Solution to exercise Fundamentele Informatica 3

Variant of 7.28b): $S \to S_1\$ \quad S_1 \to S_1 A \mid a \mid \Lambda \quad A \to Aa \mid b$

This context-free grammar does not satisfy the LL(1) property, as

$$\begin{array}{lll} LA_1(A \to b) = \{b\} & LA_1(A \to Aa) = \{b\} & \\ LA_1(S_1 \to a) = \{a\} & LA_1(S_1 \to S_1 A) = \{a, b\} & LA_1(S_1 \to \Lambda) = \{\$, b\} \end{array}$$

We eliminate left recursion for non-terminal symbols S_1 and A using new non-terminal symbols U and W, respectively:

$$\begin{array}{ll} S_1 \to aU \mid U & U \to \Lambda \mid AU \\ A \to bW & W \to \Lambda \mid aW \end{array}$$

The productions of the resulting context-free grammar are

$$S \to S_1\$ \quad S_1 \to aU \mid U \quad U \to \Lambda \mid AU \quad A \to bW \quad W \to \Lambda \mid aW$$

This grammar does satisfy the LL(1) property, as

$$\begin{array}{ll} LA_1(U \to AU) = \{b\} & LA_1(U \to \Lambda) = \{\$\} \\ LA_1(S_1 \to aU) = \{a\} & LA_1(S_1 \to U) = \{b, \$\} \\ LA_1(W \to aW) = \{a\} & LA_1(W \to \Lambda) = \{b, \$\} \end{array}$$

(RvV, 9 October 2006)

Solution to exercise Fundamentele Informatica 3

Excercise 11.12.l): Let us call the decision problem from this exercise **SubSuperSet**. We cannot apply Rice's theorem directly to **SubSuperSet**, because this decision problem has two TM's T_1 and T_2 as parameters, while Rice's theorem is about decision problems with (only) one TM as a parameter. Still, we can use the theorem to prove that **SubSuperSet** is unsolvable.

Let L_2 be any recursively enumerable language over an alphabet Σ, such that $L_2 \neq \emptyset$ and $L_2 \neq \Sigma^*$. Further, let T_2' be a TM accepting L_2. Then let **SubSuperSet**$\boldsymbol{L_2}$ be the following decision problem:

> Given a TM T, is $L(T) \subseteq L_2$ or $L_2 \subseteq L(T)$?

Because $L_2 \neq \emptyset$ and $L_2 \neq \Sigma^*$, the property of being a subset or a superset of L_2 is a non-trivial property of recursively enumerable languages (why exactly?). Hence, by Rice's theorem, the decision problem **SubSuperSet**$\boldsymbol{L_2}$ is unsolvable.

We now reduce **SubSuperSet**$\boldsymbol{L_2}$ to the decision problem **SubSuperSet** from this exercise. For this, we must define a computable transformation F from the instances of **SubSuperSet**$\boldsymbol{L_2}$ (TM's T) to instances of **SubSuperSet** (pairs of TM's (T_1, T_2)), such that T is a yes-instance of **SubSuperSet**$\boldsymbol{L_2}$, if and only if $F(T)$ is a yes-instance of **SubSuperSet**. Let TM T be an arbitrary instance of **SubSuperSet**$\boldsymbol{L_2}$. We define F by $F(T) = (T, T_2')$. Hence, $T_1 = T$ and $T_2 = T_2'$. Indeed, the function F is computable.

Now, T is a yes-instance of **SubSuperSet**$\boldsymbol{L_2}$, if and only if

$$L(T) \subseteq L_2 \text{ or } L_2 \subseteq L(T),$$

i.e., if and only if

$$L(T) \subseteq L(T_2') \text{ or } L(T_2') \subseteq L(T),$$

i.e., if and only if $F(T) = (T, T_2')$ is a yes-instance of **SubSuperSet**.

Indeed, **SubSuperSet**$\boldsymbol{L_2}$$\leq$**SubSuperSet**. Because decision problem **SubSuperSet**$\boldsymbol{L_2}$ is unsolvable, so is **SubSuperSet**.

(RvV, 6 December 2006)