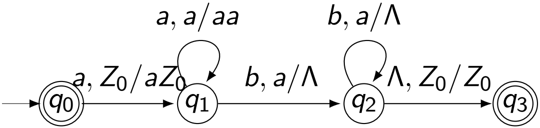
Huiswerk

- Deadline huiswerkopgave 2: woensdag 29 oktober, 23.59
- Beoordelingen huiswerkopgave 1: dinsdag 28 oktober

reg. languages	FA	reg. grammar	reg. expression
determ. cf. languages	DPDA		
cf. languages	PDA	cf. grammar	
cs. languages	LBA	cs. grammar	
re. languages	TM	unrestr. grammar	

$AnBn = \{ \; a^n b^n \mid n \geq 0 \; \}$

initial q_0, Z_0, accept $A = \{q_0, q_3\}$



[M] E 5.3

Stack in PDA contains symbols from certain alphabet.
Usual stack operations: pop, top, push
Extra possiblity: replace top element X by string α

Notation:
If stack contents is $X_1X_2X_3X_4$, then top element is X_1.
If we replace X by string α, then first symbol of α ends up at top of stack.

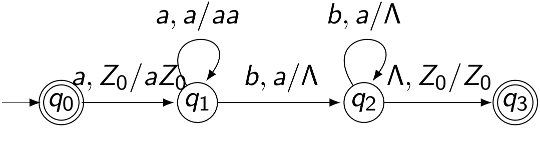
X_1
X_2
X_3
X_4

$\alpha = \Lambda$	pop
$\alpha = X$	top
$\alpha = YX$	push
$\alpha = \beta X$	push*
$\alpha = \ldots$	

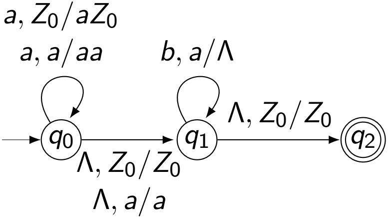
Top element X is required to do a move!

$AnBn = \{\ a^n b^n \mid n \geq 0\ \}$

initial q_0, Z_0, accept $A = \{q_0, q_3\}$



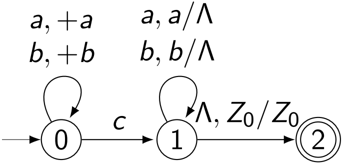
[M] E 5.3

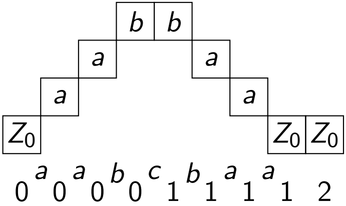


$SimplePal =$
$\{\ xcx^r \mid x \in \{a, b\}^*\ \}$

[M] Fig 5.5

$SimplePal =$
$\{ \; xcx^r \mid x \in \{a, b\}^* \; \}$



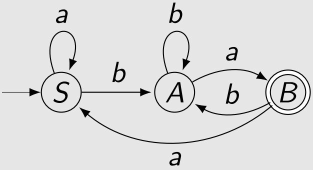


[M] Fig 5.5

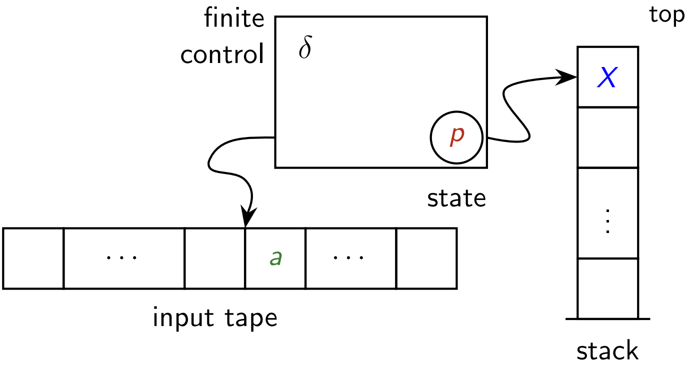
notation. . .

From lecture 6:
systematic approach

Example



axiom S	initial state
$S \rightarrow bA \mid aS$	transitions
$A \rightarrow bA \mid aB$	
$B \rightarrow bA \mid aS$	
$B \rightarrow \Lambda$	accepting state


finite
control
δ
p
state
top
X
⋮
stack
...
a
...
input tape

From lecture 1:

Definition (FA)

[deterministic] finite automaton 5-tuple $M = (Q, \Sigma, q_0, A, \delta)$,
- Q finite set *states*;
- Σ finite *input alphabet*;
- $q_0 \in Q$ *initial* state;
- $A \subseteq Q$ *accepting* states;
- $\delta : Q \times \Sigma \to Q$ *transition function*.

[M] D 2.11 Finite automaton

[L] D 2.1 Deterministic finite accepter, has 'final' states

Definition

PDA 7-tuple $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$

Q	*states*	p, q	
Σ	*input alphabet*	a, b	w, x
Γ	*stack alphabet*	a, b, A, B	α
$q_0 \in Q$	*initial state*		
$Z_0 \in \Gamma$	*initial stack symbol*		
$A \subseteq Q$	*accepting states*		

$\delta : \ldots \to \ldots$

transition function

$$\big(\underbrace{\overset{\text{from}}{p} \quad \underset{\text{read}}{a} \quad \underset{\text{pop}}{X}}_{\text{before}} \quad \underbrace{\overset{\text{to}}{q} \quad \underset{\text{push}}{\alpha}}_{\text{after}} \big)$$

Definition

PDA 7-tuple $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$

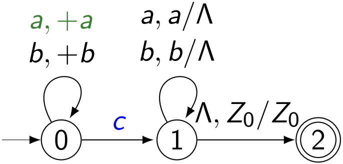
Q	*states*	p, q	
Σ	*input alphabet*	a, b	w, x
Γ	*stack alphabet*	a, b, A, B	α
$q_0 \in Q$	*initial state*		
$Z_0 \in \Gamma$	*initial stack symbol*		
$A \subseteq Q$	*accepting states*		

$\delta : Q \times (\Sigma \cup \{\Lambda\}) \times \Gamma \to 2^{Q \times \Gamma^*}$

transition function (finite)

In principle, Z_0 may be removed from the stack, but often it isn't.

$SimplePal =$
$\{ \ xcx^r \mid x \in \{a, b\}^* \ \}$



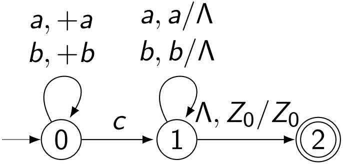
$Q = \{0, 1, 2\}$
$\Sigma = \{a, b, c\}$
$\Gamma = \{a, b, Z_0\}$
$q_0 = 0$
$Z_0 = Z_0$
$A = \{2\}$

Transition table:

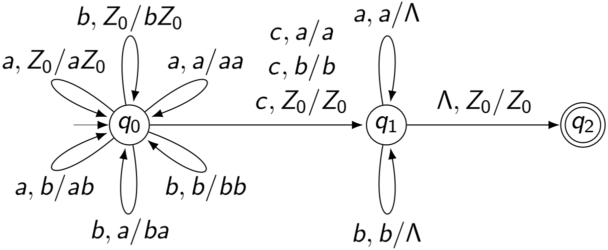
State p	Input σ	Stack Symbol X	Move(s) $\delta(p, \sigma, X)$
0	a	Z_0	$(0, aZ_0)$
0	a	a	$(0, aa)$
0	a	b	$(0, ab)$
0	b	Z_0	$(0, bZ_0)$
0	b	a	$(0, ba)$
0	b	b	$(0, bb)$
0	c	Z_0	$(1, Z_0)$
0	c	a	$(1, a)$
0	c	b	$(1, b)$
1	a	a	$(1, \Lambda)$
1	b	b	$(1, \Lambda)$
1	Λ	Z_0	$(2, Z_0)$
(all other combinations)			none

transition $(q, \alpha) \in \delta(p, a, A)$ $\quad \textcircled{p} \xrightarrow{a, A/\alpha} \textcircled{q}$

intuitive	formalized as		convention
pop A	$(q, \Lambda) \in \delta(p, a, A)$	$\alpha = \Lambda$	$\textcircled{p} \xrightarrow{a, A/\Lambda} \textcircled{q}$
push A	$(q, AX) \in \delta(p, a, X)$	for all $X \in \Gamma$	$\textcircled{p} \xrightarrow{a, +A} \textcircled{q}$
read a	$(q, X) \in \delta(p, a, X)$	for all $X \in \Gamma$	$\textcircled{p} \xrightarrow{a} \textcircled{q}$



The same PDA twice.
First our version, where we allow some shortcuts in notation.
Second as depicted in the book.



[M] Fig 5.5

Notation

Incorrect notations:

$$p \xrightarrow{\sigma, \Lambda/\alpha} q$$

top stack symbol required

$$p \xrightarrow{\sigma, XY/\alpha} q$$

remove/consider one stack symbol at a time

$M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$

configuration (q, x, α) $\quad q \in Q,\ x \in \Sigma^*,\ \alpha \in \Gamma^*$

state, remaining input, stack with top left

step $(p, ax, B\alpha) \vdash_M (q, x, \beta\alpha)$ $\quad$ when $(q, \beta) \in \delta(p, a, B)$

$\vdash_M^n \qquad \vdash_M^* \qquad \vdash \qquad \vdash^n \qquad \vdash^*$

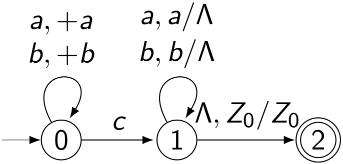
Definition

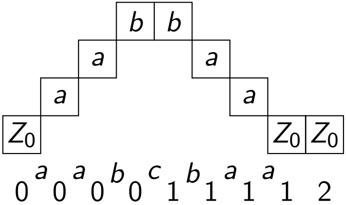
String x accepted by M (by *final state*), if
$(q_0, x, Z_0) \vdash^* (q, \Lambda, \alpha)$ for some $q \in A$, and some $\alpha \in \Gamma^*$
Language accepted by M (by *final state*)
$L(M) = \{\ x \in \Sigma^* \mid x \text{ accepted by } M\ \}$

read complete input, end in accepting state, some path

[M] D 5.2

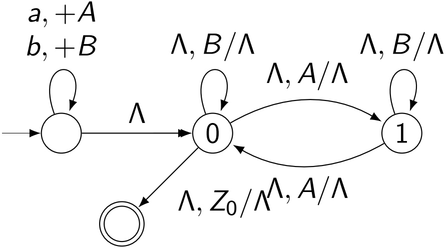
$SimplePal = \{ \, xcx^r \mid x \in \{a, b\}^* \, \}$



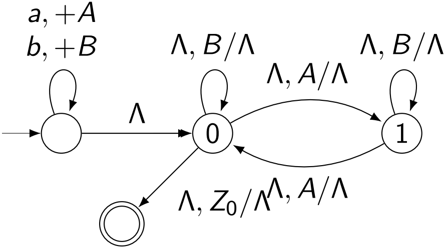


$$
\begin{aligned}
&(0, aabcbaa, Z_0) \vdash \\
&(0, abcbaa, aZ_0) \vdash \\
&(0, bcbaa, aaZ_0) \vdash \\
&(0, cbaa, baaZ_0) \vdash \\
&(1, baa, baaZ_0) \vdash \\
&(1, aa, aaZ_0) \vdash \\
&(1, a, aZ_0) \vdash \\
&(1, \Lambda, Z_0) \vdash \\
&(2, \Lambda, Z_0)
\end{aligned}
$$

[M] Fig 5.5

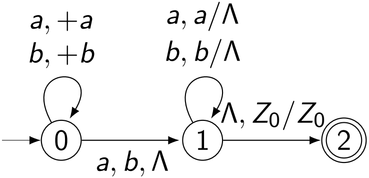


$a, +A$
$b, +B$
$\Lambda, B/\Lambda$
$\Lambda, A/\Lambda$
$\Lambda, B/\Lambda$
Λ
0
1
$\Lambda, Z_0/\Lambda$
$\Lambda, A/\Lambda$

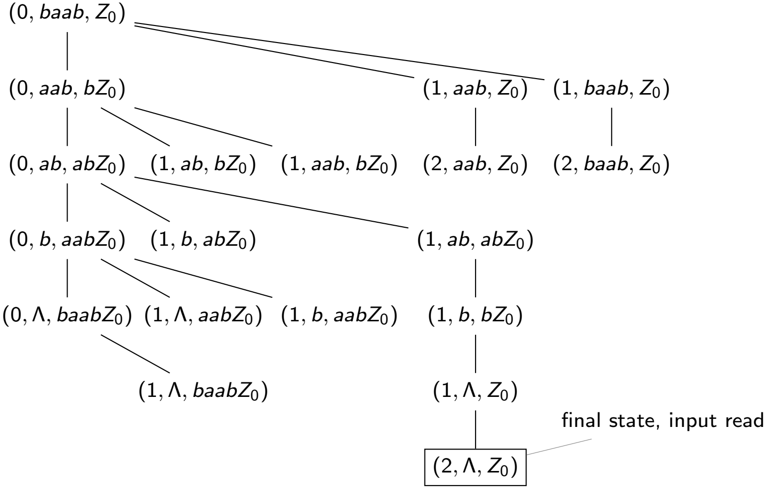


Λ-computations can be very long in PDA, they can even loop.
In the example the input is read and stored on the stack, and at the end of the input it is verified that the string contains an even number of a's.

Pal $\{ y \in \{a, b\}^* \mid y = y^r \}$



$Q = \{0, 1, 2\}$
$\Sigma = \{a, b\}$
$\Gamma = \{a, b, Z_0\}$
$q_0 = 0$
$Z_0 = Z_0$
$A = \{2\}$


$(0, baab, Z_0)$
$(0, aab, bZ_0)$
$(1, aab, Z_0)$
$(1, baab, Z_0)$
$(0, ab, abZ_0)$
$(1, ab, bZ_0)$
$(1, aab, bZ_0)$
$(2, aab, Z_0)$
$(2, baab, Z_0)$
$(0, b, aabZ_0)$
$(1, b, abZ_0)$
$(1, ab, abZ_0)$
$(0, \Lambda, baabZ_0)$
$(1, \Lambda, aabZ_0)$
$(1, b, aabZ_0)$
$(1, b, bZ_0)$
$(1, \Lambda, baabZ_0)$
$(1, \Lambda, Z_0)$
final state, input read
$(2, \Lambda, Z_0)$

[M] Fig 5.9

Note

ABOVE

Non-determinism at work. The PDA for palindromes cannot see what is the middle of the input string, and has to guess. Only one of the guesses leads to an accepting configuration.

intuitively: no choice for what to do on next input

for each state and stack symbol
– on each symbol/Λ at most one transition
– not both symbol and Λ-transition

Definition

DPDA
$\delta(q, \sigma, X) \cup \delta(q, \Lambda, X)$ at most one element for each $q \in Q, \sigma \in \Sigma, X \in \Gamma$

[M] Def 5.10

DPDA $\approx$ DCFL $=$ class of deterministic context-free languages

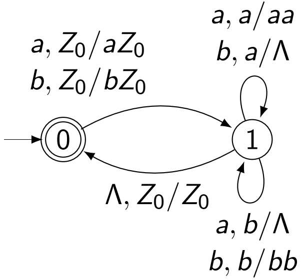
DPDA for *Balanced*

$Balanced = \{$balanced strings of brackets [and]$\}$

[M] E 5.11

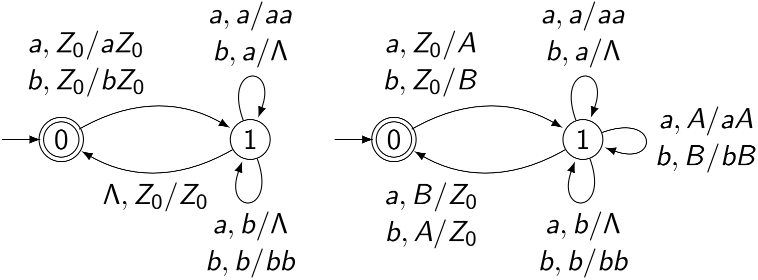
DPDA for AEqB

[M] E 5.13



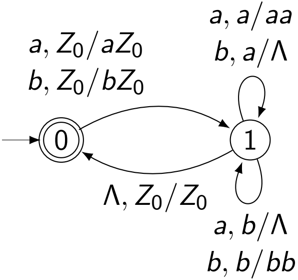
Without Λ-transitions...

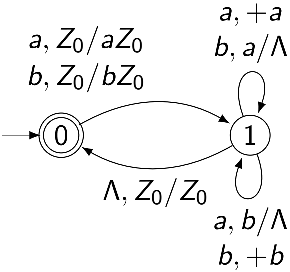
[M] E 5.13



Some nondeterminism. . .

[M] E 5.13

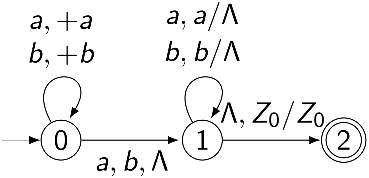




Computations for $x = abbaab\ldots$

[M] E 5.13

Pal $\{ \, y \in \{a, b\}^* \mid y = y^r \, \}$



$Q = \{0, 1, 2\}$
$\Sigma = \{a, b, c\}$
$\Gamma = \{a, b, Z_0\}$
$q_0 = 0$
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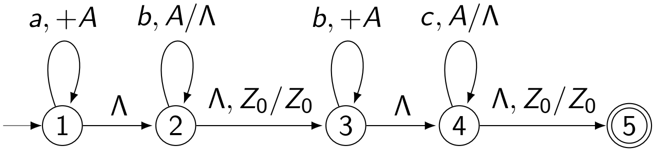
Theorem

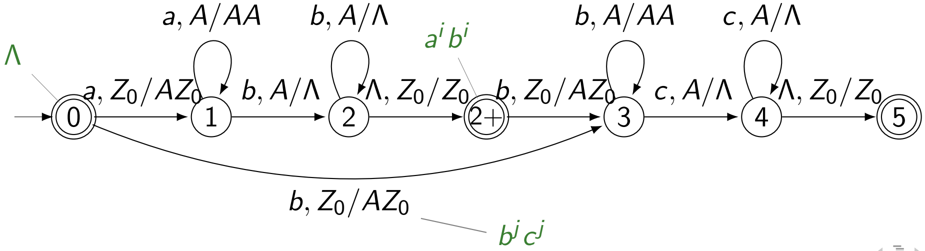
The language Pal *cannot be accepted by a deterministic pushdown automaton.*

The proof of this result does not have to be known for the exam.

[M] Thm 5.16

$S \to AB$
$A \to aAb \mid \Lambda$
$B \to bBc \mid \Lambda$





$\{ a^i b^j \mid i \neq j \}$