Fundamentele Informatica 1 (I&E)

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http://www.liacs.leidenuniv.nl/~vlietrvan1/fi1ie/

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5. Pushdown Automata

5.3. A PDA from a Given CFG

Exercise 5.8.

Give transition diagrams for PDAs accepting each of the following languages.

a0. $\{a^i b^{2i} \mid i \geq 0\}$

a. $\{a^i b^j \mid i \leq j \leq 2i\}$

a1. $\{a^i b^j \mid i < j < 2i\}$

a2. $\{a^{2j} b^j \mid j \geq 0\}$

a3. $\{a^i b^j \mid j \leq i \leq 2j\}$

b. $\{x \in \{a, b\}^* \mid n_a(x) < n_b(x) < 2n_a(x)\}$

5.3. A PDA from a Given CFG

reg. languages	FA	reg. grammar	reg. expression
determ. cf. languages	DPDA		
cf. languages	PDA	cf. grammar	
re. languages	TM	unrestr. grammar	

The Nondeterministic Top-Down PDA

Part of a slide from lecture 7:

Example 4.2. The language *Expr*

$$S \to a \mid S + S \mid S * S \mid (S)$$

Example.
Simplified Algebraic Expressions

Let G be CFG with productions

$$\begin{array}{rcl} S & \rightarrow & S + T \mid T \\ T & \rightarrow & T * a \mid a \end{array}$$

- Leftmost derivation of $a + a * a$ . . .

Example.
Simplified Algebraic Expressions

Let G be CFG with productions

$$\begin{array}{rcl} S & \rightarrow & S+T \mid T \\ T & \rightarrow & T*a \mid a \end{array}$$

- Leftmost derivation of $a+a*a$ ...
- Corresponding derivation tree ...
- Simulating leftmost derivation on stack
  - Applying productions ...
  - Matching terminals ...
- Corresponding PDA ...
- Initial/final move ...

Definition 5.17. The Nondeterministic Top-Down PDA $NT(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic top-down PDA corresponding to G is $NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

$$Q = \dots \qquad A = \dots \qquad \Gamma = \dots$$

The initial move of $NT(G)$ is the Λ-transition

$$\delta(q_0, \Lambda, Z_0) = \dots$$

and the only move to the accepting state is the Λ-transition

$$\delta(q_1, \Lambda, Z_0) = \dots$$

The moves from q_1 are the following:
For every $A \in V$, $\delta(q_1, \Lambda, A) = \dots$
For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \dots$

Definition 5.17. The Nondeterministic Top-Down PDA $NT(G)$

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic top-down PDA corresponding to G is $NT(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

$$Q = \{q_0, q_1, q_2\} \quad A = \{q_2\} \quad \Gamma = V \cup \Sigma \cup \{Z_0\}$$

The initial move of $NT(G)$ is the Λ-transition

$$\delta(q_0, \Lambda, Z_0) = \{(q_1, SZ_0)\}$$

and the only move to the accepting state is the Λ-transition

$$\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$$

The moves from q_1 are the following:
For every $A \in V$, $\delta(q_1, \Lambda, A) = \{(q_1, \alpha) \mid A \to \alpha \text{ is a production in } G\}$
For every $\sigma \in \Sigma$, $\delta(q_1, \sigma, \sigma) = \{(q_1, \Lambda)\}$

Example 5.19. The Language *Balanced*

$$S \to [S] \mid SS \mid \Lambda$$

(Succesful) computation for $x =$ [[ ]] [ ] . . .

Theorem 5.18.

If G is a context-free grammar, then the nondeterministic top-down PDA $NT(G)$ accepts the language $L(G)$.

Proof. . .

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If G is a context-free grammar, then the nondeterministic top-down PDA $NT(G)$ accepts the language $L(G)$.

Proof. . .

Claim:
for $i = 0, 1, \ldots, m$, there is a sequence of moves of $NT(G)$, such that

- $NT(G)$ has read $x_0 x_1 \ldots x_i$, and
- stack contents of $NT(G)$ is $A_i \alpha_i Z_0$

The details of the proof of this result do not have to be known for the exam.

Exercise 5.33.

Under what circumstances is the top-down PDA $NT(G)$ deterministic?
(For what kind of grammar G, and what kind of language, could this happen?)

Study this exercise yourself

The Nondeterministic Bottom-Up PDA

Example 5.24. Simplified Algebraic Expressions

Let G be CFG with productions

$$
\begin{array}{rcl}
S & \to & S + T \;\mid\; T \\
T & \to & T * a \;\mid\; a
\end{array}
$$

- Rightmost derivation of $a + a * a$ . . .

Example 5.24. Simplified Algebraic Expressions

Let G be CFG with productions

$$\begin{aligned} S &\rightarrow S + T \mid T \\ T &\rightarrow T * a \mid a \end{aligned}$$

- Rightmost derivation of $a + a * a$ ...
- Corresponding derivation tree ...
- Simulating bottom-up construction tree on stack
  - Shifting terminals ...
  - Reducing by productions ...
- Corresponding PDA ...
- Final moves ...
- Stack + remaining input ...

Definition 5.22. The Nondeterministic Bottom-Up PDA *NB*(G)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic bottom-up PDA corresponding to G is *NB*$(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.

$\Gamma = \ldots$

Definition 5.22. The Nondeterministic Bottom-Up PDA *NB*(G)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar.
The nondeterministic bottom-up PDA corresponding to G is *NB*$(G) = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$, defined as follows:

Q contains the initial state q_0, the state q_1, and the (only) accepting state q_2, together with other states to be described shortly.

$\Gamma = V \cup \Sigma \cup \{Z_0\}$

Definition 5.22. The Nondeterministic Bottom-Up PDA *NB*(G) (continued)

For every $\sigma \in \Sigma$ and every $X \in \Gamma$, $\delta(q_0, \sigma, X) = \{(q_0, \sigma X)\}$. This is a *shift* move.

For every production $B \to \alpha$ in G, and every nonnull string $\beta \in \Gamma^*$,
$(q_0, \Lambda, \alpha^r \beta) \vdash^* (q_0, B\beta)$,
where this *reduction* is a sequence of one or more moves in which, if there is more than one, the intermediate configurations involve other states that are specific to this sequence and appear in no other moves of *NB*(G).

One of the elements of $\delta(q_0, \Lambda, S)$ is (q_1, Λ),
and $\delta(q_1, \Lambda, Z_0) = \{(q_2, Z_0)\}$.

Theorem 5.23.

If G is a context-free grammar, then the nondeterministic bottom-up PDA $NB(G)$ accepts the language $L(G)$.

The proof of this result does not have to be known for the exam.

Exercise 5.33. (continued)

Can the bottom-up PDA $NB(G)$ ever be deterministic? Explain.

Study this exercise yourself