!

Exercise 5.17.

Show that if L is accepted by a PDA, then L is accepted by a PDA in which every move

- * either pops something from the stack (i.e., removes a stack symbol without putting anything else on the stack);
- * or pushes a single symbol onto the stack on top of the symbol that was previously on top;
- * or leaves the stack unchanged.

Hence, each action on the stack due to a move in the PDA has one of the following forms:

- * either X/Λ (with $X \in \Gamma$),
- * or X/YX (with $X, Y \in \Gamma$),
- * or X/X (with $X \in \Gamma$).