Fundamentele Informatica 3

voorjaar 2012

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Rudy van Vliet
kamer 124 Snellius, tel. 071-527 5777
rvvliet(at)liacs.nl

college 3, 20 februari 2012

5. Pushdown Automata

5.3. A PDA from a Given CFG

5.4. A CFG from a Given PDA

5.3. A PDA from a Given CFG

Example 5.19. The Language *Balanced*

$$S \to [S] \mid SS \mid \Lambda$$

A derivation of [[ ] [ ]] . . .

Definition 5.17. The Nondeterministic Top-Down PDA $NT(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic top-down PDA corresponding to G is $NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

$$Q = \{q_0, q_1, q_2\} \quad A = \{q_2\} \quad \Gamma = V \cup \Sigma \cup \{Z_0\}$$

The initial move of $NT(G)$ is the Λ-transition

$$\delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$$

and the only move to the accepting state is the Λ-transition

$$\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$$

The moves from q_1 are the following:
For every $A \in V$, $\delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha$ is a production in $G\}$
For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$

Successful Computation of Top-Down PDA for $x = [\ [\ ]\ [\ ]\ ]$:

$(q_0, [\ [\ ]\ [\ ]\ ], Z_0) \vdash (q_1, [\ [\ ]\ [\ ]\ ], S_1 Z_0) \vdash (q_1, [\ [\ ]\ [\ ]\ ], [S_2] Z_0)$

$\vdash (q_1, [\ ]\ [\ ]\ ], S_2] Z_0) \vdash (q_1, [\ ]\ [\ ]\ ], S_3 S_5] Z_0) \vdash (q_1, [\ ]\ [\ ]\ ], [S_4] S_5] Z_0)$

$\vdash (q_1,]\ [\ ]\ ], S_4] S_5] Z_0) \vdash (q_1,]\ [\ ]\ ],] S_5] Z_0) \vdash (q_1, [\ ]\ ], S_5] Z_0)$

$\vdash (q_1, [\ ]\ ], [S_6]\ ] Z_0) \vdash (q_1,]\ ], S_6]\ ] Z_0) \vdash (q_1,]\ ],]\ ] Z_0)$

$\vdash (q_1,],] Z_0) \vdash (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)$

Successful Computation of Top-Down PDA for $x = [\,[\,]\,[\,]\,]$:

$(q_0, [\,[\,]\,[\,]\,], Z_0) \vdash (q_1, [\,[\,]\,[\,]\,], S_1 Z_0) \vdash (q_1, [\,[\,]\,[\,]\,], [S_2]Z_0)$

$\vdash (q_1, [\,]\,[\,]\,], S_2]Z_0) \vdash (q_1, [\,]\,[\,]\,], S_3 S_5]Z_0) \vdash (q_1, [\,]\,[\,]\,], [S_4]S_5]Z_0)$

$\vdash (q_1,]\,[\,]\,], S_4]S_5]Z_0) \vdash (q_1,]\,[\,]\,],]S_5]Z_0) \vdash (q_1, [\,]\,], S_5]Z_0)$

$\vdash (q_1, [\,]\,], [S_6]\,]Z_0) \vdash (q_1,]\,], S_6]\,]Z_0) \vdash (q_1,]\,],]\,]Z_0)$

$\vdash (q_1,],]Z_0) \vdash (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)$

Successful Computation of Top-Down PDA for $x = [\ [\ ]\ [\ ]\ ]$:

$(q_0, [\ [\ ]\ [\ ]\ ], Z_0) \vdash (q_1, [\ [\ ]\ [\ ]\ ], S_1 Z_0) \vdash (q_1, [\ [\ ]\ [\ ]\ ], [S_2]Z_0)$

$\vdash (q_1, [\ ]\ [\ ]\ ], S_2]Z_0) \vdash (q_1, [\ ]\ [\ ]\ ], S_3 S_5]Z_0) \vdash (q_1, [\ ]\ [\ ]\ ], [S_4]S_5]Z_0)$

$\vdash (q_1,]\ [\ ]\ ], S_4]S_5]Z_0) \vdash (q_1,]\ [\ ]\ ],]S_5]Z_0) \vdash (q_1, [\ ]\ ], S_5]Z_0)$

$\vdash (q_1, [\ ]\ ], [S_6]\ ]Z_0) \vdash (q_1,]\ ], S_6]\ ]Z_0) \vdash (q_1,]\ ],]\ ]Z_0)$

$\vdash (q_1,],]Z_0) \vdash (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)$

Order in which symbols are removed from stack, is preorder (‘WLR’)

Theorem 5.18.

If G is a context-free grammar, then the nondeterministic top-down PDA $NT(G)$ accepts the language $L(G)$.

The details of the proof of this result do not have to be known for the exam.

The Nondeterministic Bottom-Up PDA

Example 5.24. Simplified Algebraic Expressions

$$S \to S + T \mid T \quad T \to T * a \mid a$$

A derivation of $a + a * a$ ...

Example 5.24. Simplified Algebraic Expressions

1. $S \to S + T$
2. $S \to T$
3. $T \to T * a$
4. $T \to a$

The leftmost derivation and the rightmost derivation of $a + a * a$

...

Successful Computation of Bottom-Up PDA for $x = a + a * a$:

$(q_0, a + a * a, Z_0) \vdash (q_0, + a * a, aZ_0) \vdash (q_0, + a * a, T_3 Z_0)$

$\vdash (q_0, + a * a, S_2 Z_0) \vdash (q_0, a * a, + S_2 Z_0) \vdash (q_0, * a, a + S_2 Z_0)$

$\vdash (q_0, * a, T_5 + S_2 Z_0) \vdash (q_0, a, * T_5 + S_2 Z_0) \vdash (q_0, \Lambda, a * T_5 + S_2 Z_0)$

$\vdash (q_{3,1}, \Lambda, * T_5 + S_2 Z_0) \vdash (q_{3,2}, \Lambda, T_5 + S_2 Z_0) \vdash (q_0, \Lambda, T_4 + S_2 Z_0)$

$\vdash (q_{1,1}, \Lambda, + S_2 Z_0) \vdash (q_{1,2}, \Lambda, S_2 Z_0) \vdash (q_0, \Lambda, S_1 Z_0)$

$\vdash (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)$

Successful Computation of Bottom-Up PDA for $x = a + a * a$:

$(q_0, a + a * a, Z_0) \vdash (q_0, + a * a, aZ_0) \vdash (q_0, + a * a, T_3 Z_0)$

$\vdash (q_0, + a * a, S_2 Z_0) \vdash (q_0, a * a, + S_2 Z_0) \vdash (q_0, * a, a + S_2 Z_0)$

$\vdash (q_0, * a, T_5 + S_2 Z_0) \vdash (q_0, a, * T_5 + S_2 Z_0) \vdash (q_0, \Lambda, a * T_5 + S_2 Z_0)$

$\vdash (q_{3,1}, \Lambda, * T_5 + S_2 Z_0) \vdash (q_{3,2}, \Lambda, T_5 + S_2 Z_0) \vdash (q_0, \Lambda, T_4 + S_2 Z_0)$

$\vdash (q_{1,1}, \Lambda, + S_2 Z_0) \vdash (q_{1,2}, \Lambda, S_2 Z_0) \vdash (q_0, \Lambda, S_1 Z_0)$

$\vdash (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)$

Successful Computation of Bottom-Up PDA for $x = a + a * a$:

$(q_0, a + a * a, Z_0) \vdash (q_0, + a * a, aZ_0) \vdash (q_0, + a * a, T_3Z_0)$

$\vdash (q_0, + a * a, S_2Z_0) \vdash (q_0, a * a, +S_2Z_0) \vdash (q_0, * a, a + S_2Z_0)$

$\vdash (q_0, * a, T_5 + S_2Z_0) \vdash (q_0, a, *T_5 + S_2Z_0) \vdash (q_0, \Lambda, a * T_5 + S_2Z_0)$

$\vdash (q_{3,1}, \Lambda, * T_5 + S_2Z_0) \vdash (q_{3,2}, \Lambda, T_5 + S_2Z_0) \vdash (q_0, \Lambda, T_4 + S_2Z_0)$

$\vdash (q_{1,1}, \Lambda, + S_2Z_0) \vdash (q_{1,2}, \Lambda, S_2Z_0) \vdash (q_0, \Lambda, S_1Z_0)$

$\vdash (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)$

Order in which symbols are move onto stack, is postorder (‘LRW’)

Definition 5.22. The Nondeterministic Bottom-Up PDA *NB*(G)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic bottom-up PDA corresponding to G is
NB$(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.

$\Gamma = \ldots$

Definition 5.22. The Nondeterministic Bottom-Up PDA *NB*(G)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic bottom-up PDA corresponding to G is *NB*$(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.

$\Gamma = V \cup \Sigma \cup \{Z_0\}$

Definition 5.22. The Nondeterministic Bottom-Up PDA *NB*(G) (continued)

For every $\sigma \in \Sigma$ and every $X \in \Gamma$, $\delta(q_0, \sigma, X) = \{(q_0, \sigma X)\}$. This is a *shift* move.

For every production $B \to \alpha$ in G, and every nonnull string $\beta \in \Gamma^*$,
$(q_0, \Lambda, \alpha^r \beta) \vdash^* (q_0, B\beta)$,
where this *reduction* is a sequence of one or more moves in which, if there is more than one, the intermediate configurations involve other states that are specific to this sequence and appear in no other moves of *NB*(G).

One of the elements of $\delta(q_0, \Lambda, S)$ is (q_1, Λ),
and $\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$.

Theorem 5.23.

If G is a context-free grammar, then the nondeterministic bottom-up PDA $\mathit{NB}(G)$ accepts the language $L(G)$.

The details of the proof of this result do not have to be known for the exam.

5.4. A CFG from a Given PDA

Definition 5.27. Acceptance by Empty Stack

If M is a PDA with input alphabet Σ, initial state q_1, and initial stack symbol Z_1, then M accepts a language L by empty stack if $L = L_e(M)$, where

$$L_e(M) = \{x \in \Sigma^* \mid \ (q_1, x, Z_1) \vdash_M^* (q, \Lambda, \Lambda) \text{ for some state } q\}$$

Theorem 5.28.

If $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ is a PDA,
then there is another PDA M_1 such that $L_e(M_1) = L(M)$.

Sketch of Proof.

Exercise 5.21.

Prove the converse of Theorem 5.28:

If there is a PDA $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ accepting L by empty stack (that is, $x \in L$, if and only if $(q_0, x, Z_0) \vdash_M^* (q, \Lambda, \Lambda)$ for some state q), then there is a PDA M_1 accepting L by final state (i.e., the ordinary way).