From exercise class 12:

Exercise 7.24.

In Example 7.5, a TM is given that accepts the language
$\{xx \ \mid \ x \in \{a,b\}^*\}$.

Draw a TM with tape alphabet $\Gamma = \{a,b\}$ that accepts this language.

Exercise 7.32.

Describe informally how to construct a TM T that enumerates the set of palindromes over $\{0, 1\}$ in canonical order.

In other words, T loops forever, and for every positive integer n, there is some point at which the initial portion of T's tape contains the string

$$\Delta\Delta 0\Delta 1\Delta 00\Delta 11\Delta 000\Delta \ldots \Delta x_n$$

where x_n is the nth palindrome in canonical order, and this portion of the tape is never subsequently changed.

Exercise 7.33.

Suppose you are given a Turing machine T (you have the transition diagram), and you are watching T processing an input string. At each step you can see the configuration of the TM: the state, the tape contents, and the tape head position.

a. Suppose that for some n, the tape head does not move past square n while you are watching. If the pattern continues, will you be able to conclude at some point that the TM is in an infinite loop? If so, what is the longest you might need to watch in order to draw this conclusion?

b. Suppose that in each move you observe, the tape head moves right. If the pattern continues, will you be able to conclude at some point that the TM is in an infinite loop? If so, what is the longest you might need to watch in order to draw this conclusion?

Exercise 7.34.

In each of the following cases, show that the language accepted by the TM T is regular.

a. There is an integer n such that no matter what the input string is, T never moves its tape head to the right of square n.

b. For every $n \geq 0$ and every input of length n, T begins by making $n+1$ moves in which the tape head is moved right each time, and thereafter T does not move the tape head to the left of square $n+1$.

A slide from lecture 13:

Definition 8.1. Accepting a Language and Deciding a Language

A Turing machine T with input alphabet Σ accepts a language $L \subseteq \Sigma^*$,
if $L(T) = L$.

T *decides* L,
if T computes the characteristic function $\chi_L : \Sigma^* \to \{0, 1\}$

A language L is *recursively enumerable*,
if there is a TM that accepts L,

and L is *recursive*,
if there is a TM that decides L.

A slide from lecture 13:

Theorem 8.4. If L_1 and L_2 are both recursively enumerable languages over Σ, then $L_1 \cup L_2$ and $L_1 \cap L_2$ are also recursively enumerable.

Proof. . .

Exercise 8.1.

Show that if L_1 and L_2 are recursive languages, then $L_1 \cup L_2$ and $L_1 \cap L_2$ are also.

Exercise 8.3.

Is the following statement true or false?

If $L_1, L_2, \ldots$ are any recursively enumerable subsets of Σ^*, then $\cup_{i=1}^{\infty} L_i$ is recursively enumerable.

Give reasons for your answer.

A slide from lecture 13:

Theorem 8.7. If L is a recursively enumerable language, and its complement L' is also recursively enumerable, then L is recursive
(and therefore, by Theorem 8.6, L' is recursive).

Proof. . .

Exercise 8.4.

Suppose $L_1, L_2, \ldots, L_k$ form a partition of Σ^*: in other words, their union is Σ^* and any two of them are disjoint.

Show that if each L_i is recursively enumerable, then each L_i is recursive.

Exercise 8.8.

Suppose L is recursively enumerable but not recursive. Show that if T is a TM accepting L, there must be infinitely many input strings for which T loops forever.