Fundamentele Informatica 3

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`http://www.liacs.leidenuniv.nl/~vlietrvan1/fi3/`

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9. Undecidable Problems
9.3. More Decision Problems Involving Turing Machines

A slide from lecture 7a:

Accepts-Λ: Given a TM T, is $\Lambda \in L(T)$?

Theorem 9.9. The following five decision problems are undecidable.

5. *WritesSymbol*:
   Given a TM T and a symbol a in the tape alphabet of T, does T ever write a if it starts with an empty tape ?

Proof.

5. Prove that *Accepts-Λ* $\leq$ *WritesSymbol* . . .

A slide from lecture 7a:

AtLeast10MovesOn-Λ:
Given a TM T, does T make at least ten moves on input Λ ?

WritesNonblank: Given a TM T, does T ever write a nonblank symbol on input Λ ?

Theorem 9.10.
The decision problem *WritesNonblank* is decidable.

Proof. . .

Exercise 9.12.

For each decision problem below, determine whether it is decidable or undecidable, and prove your answer.

a. Given a TM T, does it ever reach a nonhalting state other than its initial state if it starts with a blank tape?

Definition 9.11. A Language Property of TMs

A property R of Turing machines is called a *language property* if, for every Turing machine T having property R, and every other TM T_1 with $L(T_1) = L(T)$, T_1 also has property R.

A language property of TMs is *nontrivial* if there is at least one TM that has the property and at least one that doesn't.

In fact, a language property is a property *of the languages accepted by* TMs.

Example of nontrivial language property:

2. *AcceptsSomething*:
   Given a TM T, is there at least one string in $L(T)$?

Theorem 9.12. Rice's Theorem

If R is a nontrivial language property of TMs, then the decision problem

P_R: Given a TM T, does T have property R ?

is undecidable.

Proof...

Prove that *Accepts*-$\Lambda \leq P_R$ ...

(or that *Accepts*-$\Lambda \leq P_{\text{not}-R}$ ...)

Examples of decision problems to which Rice's theorem can be applied:

1. *Accepts-L*: Given a TM T, is $L(T) = L$? (assuming . . .)
2. *AcceptsSomething*:
   Given a TM T, is there at least one string in $L(T)$?
3. *AcceptsTwoOrMore*:
   Given a TM T, does $L(T)$ have at least two elements ?
4. *AcceptsFinite*: Given a TM T, is $L(T)$ finite ?
5. *AcceptsRecursive*:
   Given a TM T, is $L(T)$ recursive ? (note that . . .)

All these problems are undecidable.

Rice's theorem cannot be applied (directly)

- if the decision problem does not involve just one TM

Equivalent: Given two TMs T_1 and T_2, is $L(T_1) = L(T_2)$

Rice's theorem cannot be applied (directly)

• if the decision problem does not involve just one TM
Equivalent: Given two TMs T_1 and T_2, is $L(T_1) = L(T_2)$

• if the decision problem involves the *operation* of the TM
WritesSymbol: Given a TM T and a symbol a in the tape alphabet of T, does T ever write a if it starts with an empty tape ?
WritesNonblank: Given a TM T, does T ever write a nonblank symbol on input Λ ?

• if the decision problem involves a *trivial* property
Accepts-NSA: Given a TM T, is $L(T) =$ *NSA* ?

Exercise 9.23. Show that the property “accepts its own encoding” is not a language property of TMs.

Part of a slide from lecture 4:

Definition 7.33. An Encoding Function (continued)

For each move m of T of the form $\delta(p, \sigma) = (q, \tau, D)$

$$e(m) = 1^{n(p)}01^{n(\sigma)}01^{n(q)}01^{n(\tau)}01^{n(D)}0$$

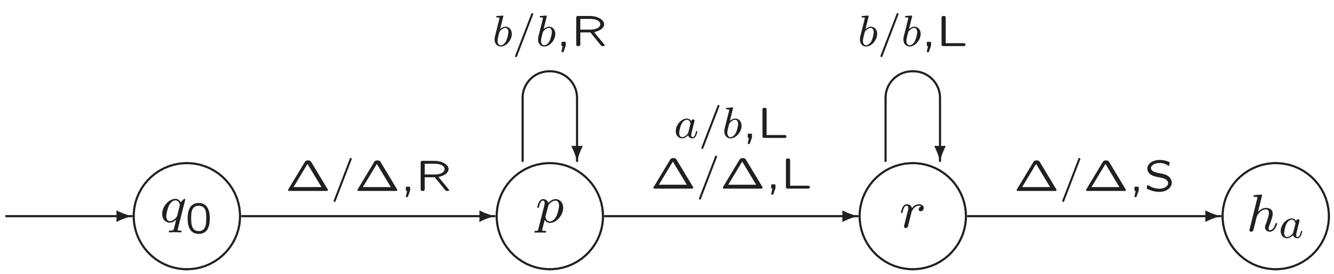
We list the moves of T in some order as $m_1, m_2, \ldots, m_k$, and we define

$$e(T) = e(m_1)0e(m_2)0 \ldots 0e(m_k)0$$

Exercise 9.23. Show that the property "accepts its own encoding" is not a language property of TMs.

A slide from lecture 4:

Example 7.34. A Sample Encoding of a TM



<u>1</u>11010111101010 0 111101110111101111010 0
111101101111101110110 0 11110101111010110 0
1111101110111110111 0110 0 1111101010101110 0

Exercise 9.12.
For each decision problem below, determine whether it is decidable or undecidable, and prove your answer.

b. Given a TM T and a nonhalting state q of T, does T ever enter state q when it begins with a blank tape?

e. Given a TM T, is there a string it accepts in an even number of moves?

j. Given a TM T, does T halt within ten moves on every string?

l. Given a TM T, does T eventually enter every one of its nonhalting states if it begins with a blank tape?

Exercise 9.13.

In this problem TMs are assumed to have input alphabet $\{0,1\}$. For a finite set $S \subseteq \{0,1\}^*$, P_S denotes the decision problem: Given a TM T, is $S \subseteq L(T)$?

a. Show that if $x, y \in \{0,1\}^*$, then $P_{\{x\}} \leq P_{\{y\}}$.

b. Show that if $x, y, z \in \{0,1\}^*$, then $P_{\{x\}} \leq P_{\{y,z\}}$.

c. Show that if $x, y, z \in \{0,1\}^*$, then $P_{\{x,y\}} \leq P_{\{z\}}$.

d. Is it true that for every two finite subsets S and U of $\{0,1\}^*$, $P_S \leq P_U$.