

Exercise 4.15/4.19.

Describe the language generated by the CFG with productions

$$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$$

Motivate your answer.

Exercise 4.34. ♣

Show that the CFG with productions

$$S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$$

is ambiguous.

Exercise 4.36. ♣

In each case below, decide whether the grammar is ambiguous or not, and prove your answer.

b. $S \rightarrow SS \mid bS \mid a$

c. $S \rightarrow SaS \mid b$

e. $S \rightarrow TT \quad T \rightarrow aT \mid Ta \mid b$

f. $S \rightarrow aSa \mid bSb \mid aAb \mid bAa \quad A \rightarrow aAa \mid bAb \mid a \mid b \mid \Lambda$

g. $S \rightarrow aT \mid bT \mid \Lambda \quad T \rightarrow aS \mid bS$

Exercise 4.38.

In each case below, show that the grammar is ambiguous, and find an equivalent unambiguous grammar.

a. ♣ $S \rightarrow SS \mid a \mid b$

b. ♣ $S \rightarrow ABA \quad A \rightarrow aA \mid \Lambda \quad B \rightarrow bB \mid \Lambda$

c. ♣ $S \rightarrow aSb \mid aaSb \mid \Lambda$

d. $S \rightarrow aSb \mid abS \mid \Lambda$

From lecture 3:

Exercise 4.45. ♠

Show that the CFG below is unambiguous.

b. The CFG with productions $S \rightarrow (S)S \mid \Lambda$

Exercise.

Let G be a context-free grammar with start variable S and the following productions:

$$S \rightarrow aSbS \mid bSaS \mid \Lambda$$

a. Show that $L(G) = AEqB = \{x \in \{a,b\}^* \mid n_a(x) = n_b(x)\}$. That is, argue why $L(G) \subseteq AEqB$ and why $AEqB \subseteq L(G)$. You do not have to give formal proofs.

b. Show that G is ambiguous, by giving a string $x \in L(G)$ and two different derivation trees for x in G .

c. ♣ Give an unambiguous context-free grammar for $AEqB$.

From lecture 2:

Example 4.10.

Find a context-free grammar generating the language

$$\{a^i b^j c^k \mid j \neq i + k\}$$

Hint: Consider the cases $j > i + k$ and $j < i + k$ separately.

Exercise. *From exam Automata Theory, 19 December, 2024*

Let

$$L = \{a^i b^j a^k \mid i, j, k \geq 0 \text{ and } j < i + k\}$$

- (a) Give the first six elements in the canonical (shortlex) order of L .
- (b) Give a context-free grammar G , such that $L(G) = L$. Try to ensure that G is unambiguous.

If your context-free grammar is ambiguous, then give two different derivation trees for a string $x \in L$.

Exercise 4.7.

Describe the language generated by the CFG with productions

$$S \rightarrow ST \mid \Lambda \quad T \rightarrow aS \mid bT \mid b$$

Motivate your answer.