Fundamentele Informatica 3

voorjaar 2018

`http://www.liacs.leidenuniv.nl/~vlietrvan1/fi3/`

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7. Turing Machines
7.4. Combining Turing Machines
7.5. Multitape Turing Machines
7.7. Nondeterministic Turing Machines

7.4. Combining Turing Machines

Example.

A TM for $f(x) = a^{n_a(x)}$

$x = aababba$

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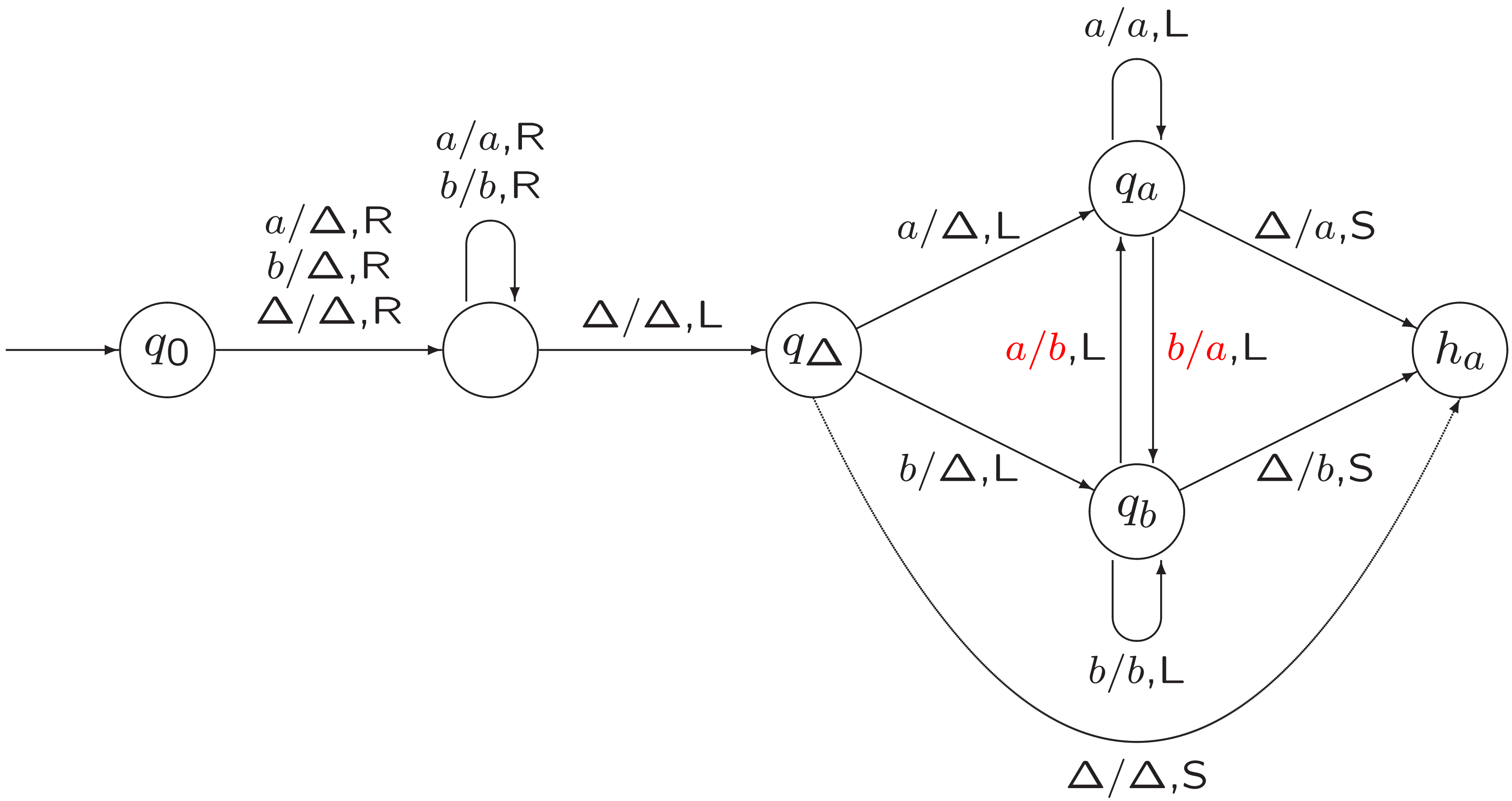
$x = aababba$

$$
\begin{array}{cccccccc}
\underline{\Delta} & a & a & b & a & b & b & a \\
\Delta & a & a & \underline{\Delta} & a & b & b & a \\
\Delta & a & a & \underline{a} & b & b & a & \Delta \\
\Delta & a & a & a & \underline{\Delta} & b & a & \Delta \\
\Delta & a & a & a & \underline{b} & a & \Delta & \Delta \\
\Delta & a & a & a & \underline{\Delta} & a & \Delta & \Delta \\
\Delta & a & a & a & \underline{a} & \Delta & \Delta & \Delta \\
\underline{\Delta} & a & a & a & a & \Delta & \Delta & \Delta
\end{array}
$$


q_0
a/Δ,R
b/Δ,R
Δ/Δ,R
a/a,R
b/b,R
Δ/Δ,L
q_Δ
a/Δ,L
b/Δ,L
q_a
a/a,L
q_b
b/b,L
a/b,L
b/a,L
Δ/a,S
Δ/b,S
h_a
Δ/Δ,S

Example 7.20. Inserting and Deleting a Symbol

Delete: from $y\underline{\sigma}z$ to $y\underline{z}$

Insert(σ): from $y\underline{z}$ to $y\underline{\sigma}z$

N.B.: z does not contain blanks

TM T_1 computes f

TM T_2 computes g

TM T_1T_2 computes . . .

$$T_1 \longrightarrow T_2$$

Example 7.17. Finding the Next Blank or the Previous Blank

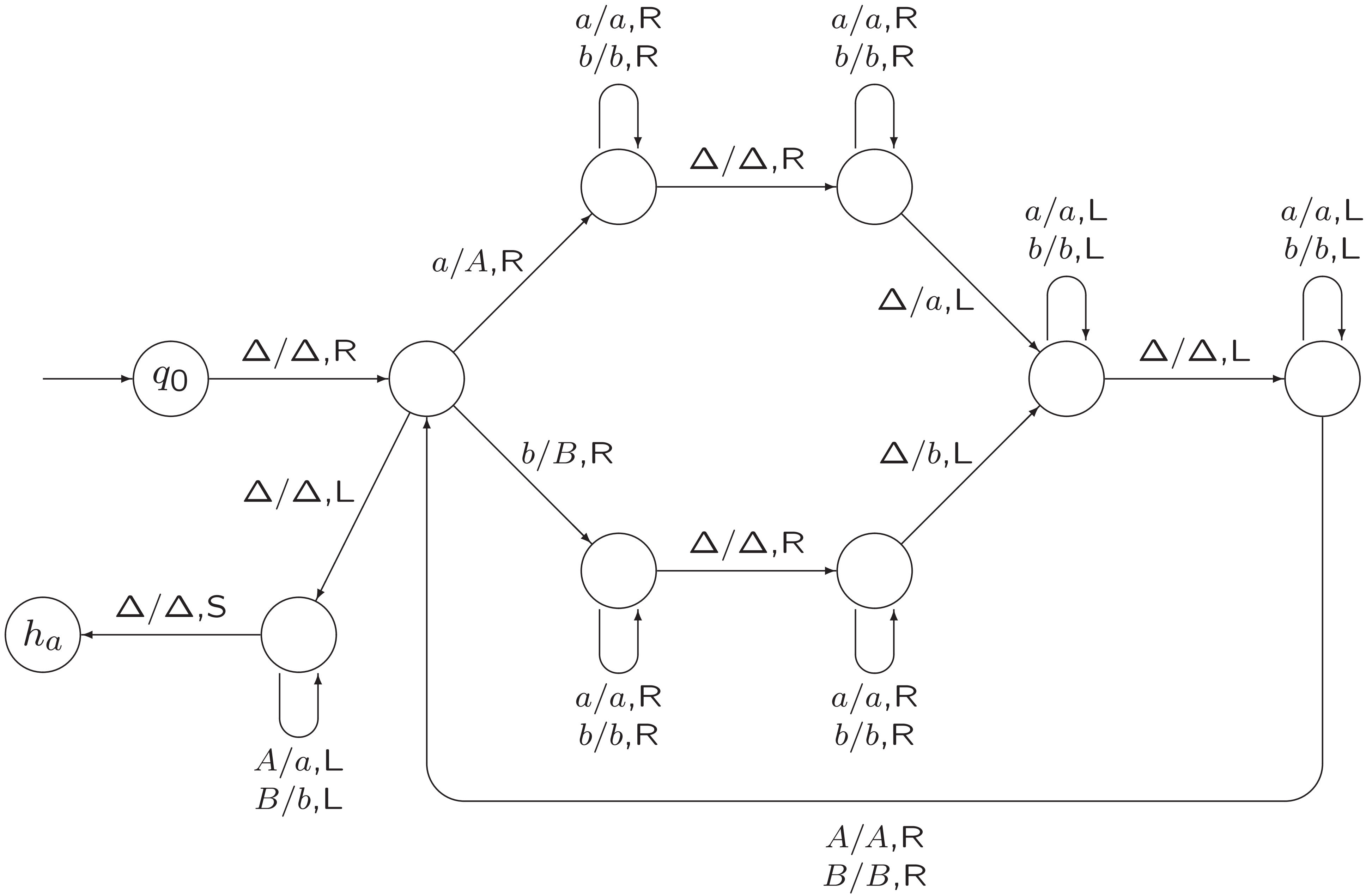
NB

PB

Example 7.18. Copying a String

Copy: from $\underline{\Delta}x$ to $\underline{\Delta}x\Delta x$

$x = abaa$



a/a,R
b/b,R
a/a,R
b/b,R
Δ/Δ,R
a/a,L
b/b,L
a/a,L
b/b,L
a/A,R
Δ/a,L
q_0
Δ/Δ,R
Δ/Δ,L
b/B,R
Δ/b,L
Δ/Δ,L
Δ/Δ,R
h_a
Δ/Δ,S
a/a,R
b/b,R
a/a,R
b/b,R
A/a,L
B/b,L
A/A,R
B/B,R

A slide from lecture 2

Example 7.10. The Reverse of a String

$$\underline{\Delta}\, a\ a\ b\ a\ b$$
$$\Delta A\, a\ b\ a\ b$$
$$\Delta A\, a\ b\ a\, A$$
$$\Delta B\, a\ b\ a\, A$$
$$\Delta BA\, b\ a\, A$$
$$\Delta BA\, b\, AA$$
$$\Delta BA\, b\, AA$$
$$\Delta BABAA$$
$$\underline{\Delta}\, b\ a\ b\ a\ a$$

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\underline{\Delta}} x \Delta y$ if $x = y$,
and reject if $x \neq y$

An exercise from exercise class 2

Exercise 7.17.

For each case below, draw a TM that computes the indicated function.

e. $E : \{a, b\}^* \times \{a, b\}^* \rightarrow \{0, 1\}$
defined by $E(x, y) = 1$ if $x = y$, $E(x, y) = 0$ otherwise.

Example 7.25. Accepting the Language of . . .

$$Copy \rightarrow NB \rightarrow R \rightarrow PB \rightarrow Equal$$

Example 7.25. Accepting the Language of Palindromes

$$Copy \to NB \to R \to PB \to Equal$$

Example 7.21. Erasing the Tape

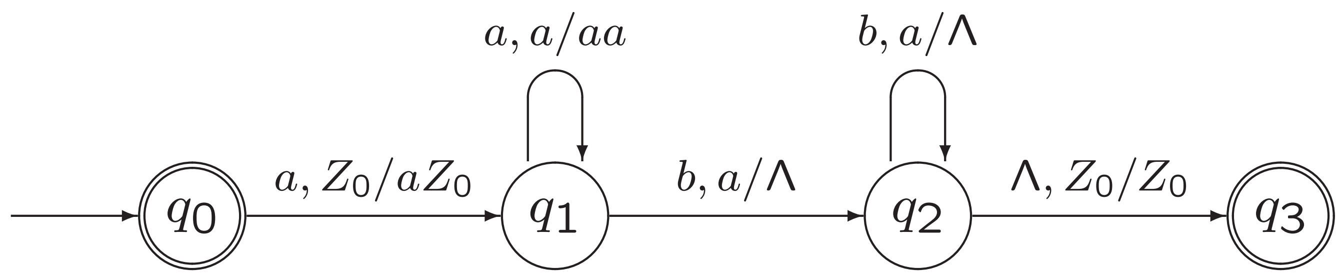
From the current position to the right

Many notations for composition

7.5. Multitape Turing Machines

Example 5.3. A PDA Accepting the Language *AnBn*

$$AnBn = \{a^i b^i \mid i \geq 0\}$$



Part of a slide from exercise class 1

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

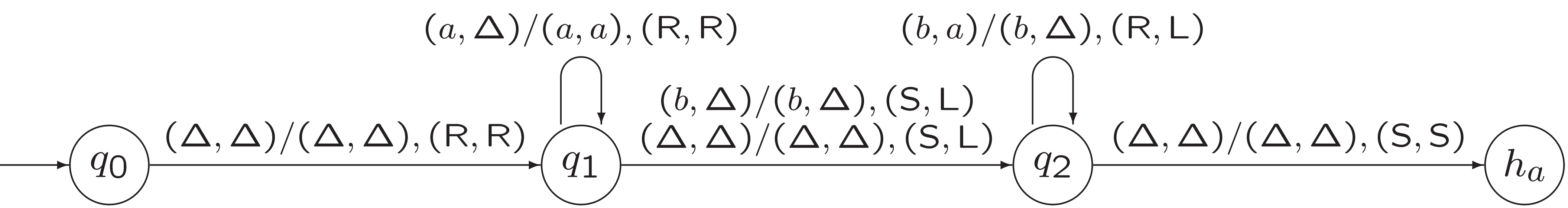
With two tapes. . .

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. *AnBn* $= \{a^i b^i \mid i \geq 0\}$

With two tapes:



A slide from exercise class 1

Exercise 7.4.

For each of the following languages, draw a transition diagram for a Turing machine that accepts that language.

a. $AnBn = \{a^i b^i \mid i \geq 0\}$

We could also use the portion of the tape to the right of the input, to simulate the stack of a deterministic pushdown automaton (works for any deterministic PDA!)

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\Delta} x \Delta y$ if $x = y$,
and reject if $x \neq y$

2-tape TM. . .

Theorem 7.26. (informal)
For every 2-tape TM T, there is an ordinary 1-tape TM T_1, which for every input x,

- simulates the computation of T for x,
- accepts (rejects) x, if and only if T accepts (rejects) x,
- on acceptance, leaves the same output on its tape as T leaves on its first tape.

The proof of this result does not have to be known for the exam.

Corollary 7.27.
Every language that is accepted by a 2-tape TM can be accepted by an ordinary 1-tape TM,
and every function that is computed by a 2-tape TM can be computed by an ordinary TM.

This generalizes to k-tape TMs for $k \geq 3$.

7.7. Nondeterministic Turing Machines

A slide from lecture 2

Definition 7.1. Turing machines

A Turing machine (TM) is a 5-tuple $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

Q is a finite set of states. The two *halt* states h_a and h_r are not elements of Q.

Σ, the input alphabet, and Γ, the tape alphabet, are both finite sets, with $\Sigma \subseteq \Gamma$. The *blank* symbol Δ is not an element of Γ.

q_0, the initial state, is an element of Q.

δ is the transition function:

$$\delta : Q \times (\Gamma \cup \{\Delta\}) \to (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\}) \times \{R, L, S\}$$

Nondeterministic Turing machine.

There may be more than one move for a state-symbol pair.

Same notation:

$$wpax \vdash_T yqbz \qquad wpax \vdash_T^* yqbz$$

A string x is accepted by T if

$$q_0 \Delta x \vdash_T^* w h_a y$$

for some strings $w, y \in (\Gamma \cup \{\Delta\})^*$.

NTM useful for accepting languages, for producing output, but not for computing function.

Example 7.28. The Set of Composite Natural Numbers.

Use *G2*

Example 7.28. The Set of Composite Natural Numbers.

$$NB \to G2 \to NB \to G2 \to PB \to M \to PB \to Equal$$

Take $x = 1^{15}$

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then

$$P(L) = \{x \in \Sigma^* \mid \; xy \in L \text{ for some } y \in \Sigma^*\}$$

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then

$$P(L) = \{x \in \Sigma^* \mid \ xy \in L \text{ for some } y \in \Sigma^*\}$$

Deterministic TM accepting $P(L)$ may execute following algorithm for input x:

$y = \Lambda$;
while (T does not accept xy)
    y is next string in Σ^* (in canonical order);
accept;

but. . .

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then

$$P(L) = \{x \in \Sigma^* \mid \ xy \in L \text{ for some } y \in \Sigma^*\}$$

$$\textit{NB} \to \textit{G} \to \textit{Delete} \to \textit{PB} \to T$$