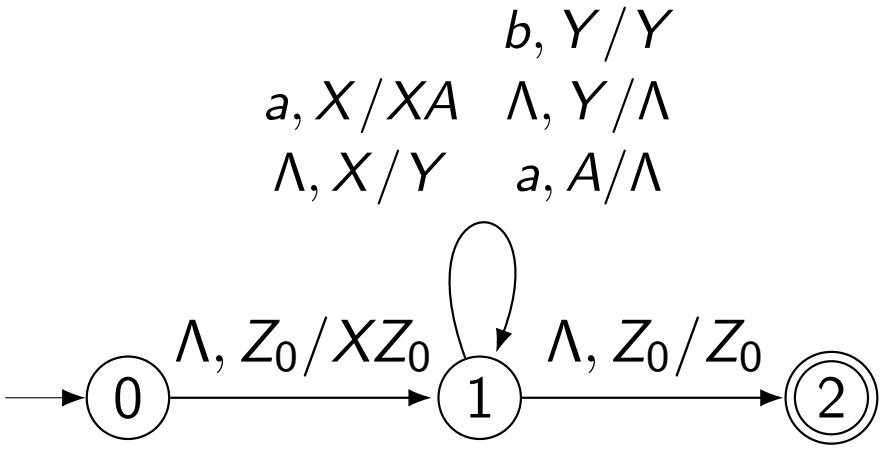
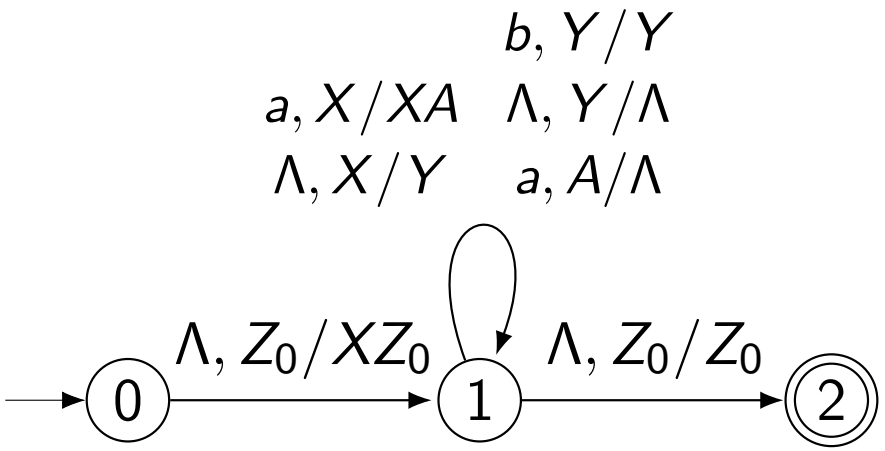
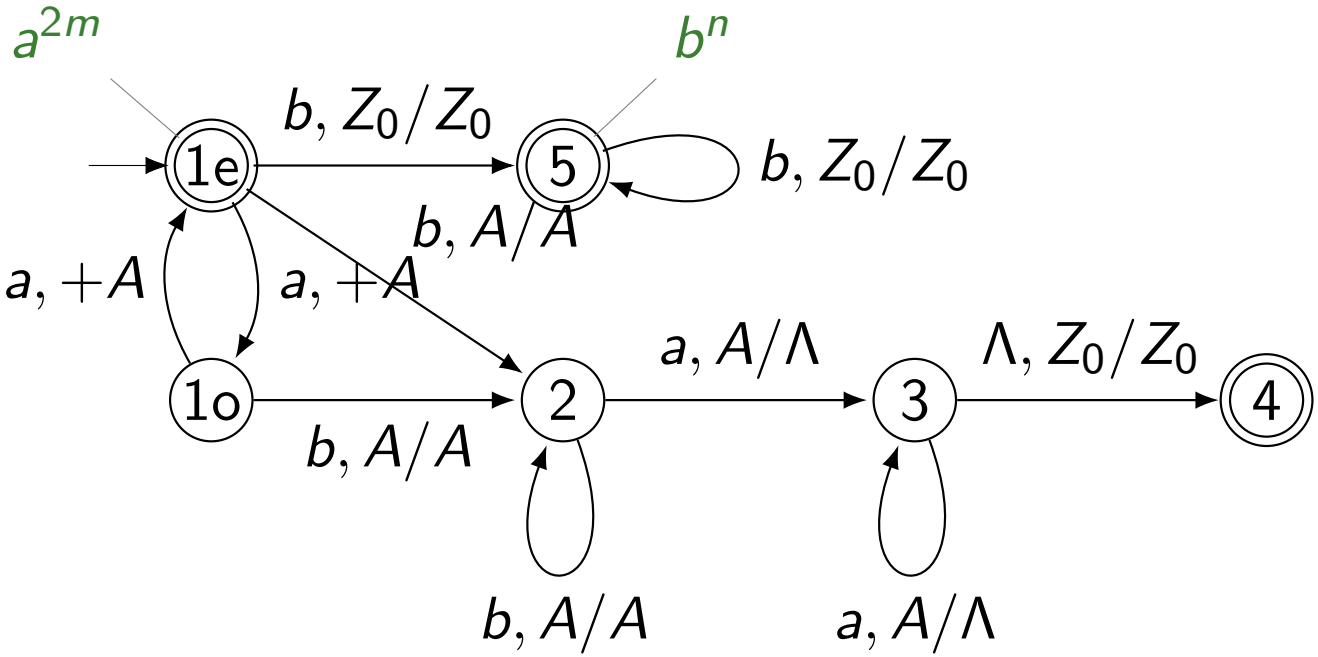
Huiswerk

- Eind deze week: huiswerkopgave 3


b, Y/Y
a, X/XA
Λ, Y/Λ
Λ, X/Y
a, A/Λ
Λ, Z0/XZ0
Λ, Z0/Z0
0
1
2


b, Y/Y
a, X/XA
Λ, Y/Λ
Λ, X/Y
a, A/Λ
Λ, Z0/XZ0
Λ, Z0/Z0
0
1
2


a2m
bn
b, Z0/Z0
b, Z0/Z0
1e
5
b, A/A
a, +A
a, +A
a, A/Λ
Λ, Z0/Z0
1o
2
3
4
b, A/A
b, A/A
a, A/Λ

The first PDA in the previous slide (about $a^m b^n a^m$) is not deterministic. Actually it is working like a grammar: in state 1 the following productions are simulated:
$X \to aXA \mid Y$
$Y \to bY \mid \Lambda$
$A \to a$

The second automaton is deterministic. We have to distinguish the cases where $m = 0$ (state 5) and $n = 0$ (states $1e$ and $1o$).

Top-down, example

$L = \{ a^i b^j \mid i \neq j \}$

$S \to X \mid Y$ (choice!)

$X \to aXb \mid aX \mid a \quad (i > j)$

$Y \to aYb \mid Yb \mid b \quad (i < j)$

$S \Rightarrow X \Rightarrow aXb \Rightarrow aaXb \Rightarrow aaaXbb \Rightarrow aaaabb$

Example

$L = \{ a^i b^j \mid i \neq j \}$

$S \to X \mid Y$ (choice!)

$X \to aXb \mid aX \mid a$ $(i > j)$

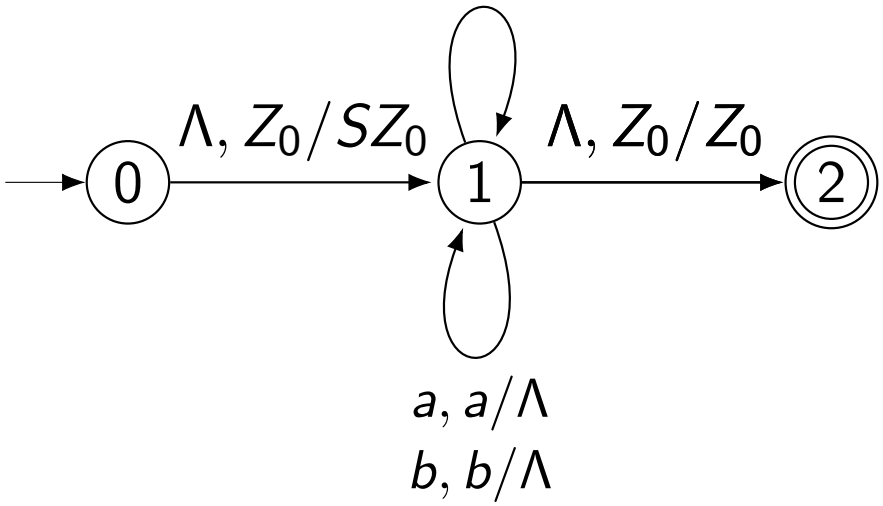
$Y \to aYb \mid Yb \mid b$ $(i < j)$

$\Lambda, S/X$ $\Lambda, S/Y$

$\Lambda, X/aXb$ $\Lambda, Y/aYb$

$\Lambda, X/aX$ $\Lambda, Y/Yb$

$\Lambda, X/a$ $\Lambda, Y/b$



CFG $G = (V, \Sigma, S, P)$

Definition (Nondeterministic Top-Down PDA)

$NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, as follows:

- $Q = \{q_0, q_1, q_2\}$
- $A = \{q_2\}$
- $\Gamma = V \cup \Sigma \cup \{Z_0\}$
- start $\quad \delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$
- *expand* $\quad \delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha \text{ in } P\} \quad$ for $A \in V$
- *match* $\quad \delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\} \quad$ for $\sigma \in \Sigma$
- finish $\quad \delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\} \quad$ check empty stack

[M] Def 5.17

From lecture 7:

$AEqB = \{ x \in \{a, b\}^* \mid n_a(x) = n_b(x) \}$

$aaabbb, ababab, aababb, \ldots$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$

A generates $n_a(x) = n_b(x) + 1$

B generates $n_a(x) + 1 = n_b(x)$

$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow \ldots$ (different options)

(1) $aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow aababB \Rightarrow aababbS \Rightarrow aababb$

(2) $aabaBB \Rightarrow aababSB \Rightarrow aababB \Rightarrow aababbS \Rightarrow aababb$

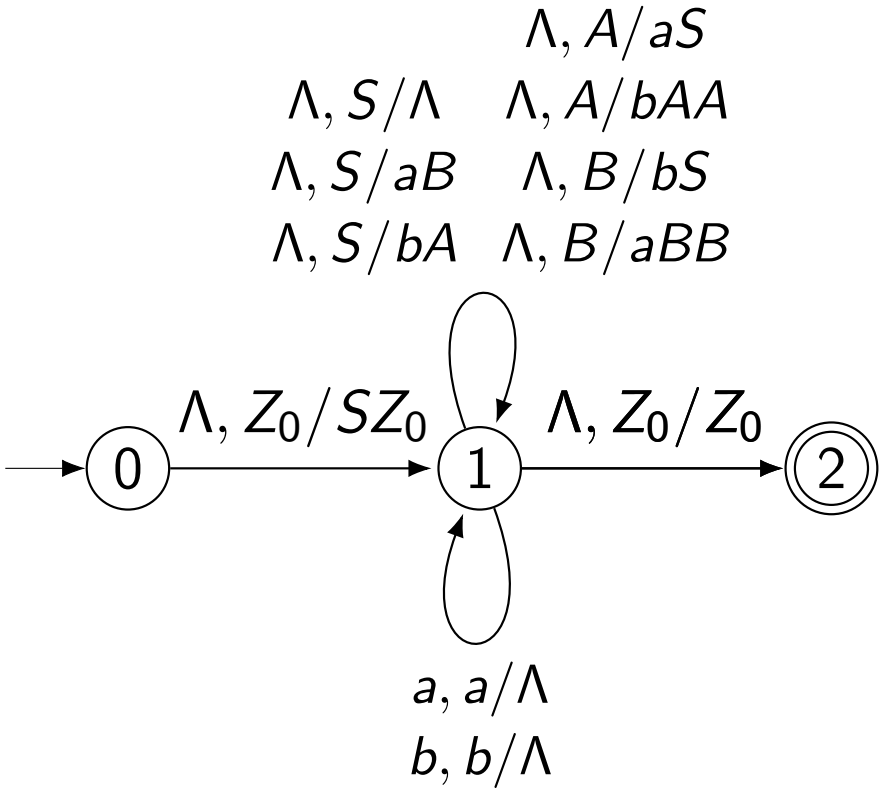
(2') $aabaBB \Rightarrow aabaBbS \Rightarrow aababSbS \Rightarrow aababSb \Rightarrow aababb$

[M] E 4.8

Example

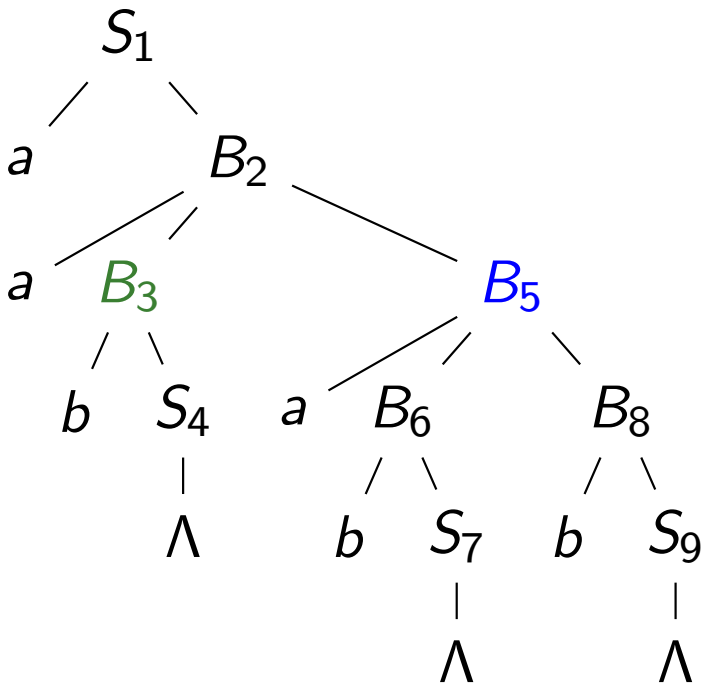
$$AEqB = \{\ x \in \{a, b\}^* \mid n_a(x) = n_b(x)\ \}$$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$

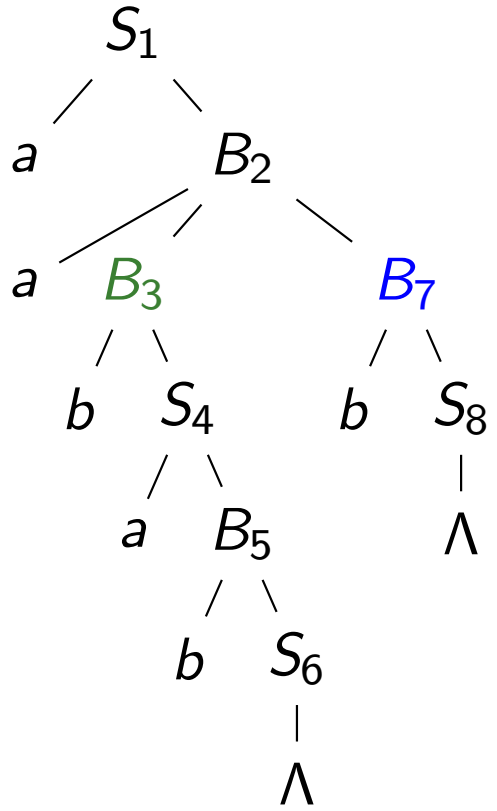


Derivation tree & leftmost derivations

From lecture 7:



$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow$
$aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow$
$aababB \Rightarrow aababbS \Rightarrow aababb$

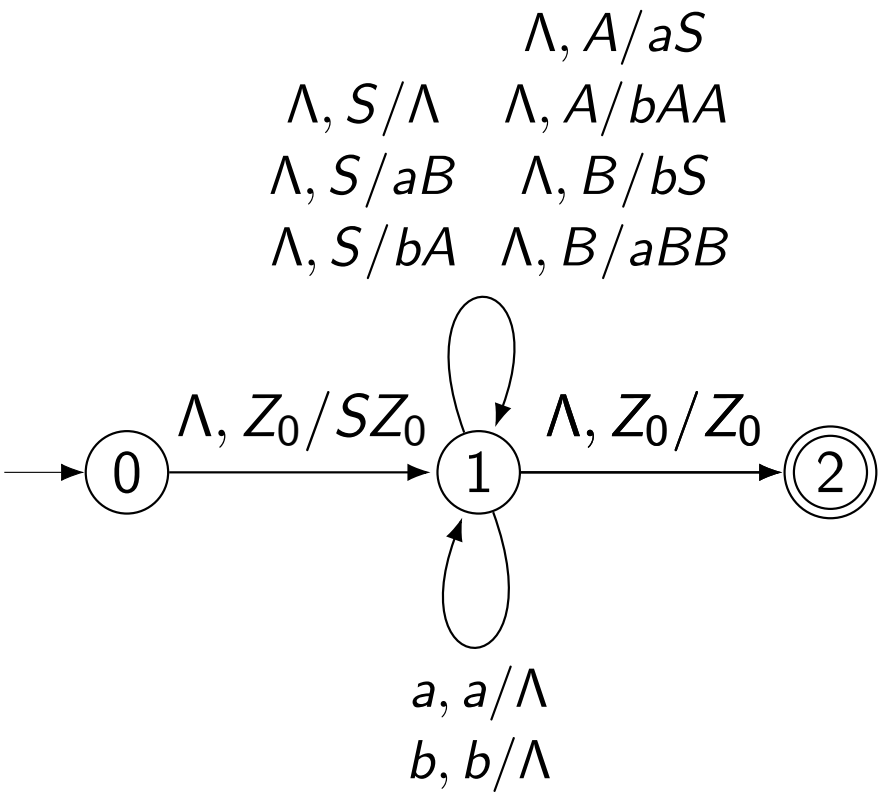


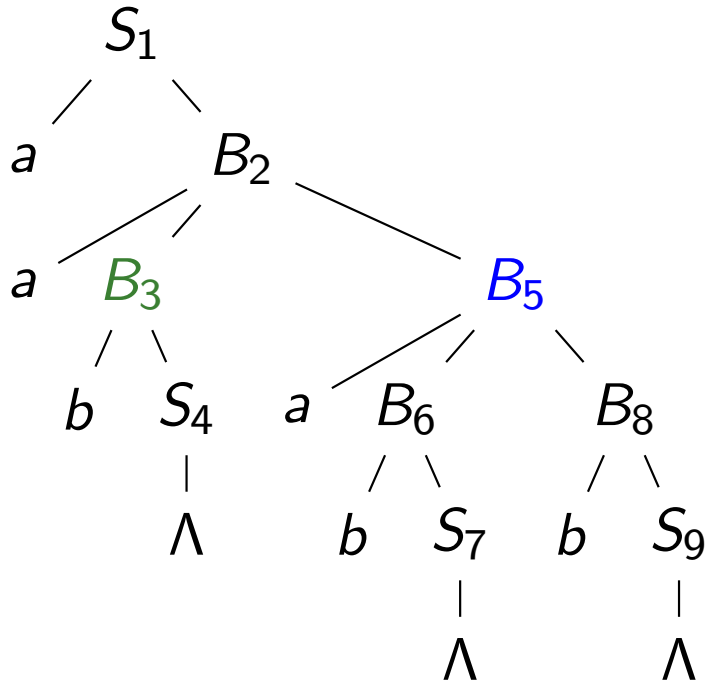
$S \Rightarrow aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow$
$aabaBB \Rightarrow aababSB \Rightarrow aababB \Rightarrow$
$aababbS \Rightarrow aababb$

Top-down = expand-match

$$AEqB = \{\, x \in \{a,b\}^* \mid n_a(x) = n_b(x) \,\}$$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$





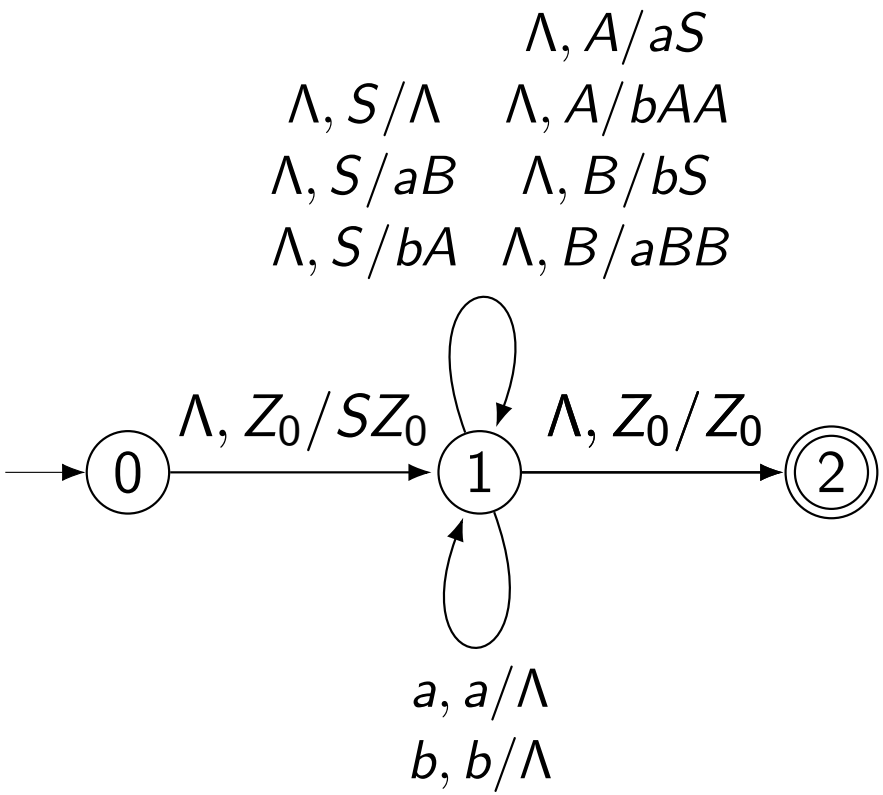
q_0 $aababb$ Z_0

...

Top-down = expand-match

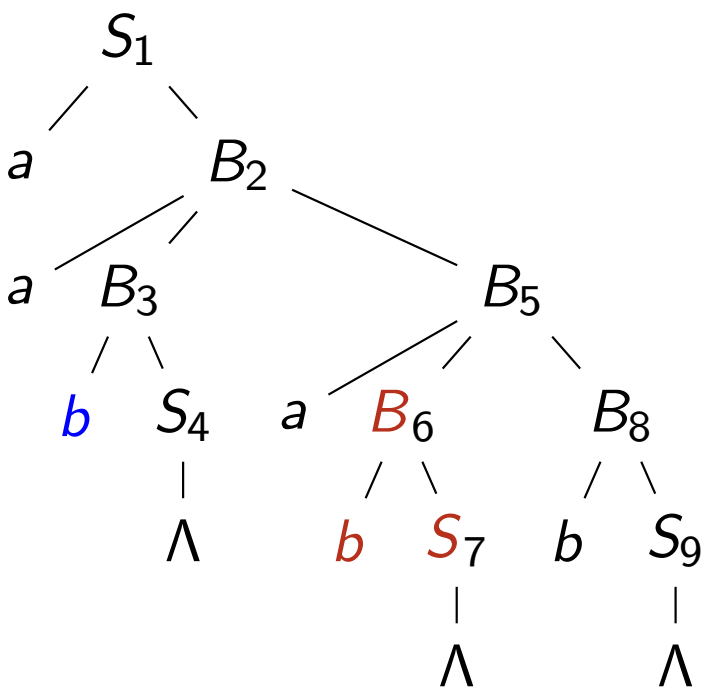
$$AEqB = \{\, x \in \{a,b\}^* \mid n_a(x) = n_b(x) \,\}$$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$



q_0	$aababb$	Z_0	
q_1	$aababb$	$S\ Z_0$	$1 : S \to aB$
q_1	$aababb$	$aB\ Z_0$	match a
q_1	$a\ ababb$	$B\ Z_0$	$2 : B \to aBB$
q_1	$a\ ababb$	$aBB\ Z_0$	match a
q_1	$aa\ babb$	$BB\ Z_0$	$3 : B \to bS$
q_1	$aa\ babb$	$bSB\ Z_0$	match b
q_1	$aab\ abb$	$SB\ Z_0$	$4 : S \to \Lambda$
q_1	$aab\ abb$	$B\ Z_0$	$5 : B \to aBB$
q_1	$aab\ abb$	$aBB\ Z_0$	match a
q_1	$aaba\ bb$	$BB\ Z_0$	$6 : B \to bS$
q_1	$aaba\ bb$	$bSB\ Z_0$	match b
q_1	$aabab\ b$	$SB\ Z_0$	$7 : S \to \Lambda$
q_1	$aabab\ b$	$B\ Z_0$	$8 : B \to bS$
q_1	$aabab\ b$	$bS\ Z_0$	match b
q_1	$aababb$	$S\ Z_0$	$9 : S \to \Lambda$
q_1	$aababb$	Z_0	
q_2	$aababb$	Z_0	

Top-down = expand-match

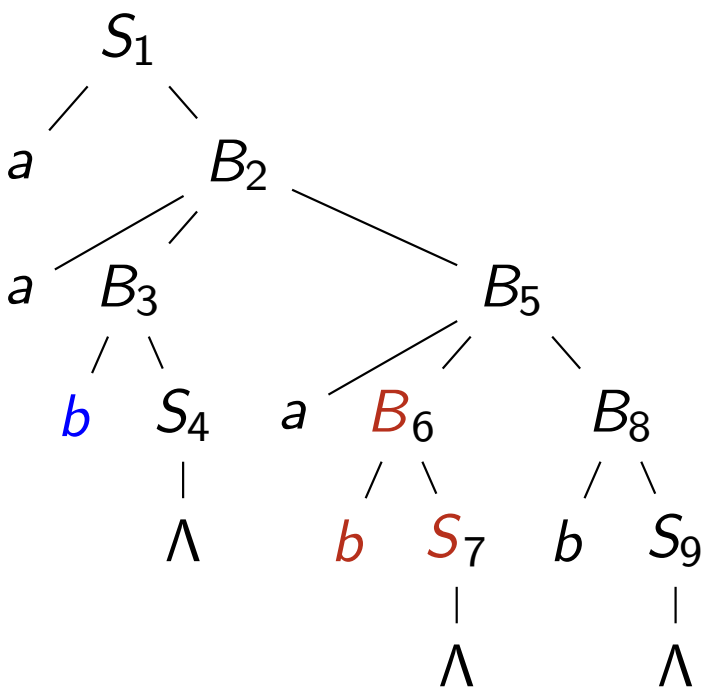


preorder: leftmost

$S \overset{\ell}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow$
$aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow$
$aababB \Rightarrow aababbS \Rightarrow aababb$

q_0	$aababb$	Z_0	
q_1	$aababb$	$S\,Z_0$	$1 : S \to aB$
q_1	$aababb$	$aB\,Z_0$	match a
q_1	$a\,ababb$	$B\,Z_0$	$2 : B \to aBB$
q_1	$a\,ababb$	$aBB\,Z_0$	match a
q_1	$aa\,babb$	$BB\,Z_0$	$3 : B \to bS$
q_1	$aa\,babb$	$bSB\,Z_0$	match b
q_1	$aab\,abb$	$SB\,Z_0$	$4 : S \to \Lambda$
q_1	$aab\,abb$	$B\,Z_0$	$5 : B \to aBB$
q_1	$aab\,abb$	$aBB\,Z_0$	match a
q_1	$aaba\,bb$	$BB\,Z_0$	$6 : B \to bS$
q_1	$aaba\,bb$	$bSB\,Z_0$	match b
q_1	$aabab\,b$	$SB\,Z_0$	$7 : S \to \Lambda$
q_1	$aabab\,b$	$B\,Z_0$	$8 : B \to bS$
q_1	$aabab\,b$	$bS\,Z_0$	match b
q_1	$aababb$	$S\,Z_0$	$9 : S \to \Lambda$
q_1	$aababb$	Z_0	
q_2	$aababb$	Z_0	

Top-down = expand-match



preorder: leftmost

$S \overset{\ell}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aabSB \Rightarrow$
$aabB \Rightarrow aabaBB \Rightarrow aababSB \Rightarrow$
$aababB \Rightarrow aababbS \Rightarrow aababb$

q_0	$aababb$	Z_0	
q_1	$aababb$	$S\ Z_0$	$1 : S \to aB$
q_1	$aababb$	$aB\ Z_0$	match a
q_1	$a\ ababb$	$B\ Z_0$	$2 : B \to aBB$
q_1	$a\ ababb$	$aBB\ Z_0$	match a
q_1	$aa\ babb$	$BB\ Z_0$	$3 : B \to bS$
q_1	$aa\ babb$	$bSB\ Z_0$	match b
q_1	$aab\ abb$	$SB\ Z_0$	$4 : S \to \Lambda$
q_1	$aab\ abb$	$B\ Z_0$	$5 : B \to aBB$
q_1	$aab\ abb$	$aBB\ Z_0$	match a
q_1	$aaba\ bb$	$BB\ Z_0$	$6 : B \to bS$
q_1	$aaba\ bb$	$bSB\ Z_0$	match b
q_1	$aabab\ b$	$SB\ Z_0$	$7 : S \to \Lambda$
q_1	$aabab\ b$	$B\ Z_0$	$8 : B \to bS$
q_1	$aabab\ b$	$bS\ Z_0$	match b
q_1	$aababb$	$S\ Z_0$	$9 : S \to \Lambda$
q_1	$aababb$	Z_0	
q_2	$aababb$	Z_0	

Theorem

If G is a context-free grammar, then the nondeterministic top-down PDA $\mathrm{NT}(G)$ *accepts the language* $L(G)$.

The details of the proof of this result do not have to be known for the exam.

Intuition $L(G) \subseteq L(NT(G))\ldots$:

- wish to simulate derivation on stack
- match terminals when on top of stack
- topmost variable first, so leftmost derivation

[M] Th 5.18

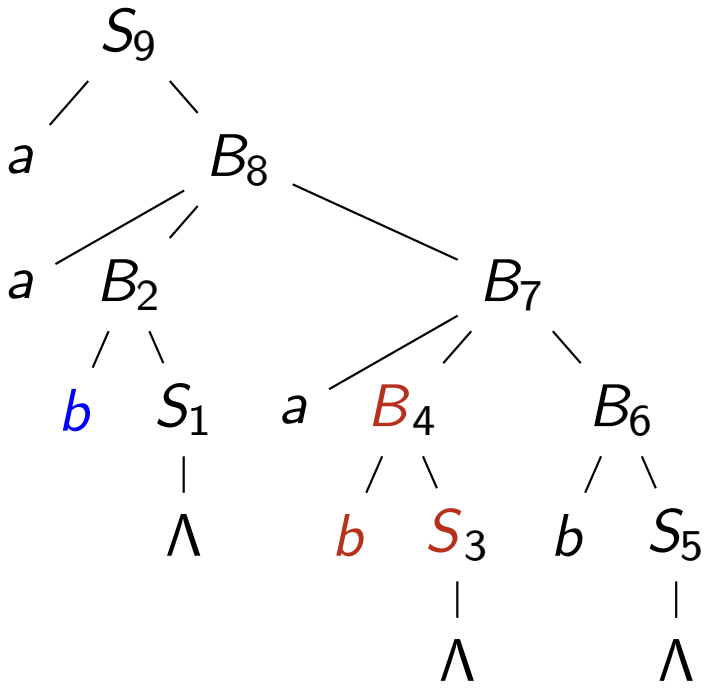
Bottom-up = shift-reduce

$$S \to abc$$

Bottom-up = shift-reduce

$$AEqB = \{\, x \in \{a,b\}^* \mid n_a(x) = n_b(x) \,\}$$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$



	stack^r	input	
q_0	Z_0	$aababb$	shift a
$\ldots$			

Bottom-up = shift-reduce

$$AEqB = \{\, x \in \{a,b\}^* \mid n_a(x) = n_b(x) \,\}$$

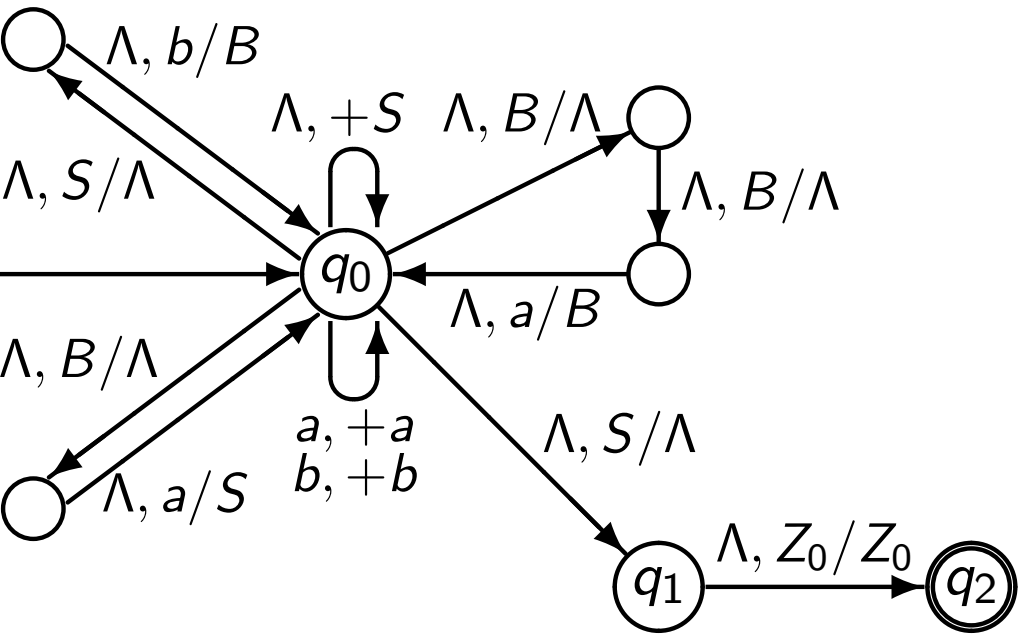
$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

Bottom-up = shift-reduce

$$AEqB = \{\, x \in \{a,b\}^* \mid n_a(x) = n_b(x) \,\}$$

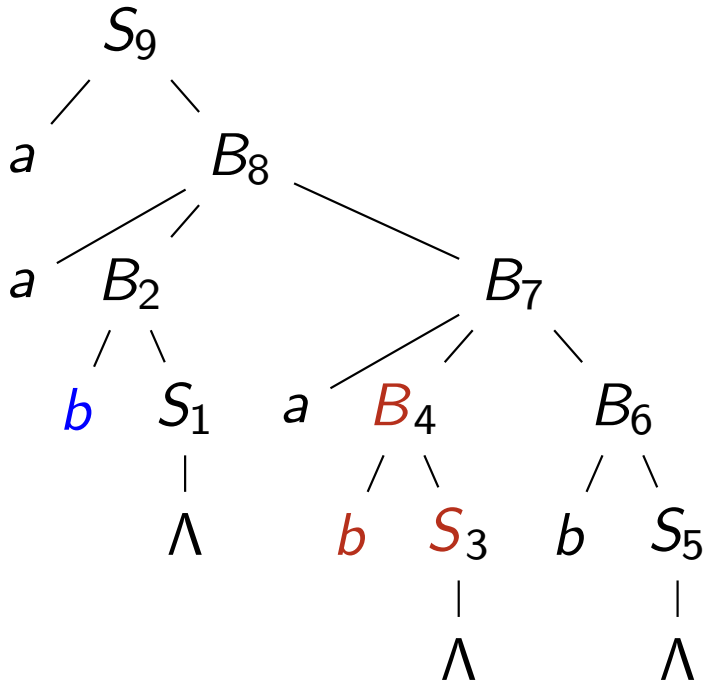
$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$



+ states/transitions for other productions

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

Bottom-up = shift-reduce

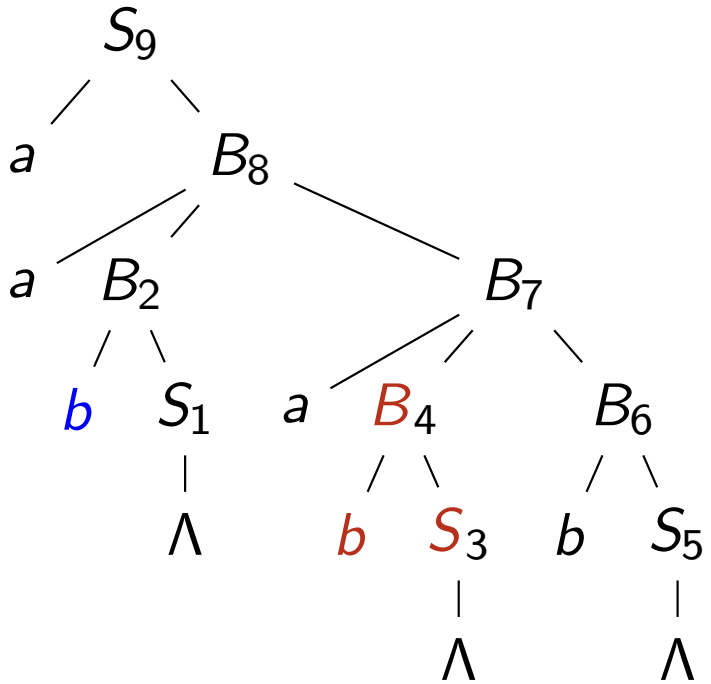


postorder: rightmost

$S \overset{r}{\Rightarrow}_9 aB \Rightarrow_8 aaBB \Rightarrow_7 aaBaBB \Rightarrow_6$
$aaBaBbS \Rightarrow_5 aaBaBb \Rightarrow_4 aaBabSb$
$\Rightarrow_3 aaBabb \Rightarrow_2 aabSabb \Rightarrow_1 aababb$

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

Bottom-up = shift-reduce



postorder: rightmost, in reverse

$S \overset{r}{\Rightarrow}_9 aB \Rightarrow_8 aaBB \Rightarrow_7 aaBaBB \Rightarrow_6$
$aaBaBbS \Rightarrow_5 aaBaBb \Rightarrow_4 aaBabSb$
$\Rightarrow_3 aaBabb \Rightarrow_2 aabSabb \Rightarrow_1 aababb$

	stack^r	input	
q_0	Z_0	$aababb$	shift a
q_0	$Z_0\ a$	$a\ ababb$	shift a
q_0	$Z_0\ aa$	$aa\ babb$	shift b
q_0	$Z_0\ aab$	$aab\ abb$	$1 : S \to \Lambda$
q_0	$Z_0\ aabS$	$aab\ abb$	$2 : B \to bS$
q_0	$Z_0\ aaB$	$aab\ abb$	shift a
q_0	$Z_0\ aaBa$	$aaba\ bb$	shift b
q_0	$Z_0\ aaBab$	$aabab\ b$	$3 : S \to \Lambda$
q_0	$Z_0\ aaBabS$	$aabab\ b$	$4 : B \to bS$
q_0	$Z_0\ aaBaB$	$aabab\ b$	shift b
q_0	$Z_0\ aaBaBb$	$aababb$	$5 : S \to \Lambda$
q_0	$Z_0\ aaBaBbS$	$aababb$	$6 : B \to bS$
q_0	$Z_0\ aaBaBB$	$aababb$	$7 : B \to aBB$
q_0	$Z_0\ aaBB$	$aababb$	$8 : B \to aBB$
q_0	$Z_0\ aB$	$aababb$	$9 : S \to aB$
q_0	$Z_0\ S$	$aababb$	
q_1	Z_0	$aababb$	
q_2	Z_0	$aababb$	

ABOVE

To write down the construction of the shift-reduce PDA for a given CFG, we have two technical problems.

Consider a production $A \to \alpha$

First the stack (in standard notation) now contains the string α in reverse.

Second, we pop α, that is, several symbols, rather than exactly one. This can be simulated by popping the symbols one-by-one, using $|\alpha|$ separate transitions.

shift $\quad \delta(q_0, \sigma, X) = \{(q_0, \sigma X)\} \quad$ for $\sigma \in \Sigma$, $X \in \Gamma$

reduce $\quad$ '$\delta^*(q_0, \Lambda, \alpha^r) \ni (q_0, A)$' $\quad$ for $A \to \alpha$ in P

Special case: $\alpha = \Lambda$

Definition

The Nondeterministic Bottom-Up PDA $NB(G)$
Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic bottom-up PDA corresponding to G is $NB(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:
Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.
$\Gamma = \ldots$

[M] D 5.22

Definition

The Nondeterministic Bottom-Up PDA $NB(G)$
Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic bottom-up PDA corresponding to G is $NB(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:
Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.
$\Gamma = V \cup \Sigma \cup \{Z_0\}$

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Definition

The Nondeterministic Bottom-Up PDA $NB(G)$ (continued)
For every $\sigma \in \Sigma$ and every $X \in \Gamma$, $\delta(q_0, \sigma, X) = \{(q_0, \sigma X)\}$. This is a *shift* move.
For every production $B \to \alpha$ in G, and every nonnull string $\beta \in \Gamma^*$,
$(q_0, \Lambda, \alpha^r \beta) \vdash^* (q_0, \Lambda, B\beta)$,
where this *reduction* is a sequence of one or more moves in which, if there is more than one, the intermediate configurations involve other states that are specific to this sequence and appear in no other moves of $NB(G)$.
One of the elements of $\delta(q_0, \Lambda, S)$ is (q_1, Λ),
and $\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$.

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Theorem

If G is a context-free grammar, then the nondeterministic bottom-up PDA $\mathrm{NB}(G)$ *accepts the language* $L(G)$.

The details of the proof of this result do not have to be known for the exam.

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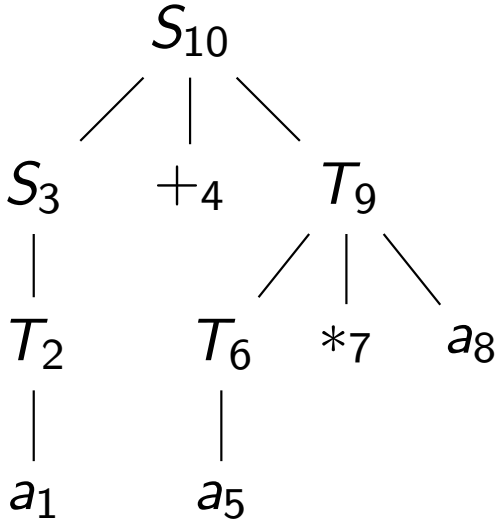
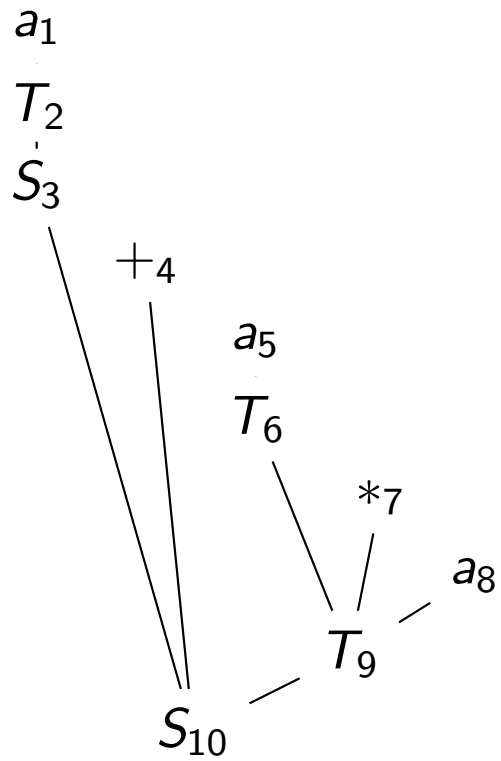
Example: algebraic expressions

shift-reduce

post-order reduction $\equiv$ rightmost derivation, bottom-up



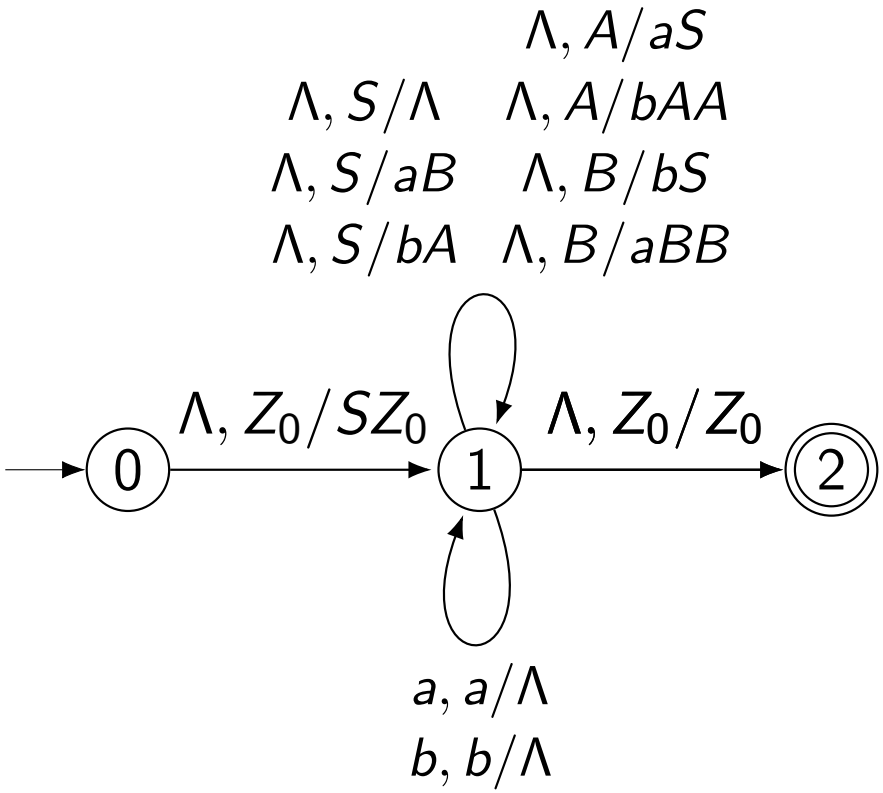
stack [reverse]	input
Z_0	$a + a * a$
$Z_0\ a_1$	$+a * a$
$Z_0\ T_2$	$+a * a$
$Z_0\ S_3$	$+a * a$
$Z_0\ S_3\ +_4$	$a * a$
$Z_0\ S_3\ +_4\ a_5$	$*a$
$Z_0\ S_3\ +_4\ T_6$	$*a$
$Z_0\ S_3\ +_4\ T_6\ *_7$	a
$Z_0\ S_3\ +_4\ T_6\ *_7\ a_8$	
$Z_0\ S_3\ +_4\ T_9$	
$Z_0\ S_{10}$	
$-$	



From lecture 9:

$$AEqB = \{\, x \in \{a, b\}^* \mid n_a(x) = n_b(x) \,\}$$

$$S \to \Lambda \mid aB \mid bA$$
$$A \to aS \mid bAA$$
$$B \to bS \mid aBB$$



5.5. Parsing: make PDA (more) deterministic by looking ahead one symbol in input.
See Compiler Construction