Fundamentele Informatica 3

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Huiswerkopgave 2,
inleverdatum 27 maart 2012, 13:45 uur

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7.4. Combining Turing Machines

Example 7.17. Finding the Next Blank or the Previous Blank

NB

PB

Example 7.20. Inserting and Deleting a Symbol

Delete: from $x\underline{\sigma}y$ to $x\underline{y}$

Insert(σ): from $x\underline{y}$ to $x\underline{\sigma}y$

N.B.: y does not contain blanks

Example 7.21. Erasing the Tape
from the current position to the right

Example 7.24. Comparing Two Strings

Equal: accept $\underline{\Delta} x \Delta y$ if $x = y$,
and reject if $x \neq y$

7.5. Multitape Turing Machines

Definition 7.1. Turing machines

A Turing machine (TM) is a 5-tuple $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

Q is a finite set of states. The two *halt* states h_a and h_r are not elements of Q.

Σ, the input alphabet, and Γ, the tape alphabet, are both finite sets, with $\Sigma \subseteq \Gamma$. The *blank* symbol Δ is not an element of Γ.

q_0, the initial state, is an element of Q.

δ is the transition function:

$$\delta : Q \times (\Gamma \cup \{\Delta\}) \to (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\}) \times \{R, L, S\}$$

2-Tape TM $T = (Q, \Sigma, \Gamma, q_0, \delta)$, where

$$\delta : Q \times (\Gamma \cup \{\Delta\})^2 \to (Q \cup \{h_a, h_r\}) \times (\Gamma \cup \{\Delta\})^2 \times \{R, L, S\}^2$$

Notation:

description of tape contents: $x\underline{\sigma}y$ or $x\underline{y}$

configuration $xqy = xqy\Delta = xqy\Delta\Delta$

initial configuration corresponding to input x: $q_0\Delta x$

In the third edition of the book, a configuration is denoted as $(q, x\underline{y})$ or $(q, x\underline{\sigma}y)$ instead of xqy or $xq\sigma y$.
This old notation is also allowed for Fundamentele Informatica 3.

Configuration of 2-tape TM is

$$(q, x_1\underline{a_1}y_1, x_2\underline{a_2}y_2)$$

Initial configuration corresponding to input string x is

$$(q_0, \underline{\Delta}x, \underline{\Delta})$$

Output will appear on first tape.

Theorem 7.26.
For every 2-tape TM $T = (Q, \Sigma, \Gamma, q_0, \delta)$, there is an ordinary 1-tape TM $T_1 = (Q_1, \Sigma, \Gamma_1, q_1, \delta_1)$ with $\Gamma \subseteq \Gamma_1$, such that

1. For every $x \in \Sigma^*$, T accepts x if and only if T_1 accepts x, and T rejects x if and only if T_1 rejects x.
   (In particular, $L(T) = L(T_1)$.)

2. For every $x \in \Sigma^*$, if

$$(q_0, \underline{\Delta}x, \underline{\Delta}) \vdash_T^* (h_a, y\underline{a}z, u\underline{b}v)$$

for some strings $y, z, u, v \in (\Gamma \cup \{\Delta\})^*$ and symbols $a, b \in \Gamma \cup \{\Delta\}$, then

$$q_1 \Delta x \vdash_{T_1}^* y h_a a z$$

Corollary 7.27.
Every language that is accepted by a 2-tape TM can be accepted by an ordinary 1-tape TM, and every function that is computed by a 2-tape TM can be computed by an ordinary TM.

7.6. The Church-Turing Thesis

Turing machine is general model of computation.

Any algorithmic procedure that can be carried out at all
(by human computer, team of humans, electronic computer)
can be carried out by a TM.
(Alonzo Church, 1930s)

Evidence for Church-Turing thesis:

1. Nature of the model.

2. Various enhancements of TM do not change computing power.

3. Other theoretical models of computation have been proposed. Various notational systems have been suggested as ways of describing computations. All of them equivalent to TM.

4. No one has suggested any type of computation that ought to be considered ‘algorithmic procedure’ and cannot be implemented on TM.

7.7. Nondeterministic Turing Machines

Definition 7.1. Turing machines

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δ is the transition function:

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Nondeterministic Turing machine.

There may be more than one move for a state-symbol pair.

Same notation:

$$wpax \vdash_T yqbz \qquad wpax \vdash_T^* yqbz$$

A string x is accepted by T if

$$q_0 \Delta x \vdash_T^* wh_a y$$

for some strings $w, y \in (\Gamma \cup \{\Delta\})^*$.

NTM useful for producing output,
but not for computing function.

Example 7.28. The Set of Composite Natural Numbers.

$$\mathit{NB} \to \mathit{G2} \to \mathit{NB} \to \mathit{G2} \to \mathit{PB} \to \mathit{M} \to \mathit{PB} \to \mathit{Equal}$$

Take $x = 1^{15}$

Example 7.30. The Language of Prefixes of Elements of L.

Let $L = L(T)$. Then

$$P(L) = \{x \in \Sigma^* \mid \ xy \in L \text{ for some } y \in \Sigma^*\}$$

$$\textit{NB} \to \textit{G} \to \textit{Delete} \to \textit{PB} \to T$$