From lecture 2:

Example 4.10.

Find a context-free grammar generating the language

$$\{a^i b^j c^k \mid j \neq i + k\}$$

Hint: Consider the cases $j > i + k$ and $j < i + k$ separately.

Exercise 4.12.

Find a context-free grammar generating the language

$$\{a^i b^j c^k \ \mid \ i \neq j + k\}$$

Exercise 4.9. Suppose that $G_1 = (V_1, \{a, b\}, S_1, P_1)$ and $G_2 = (V_2, \{a, b\}, S_2, P_2)$ are CFGs and that $V_1 \cap V_2 = \varnothing$.

a. It is easy to see that no matter what G_1 and G_2 are, the CFG $G_u = (V_u, \{a, b\}, S_u, P_u)$ defined by $V_u = V_1 \cup V_2$, $S_u = S_1$ and $P_u = P_1 \cup P_2 \cup \{S_1 \to S_2\}$ generates every string in $L(G_1) \cup L(G_2)$. Find grammars G_1 and G_2 (you can use $V_1 = \{S_1\}$ and $V_2 = \{S_2\}$) and a string $x \in L(G_u)$ such that $x \notin L(G_1) \cup L(G_2)$.

b. As in part (a), the CFG $G_c = (V_c, \{a, b\}, S_c, P_c)$ defined by $V_c = V_1 \cup V_2$, $S_c = S_1$ and $P_c = P_1 \cup P_2 \cup \{S_1 \to S_1S_2\}$ generates every string in $L(G_1)L(G_2)$.
Find grammars G_1 and G_2 (again with $V_1 = \{S_1\}$ and $V_2 = \{S_2\}$) and a string $x \in L(G_c)$ such that $x \notin L(G_1)L(G_2)$.

Exercise 4.9. (continued)

c. The CFG $G^* = (V, \{a,b\}, S, P)$ defined by $V = V_1$, $S = S_1$ and $P = P_1 \cup \{S_1 \to S_1S_1 \mid \Lambda\}$ generates every string in $L(G_1)^*$. Find a grammar G_1 with $V_1 = \{S_1\}$ and a string $x \in L(G^*)$ such that $x \notin L(G_1)^*$.

Exercise.

Suppose that $G_1 = (V_1, \{a, b\}, S_1, P_1)$ and $G_2 = (V_2, \{a, b\}, S_2, P_2)$ are CFGs.

In Theorem 4.9(1), it is proven that CFG $G_u = (V_u, \{a, b\}, S_u, P_u)$ defined by $V_u = V_1 \cup V_2$, S_u a new variable, and $P_u = P_1 \cup P_2 \cup \{S_u \to S_1 \mid S_2\}$ generates $L(G_1) \cup L(G_2)$. The proof assumed that $V_1 \cap V_2 = \varnothing$.

Give an example of two context-free grammars G_1 and G_2, with $V_1 \cap V_2 \neq \varnothing$, such that the resulting grammar G_u does not (precisely) generate $L(G_1) \cup L(G_2)$. In particular, for this example, give a string $x \in \{a, b\}^*$, such that either $x \in L(G_1) \cup L(G_2)$ and $x \notin L(G_u)$, or $x \in L(G_u)$ and $x \notin L(G_1) \cup L(G_2)$.

Exercise 4.26. ♣

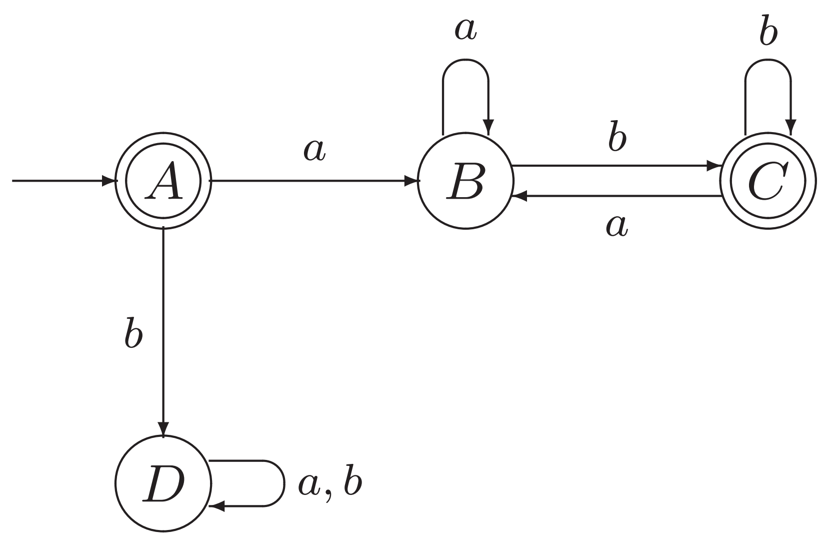
In each part, draw an NFA (which might be an FA) accepting the language generated by the CFG having the given productions.

a.

$$S \to aA \mid bC \qquad A \to aS \mid bB \qquad B \to aC \mid bA \qquad C \to aB \mid bS \mid \Lambda$$

Exercise 4.27. ♣

Find a regular grammar generating the language $L(M)$, where M is the FA shown below:



Exercise 4.22.

Show that if G is a context-free grammar in which every production has one of the forms

$$A \to aB, \quad A \to a \quad \text{and } A \to \Lambda$$

(where A and B are variables and a is a terminal), then $L(G)$ is regular.

Suggestion: construct an NFA accepting $L(G)$, in which there is a state for each variable in G and one additional state F, the only accepting state.

Exercise 4.28. ♣

Draw an NFA accepting the language generated by the grammar with productions

$$S \to abA \mid bB \mid aba \qquad A \to b \mid aB \mid bA \qquad B \to aB \mid aA$$

Exercise 4.29. ♣

Each of the following grammars, though not regular, generates a regular language. In each case, find a regular grammar generating the language.

a. $S \to SSS \mid a \mid ab$

b. $S \to AabB \quad A \to aA \mid bA \mid \Lambda \quad B \to Bab \mid Bb \mid ab \mid b$

Exercise. *From exam Automata Theory, 19 December, 2024*

Each regular language is also context-free, i.e., it can be generated by a context-free grammar. However, not all context-free languages are also regular. For each of the following context-free grammars G, with start variable S, indicate whether or not $L(G)$ is regular. You do not need to explain your answers.

(a) G has productions

$$S \to abA \mid bB \mid aba \qquad A \to b \mid aB \mid bA \qquad B \to aB \mid bA$$

(b) G has productions

$$S \to aS \mid Sb \mid a \mid b$$

(c) G has productions

$$S \to aA \mid b \qquad A \to Sb \mid a$$

(d) G has productions

$$S \to aA \mid bB \qquad A \to aB \mid bA \mid bS \qquad B \to bS \mid \Lambda$$

Exercise 4.15/4.19.

Describe the language generated by the CFG with productions

$$S \to a \mid Sa \mid bSS \mid SSb \mid SbS$$

Motivate your answer.