Fundamentele Informatica 3

voorjaar 2012

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college 4, 27 februari 2012

5. Pushdown Automata

5.4. A CFG from a Given PDA

5.5. Parsing

5.3. A PDA from a Given CFG

Definition 5.17. The Nondeterministic Top-Down PDA $NT(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic top-down PDA corresponding to G is $NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

$$Q = \{q_0, q_1, q_2\} \quad A = \{q_2\} \quad \Gamma = V \cup \Sigma \cup \{Z_0\}$$

The initial move of $NT(G)$ is the Λ-transition

$$\delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$$

and the only move to the accepting state is the Λ-transition

$$\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$$

The moves from q_1 are the following:
For every $A \in V$, $\delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha \text{ is a production in } G\}$
For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$

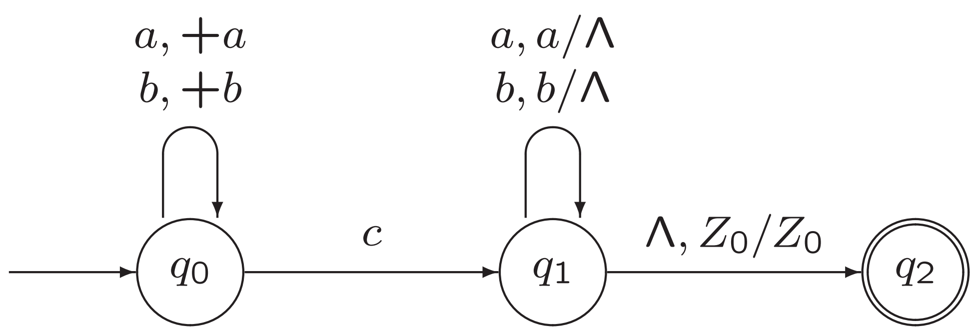
5.4. A CFG from a Given PDA

Theorem 5.28.

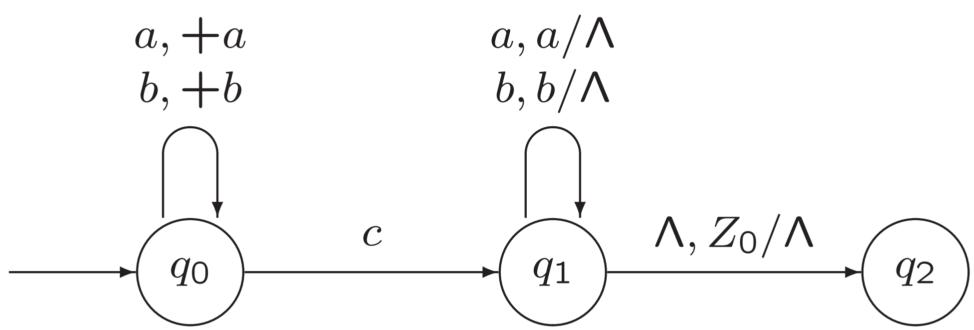
If $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ is a PDA, then there is another PDA M_1 such that $L_e(M_1) = L(M)$.

Sketch of Proof.

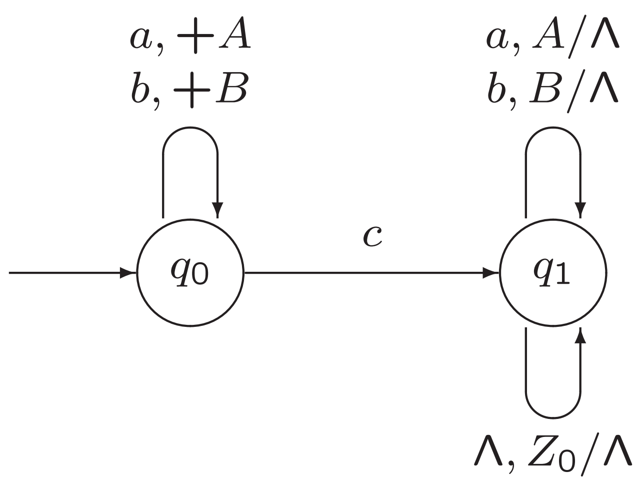
Example 5.30. A CFG from a PDA Accepting *SimplePal* by final state



Example 5.30. A CFG from a PDA Accepting *SimplePal* by empty stack



Example 5.30. A CFG from a PDA Accepting *SimplePal* by empty stack



Exercise 5.35.

Let M be the PDA from Example 5.30 accepting *SimplePal* by empty stack.

Consider the simplistic preliminary approach to obtaining a CFG described in the discussion preceding Theorem 5.29. The states of M are ignored, the variables of the grammar are the stack symbols of M,. and for every move that reads σ and replaces A on the stack by $BC \dots D$, we introduce a production $A \to \sigma BC \dots D$.

Show that although the string aa is not accepted by M, it is generated by the resulting CFG.

Theorem 5.29.

If $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ is a pushdown automaton accepting L by empty stack,
then there is a context-free grammar G such that $L = L(G)$.

Proof.

Theorem 5.29.

If $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ is a pushdown automaton accepting L by empty stack,
then there is a context-free grammar G such that $L = L(G)$.

Proof.

We define $G = (V, \Sigma, S, P)$ as follows:
V contains S as well as all possible variables of the form $[p, A, q]$, where $A \in \Gamma$ and $p, q \in Q$.

Proof of Theorem 5.29. (continued)

P contains the following productions:

1. For every $q \in Q$, the production $S \to [q_0, Z_0, q]$ is in P.

2. For every $q, q_1 \in Q$, every $\sigma \in \Sigma \cup \{\Lambda\}$, and every $A \in \Gamma$,
   if $\delta(q, \sigma, A)$ contains (q_1, Λ),
   then the production $[q, A, q_1] \to \sigma$ is in P.

3. For every $q, q_1 \in Q$, every $\sigma \in \Sigma \cup \{\Lambda\}$, every $A \in \Gamma$,
   and every $m \geq 1$,
   if $\delta(q, \sigma, A)$ contains $(q_1, B_1 B_2 \ldots B_m)$
   for some $B_1, B_2, \ldots, B_m$ in Γ,
   then for every choice of $q_2, q_3, \ldots, q_{m+1}$ in Q, the production

   $$[q, A, q_{m+1}] \to \sigma[q_1, B_1, q_2][q_2, B_2, q_3] \ldots [q_m, B_m, q_{m+1}]$$

   is in P.

Proof of Theorem 5.29. (continued)

The details of the rest of the proof do not have to be known for the exam.

5.5. Parsing

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For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$

Example 5.32. A Top-Down Parser for *Balanced*

CFG 1:

$$S \to [S] \mid SS \mid \Lambda$$

$$LA_1\ (S \to [S]) \ = \ \{\ [\ \}$$

$$LA_1\ (S \to SS) \ = \ \{\ [,\ ],\ \_\}$$

$$LA_1\ (S \to \Lambda) \ = \ \{\ [,\ ],\ \_\}$$

Example 5.32. A Top-Down Parser for *Balanced* (continued)

CFG 2:

$$S \to [S]S \mid \Lambda$$

$$\begin{aligned} LA_1\ (S \to [S]S) &= \{\ [\ \} \\ LA_1\ (S \to \Lambda) &= \{\ ], \_ \} \end{aligned}$$

Example 5.32. A Top-Down Parser for *Balanced* (continued)

CFG 3:

$$S \rightarrow S_1\$ \qquad S_1 \rightarrow [S_1]S_1 \mid \Lambda$$

$$LA_1\ (S_1 \rightarrow [S_1]S_1) \;=\; \{\ [\ \}$$

$$LA_1\ (S_1 \rightarrow \Lambda) \;=\; \{\ ], \$\}$$