From exam Automata Theory, 21 December, 2023

Let

$$L = \{\ a^i b^j \mid i \neq j\}$$

For example, $a^5 b^4 \in L$. Let us assume that $n \geq 2$ for this exercise.

For each of the following four strings x_1, x_2, x_3, x_4, indicate whether it is suitable for establishing a contradiction with the pumping lemma.

Furthermore, if x_i is indeed suitable, then derive a contradiction with the pumping lemma. If x_i is not suitable, indicate why not, for example, via a concrete decomposition uvw of x_i that does satisfy the pumping lemma.

$$x_1 = a^{n+1} b^n$$
$$x_2 = a^{n!} b^{(n+1)!}$$
$$x_3 = aaaaabbbb$$
$$x_4 = b^{2n}$$

From exam Automata Theory, 19 December, 2024

Consider the language

$$L = \{x \in \{a,b\}^* \mid n_a(x) \geq n_b(x) \geq n_a(x) - 2\}$$

Hence, L contains the strings in which there are 0, 1 or 2 fewer b's than a's. For example, $aaba \in L$ and $bbabaa \in L$, but $aabaa \notin L$ because there are too few b's, and $bba \notin L$ because there are too many b's.

Prove that the language L cannot be accepted by a finite automaton by using the pumping lemma for regular languages.

From lecture 2:

FA $M_i = (Q_i, \Sigma, q_i, A_i, \delta_i) \qquad i = 1, 2$

Product construction

Construct FA $M = (Q, \Sigma, q_0, A, \delta)$ such that

– $Q = Q_1 \times Q_2$

– $q_0 = (q_1, q_2)$

– $\delta(\ (p, q), \sigma) = (\ \delta_1(p, \sigma), \delta_2(q, \sigma)\ )$

– A as needed

Theorem 2.15 (Parallel simulation).

– $A = \{(p, q) \mid p \in A_1$ or $q \in A_2\}$, then $L(M) = L(M_1) \cup L(M_2)$

– $A = \{(p, q) \mid p \in A_1$ and $q \in A_2\}$, then $L(M) = L(M_1) \cap L(M_2)$

– $A = \{(p, q) \mid p \in A_1$ and $q \notin A_2\}$, then $L(M) = L(M_1) - L(M_2)$

Exercise 2.27.
Describe decision algorithms to answer each of the following questions.

a. ♣ Given two FAs M_1 and M_2,
are there any strings that are accepted by neither?

d. ♠ Given an FA M accepting a language L, and a string x,
is x a prefix of an element of L?

g. Given two FAs M_1 and M_2,
is $L(M_1) \subseteq L(M_2)$?

Exercise 3.21. ♣

Consider the following transition table for an NFA with states 1–5, initial state 1 and input alphabet $\{a, b\}$. There are no Λ-transitions:

q	$\delta(q, a)$	$\delta(q, b)$
1	$\{1, 2\}$	$\{1\}$
2	$\{3\}$	$\{3\}$
3	$\{4\}$	$\{4\}$
4	$\{5\}$	$\emptyset$
5	$\emptyset$	$\{5\}$

a. Draw a transition diagram of the NFA (note that the accepting states are not specified).

b. Calculate $\delta^*(1, ab)$.
Hint: first calculate $\delta^*(1, \Lambda)$, then $\delta^*(1, a)$, then $\delta^*(1, ab)$.

c. Calculate $\delta^*(1, abaab)$.

Exercise 3.24. ♣

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA with no Λ-transitions. Show that for every $q \in Q$ and every $\sigma \in \Sigma$, $\delta^*(q, \sigma) = \delta(q, \sigma)$.

Exercise 3.33. ♣

Give an example of a regular language L containing Λ that cannot be accepted by any NFA having only one accepting state and no Λ-transitions, and show that your answer is correct.

Exercise 3.22.
A transition table is given for an NFA with seven states.

q	$\delta(q,a)$	$\delta(q,b)$	$\delta(q,\Lambda)$
1	$\emptyset$	$\emptyset$	$\{2\}$
2	$\{3\}$	$\emptyset$	$\{5\}$
3	$\emptyset$	$\{4\}$	$\emptyset$
4	$\{4\}$	$\emptyset$	$\{1\}$
5	$\emptyset$	$\{6,7\}$	$\emptyset$
6	$\{5\}$	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	$\{1\}$

Find:
d. ♣ $\delta^*(1, ba)$
Hint: first calculate $\delta^*(1,\Lambda)$, then $\delta^*(1,b)$, then $\delta^*(1,ba)$.
e. $\delta^*(1, ab)$
f. $\delta^*(1, ababa)$

Exercise 3.32.

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA accepting a language L. Assume that there are no transitions to q_0, that A has only one element, q_f, and that there are no transitions from q_f.

a. Let M_1 be obtained from M by adding Λ-transitions from q_0 to every state that is reachable from q_0 in M.
(If p and q are states, q is reachable from p if there is a string $x \in \Sigma^*$ such that $q \in \delta^*(p, x)$.)
Describe (in terms of L) the language accepted by M_1.

b. Let M_2 be obtained from M by adding Λ-transitions to q_f from every state from which q_f is reachable in M.
Describe (in terms of L) the language accepted by M_2.

c. Let M_3 be obtained from M by adding both the Λ-transitions in (a) and those in (b).
Describe (in terms of L) the language accepted by M_3.