From lecture 1:

Example 4.1.

$AnBn = \{ \ a^n b^n \mid n \geq 0 \ \} \subseteq \{a, b\}^*$

Recursive definition:

- $\Lambda \in AnBn$ (basis)
- for every $x \in AnBn$, also $axb \in AnBn$ (induction)

$S \rightarrow \Lambda$
$S \rightarrow aSb$

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aa\,bb$

Exercise 4.10. Find context-free grammars generating each of the languages below.

a. ♣ $\{a^i b^j \mid i \leq j\}$

c. ♣ $\{a^i b^j \mid j = 2i\}$

e. ♣ $\{a^i b^j \mid j \leq 2i\}$

f. ♣ $\{a^i b^j \mid j < 2i\}$

d. ♣ $\{a^i b^j \mid i \leq j \leq 2i\}$

d2. $\{a^i b^j \mid i < j < 2i\}$

Exercise 4.1. ♣

In each case below, say what language (a subset of $\{a, b\}^*$) is generated by the context-free grammar with the indicated productions.

b. $S \to SS \mid bS \mid a$

c. $S \to SaS \mid b$

e. $S \to TT \qquad T \to aT \mid Ta \mid b$

g. $S \to aT \mid bT \mid \Lambda \qquad T \to aS \mid bS$

From lecture 5:

Example 4.3.

$Pal = \{x \in \{a,b\}^* \mid x^r = x\}$ (reverse)
In canonical (shortlex) order:
$\Lambda, a, b, aa, bb, aaa, aba, bab, bbb, aaaa, abba, \ldots$

Recursive definition:

- $\Lambda, a, b \in Pal$
- for every $x \in Pal$, also $axa, bxb \in Pal$

$S \to \Lambda \mid a \mid b$
$S \to aSa$
$S \to bSb$

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aababaa$

Exercise 4.3. ♣

In each case below, find a CFG generating the given language.

b. The set of even-length strings in $\{a, b\}^*$ with the two middle symbols equal.

c. The set of odd-length strings in $\{a, b\}^*$ whose first, middle, and last symbols are all the same.

Exercise 4.1. ♣

In each case below, say what language (a subset of $\{a, b\}^*$) is generated by the context-free grammar with the indicated productions.

f. $S \to aSa \mid bSb \mid aAb \mid bAa \qquad A \to aAa \mid bAb \mid a \mid b \mid \Lambda$

Exercise 4.4. ♣

In both parts below, the productions in a CFG G are given. In each part, show first that for every string $x \in L(G)$, $n_a(x) = n_b(x)$; then find a string $x \in \{a, b\}^*$ with $n_a(x) = n_b(x)$ that is not in $L(G)$.

a. $S \to SabS \mid SbaS \mid \Lambda$

b. $S \to aSb \mid bSa \mid abS \mid baS \mid Sab \mid Sba \mid \Lambda$

Exercise.

Find a context-free grammar generating the language

$$\{a^i b^j c^k \ \mid\ i = j + k\}$$

Exercise.

Find a context-free grammar generating the language

$$\{a^i b^j c^k \ \mid \ j = i + k\}$$