Exercise 3.2. Consider the two regular expressions

$$r = a^* + b^* \qquad s = ab^* + ba^* + b^*a + (a^*b)^*$$

a. Find a string corresponding to r but not to s.
b. Find a string corresponding to s but not to r.
c. Find a string corresponding to both r and s.
d. Find a string in $\{a,b\}^*$ corresponding to neither r nor s.

Exercise 3.10. ♣

a. If L is the language corresponding to the regular expression $(aab + bbaba)^*baba$, find a regular expression corresponding to $L^r = \{x^r \mid x \in L\}$.

b. Using the example in part (a) as a model, give a recursive definition (based on Definition 3.1) of the reverse e^r of a regular expression e.

c. Show that for every regular expression e, if the language L corresponds to e, then L^r corresponds to e^r.

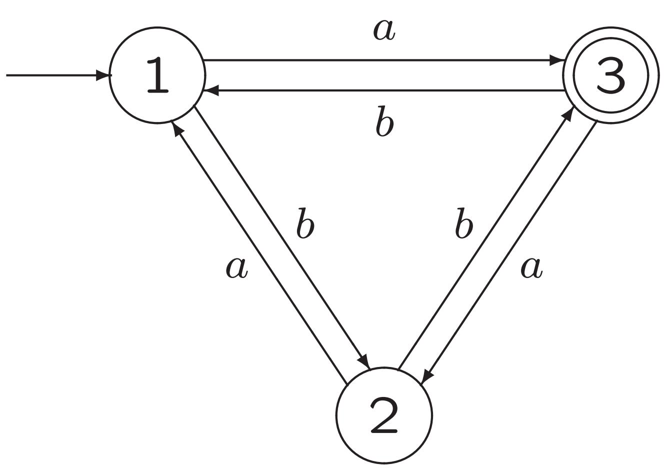
Exercise 3.41. For each of the following regular expressions, draw an NFA accepting the corresponding language, so that there is a recognizable correspondence between the regular expression and the transition diagram.

e. $(a^*bb)^* + bb^*a^*$

Exercise 3.42. For part (e) of Exercise 3.41, draw the NFA that is obtained by a literal application of Kleene's theorem, without any simplifications.

Exercise 3.51 (variant).
Use the algorithm of Brzozowski and McCluskey to find a regular expression corresponding to the FA below.

a.



From lecture 5:

Example 4.1.

AnBn $= \{\ a^n b^n \mid n \geq 0\ \} \subseteq \{a,b\}^*$

Recursive definition:

- $\Lambda \in$ *AnBn* (basis)
- for every $x \in$ *AnBn*, also $axb \in$ *AnBn* (induction)

$S \to \Lambda$
$S \to aSb$

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aa\,bb$

Exercise 4.10. Find context-free grammars generating each of the languages below.

a. ♣ $\{a^i b^j \mid i \leq j\}$

c. ♣ $\{a^i b^j \mid j = 2i\}$

e. ♣ $\{a^i b^j \mid j \leq 2i\}$

f. ♣ $\{a^i b^j \mid j < 2i\}$

d. ♣ $\{a^i b^j \mid i \leq j \leq 2i\}$

d2. $\{a^i b^j \mid i < j < 2i\}$

Exercise 4.1. ♣

In each case below, say what language (a subset of $\{a, b\}^*$) is generated by the context-free grammar with the indicated productions.

b. $S \to SS \mid bS \mid a$

c. $S \to SaS \mid b$

e. $S \to TT \qquad T \to aT \mid Ta \mid b$

g. $S \to aT \mid bT \mid \Lambda \qquad T \to aS \mid bS$

From lecture 5:

Example 4.3.

$Pal = \{x \in \{a,b\}^* \mid x^r = x\}$ (reverse)
In canonical (shortlex) order:
$\Lambda, a, b, aa, bb, aaa, aba, bab, bbb, aaaa, abba, \dots$

Recursive definition:

- $\Lambda, a, b \in Pal$
- for every $x \in Pal$, also $axa, bxb \in Pal$

$S \to \Lambda \mid a \mid b$
$S \to aSa$
$S \to bSb$

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aababaa$

Exercise 4.3. ♣

In each case below, find a CFG generating the given language.

b. The set of even-length strings in $\{a, b\}^*$ with the two middle symbols equal.

c. The set of odd-length strings in $\{a, b\}^*$ whose first, middle, and last symbols are all the same.