Pumping Lemma for context-free languages:

For every context-free language L

there exists a constant $n \geq 2$ such that

for every $u \in L$ with $|u| \geq n$

there exists a decomposition $u = vwxyz$ such that

(1) $|wy| \geq 1$,

(2) $|wxy| \leq n$,

(3) for all $m \geq 0$, $vw^m xy^m z \in L$

Example 6.4. $XX = \{xx \mid x \in \{a,b\}^*\ \}$ is not context-free.
Use $u = a^n b^n a^n b^n$

Exercise 6.4.

In the proof given in Example 6.4 using the pumping lemma, the contradiction was obtained in each case by considering the string vw^0xy^0z.

Would it have been possible instead to use vw^2xy^2z in each case? If so, give the proof in at least one case;
if not, explain why not.

Example 6.4. $XX = \{xx \mid x \in \{a,b\}^* \}$ is not context-free.
Use $u = a^n b^n a^n b^n$

Exercise 6.3.

In the pumping-lemma proof in Example 6.4, give some examples of choices of strings $u \in L$ with $|u| \geq n$ that would not work.

Exercise 6.2.

In each case below, show using the pumping lemma that the given language is not a CFL.

a. ♣ $L = \{a^i b^j c^k \mid i < j < k\}$

b. $L = \{a^{2^i} \mid i \geq 0\}$

d. $L = \{a^i b^{2i} a^i \mid i \geq 0\}$

e. ♠ $L = \{s \in \{a, b, c\}^* \mid n_a(s) = \max\{n_b(s), n_c(s)\}\ \}$

g. ♣ $L = \{a^i b^j a^i b^{i+j} \mid i, j \geq 0\}$

Exercise 6.5. ♣

For each case below, decide whether the given language is a CFL, and prove your answer.

a. $L = \{a^i b^j a^j b^i \mid i, j \geq 0\}$

c. $L = \{scs \mid s \in \{a, b\}^*\}$

d. $L = \{sts \mid s, t \in \{a, b\}^* \text{ en } |s| \geq 1\}$

g. $L =$ the set of non-balanced strings of parentheses

Exercise.

Give a context-free grammar for the language

$$\{a^i x\, y\, b^j \quad | \quad i, j \geq 0,\ x, y \in \{a, b\}^*,\ |x| = j \text{ and } |y| = i\}$$

Exercise.

Show using the pumping lemma that the language

$$L = \{a^i b^j a^i b^j \mid i, j \geq 0\}$$

is not context-free.

Exercise 6.6.

If L is a CFL, does it follow that $r(L) = \{x^r \mid x \in L\}$ is a CFL? Give a proof or a counterexample.

Exercise 6.9.

In each case below, show that the given language is a CFL but that its complement is not.

b. ♣ $\{a^i b^j c^k \quad | \quad i \neq j$ or $i \neq k\}$

a. ♣ ♠ $\{a^i b^j c^k \quad | \quad i \geq j$ or $i \geq k\}$

From exercise class 9

From exam Automata Theory, 23 December, 2021

Let

$$L = \{a^i b^j a^k \mid i, j, k \geq 0 \text{ and } i + k \leq 2j\}$$

a. Give the first six elements in the canonical (shortlex) order of L.

b. Draw a pushdown automaton M, such that $L(M) = L$.
This pushdown automaton must be directly based on the properties of the language. So it must not be the result of a standard construction to, e.g., transform a context-free grammar into a pushdown automaton.
Try to make M deterministic, and not containing any Λ-transitions.