Fundamentele Informatica 3

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5. Pushdown Automata

5.2. Deterministic Pushdown Automata

5.3. A PDA from a Given CFG

Theorem 5.16.

The language *Pal* cannot be accepted
by a deterministic pushdown automaton.

Sketch of Proof.

Assume M is DPDA for *Pal*.

Let moves of M be of forms
$\delta(p, \sigma, X) = \{(q, \Lambda)\}$ or $\delta(p, \sigma, X) = \{(q, \alpha X)\}$

M reads every string $x \in \{a, b\}^*$ completely, with one path.

There exist different strings $r, s \in \{a, b\}^*$, such that for every $z \in \{a, b\}^*$, M treats rz and sz the same way.

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For a string $x \in \{a,b\}^*$, let y_x be a string such height of stack after xy_x is minimal.

Let α_x be stack after xy_x.
(state, top stack symbol) determines how suffix z is treated.

Infinitely many strings xy_x.

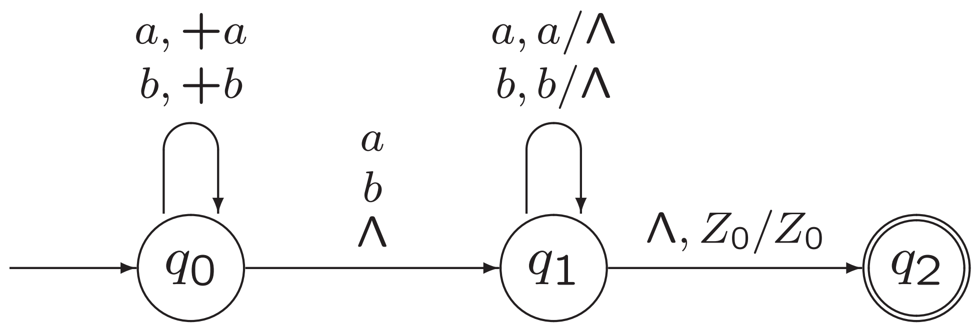
Finitely many pairs (q, X)

Different $r = uy_u$ and $s = vy_v$ arrive at same pair (q, X).

For any suffix z, rz and sz are treated the same:
$rz \in \mathit{Pal} \iff sz \in \mathit{Pal}$.

Example 5.7. A Pushdown Automaton Accepting *Pal*

$$\textit{Pal} = \{x \in \{a,b\}^* \mid x = x^r\}$$



Example 5.13. Two DPDAs accepting *AEqB*

$AEqB = \{x \in \{a, b\}^* \mid n_a(x) = n_b(x)\}$

Exercise 5.18.

For each of the following languages, give a transition diagram for a deterministic PDA that accepts that language.

a. $\{x \in \{a, b\}^* \mid n_a(x) < n_b(x)\}$

b. $\{x \in \{a, b\}^* \mid n_a(x) \neq n_b(x)\}$

Homework:

c. $\{x \in \{a, b\}^* \mid n_a(x) = 2n_b(x)\}$

d. $\{a^n b^{n+m} a^m \mid n, m \geq 0\}$

5.3. A PDA from a Given CFG

Example 5.19. The Language *Balanced*

$$S \rightarrow [S] \mid SS \mid \Lambda$$

A derivation of [[ ] [ ]] . . .

Definition 5.17. The Nondeterministic Top-Down PDA $NT(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic top-down PDA corresponding to G is $NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

$$Q = \{q_0, q_1, q_2\} \quad A = \{q_2\} \quad \Gamma = V \cup \Sigma \cup \{Z_0\}$$

The initial move of $NT(G)$ is the Λ-transition

$$\delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$$

and the only move to the accepting state is the Λ-transition

$$\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$$

The moves from q_1 are the following:
For every $A \in V$, $\delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha$ is a production in $G\}$
For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$

Exercise 5.28.

In each case below, you are given a CFG G and a string x that it generates.
1. Draw the transition diagram of the top-down PDA $NT(G)$.
2a. For $NT(G)$, trace a sequence of moves by which x is accepted, showing at each step the state, the unread input, and the stack contents.
2b. Show at the same time the corresponding leftmost derivation of x in the grammar. See Example 5.19 for a guide.

a. The grammar has productions

$$S \to S + T \mid T \quad T \to T * F \mid F \quad F \to [S] \mid a$$

and $x = [a + a * a] * a$.