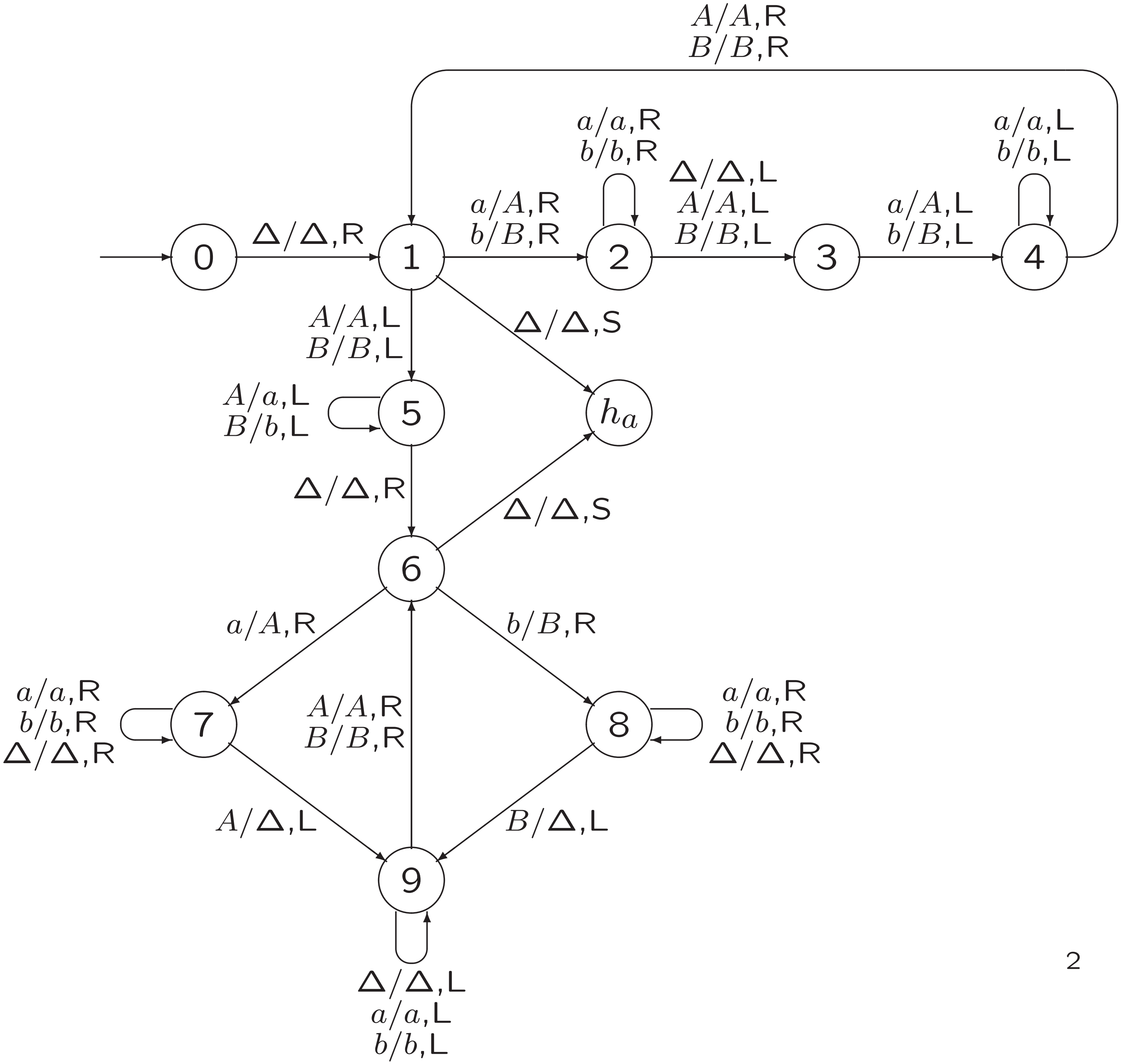
Exercise 7.1.

Trace the TM in Figure 7.6 (see next slide), accepting the language $\{xx \mid x \in \{a, b\}^*\}$, on the string $aaba$. Show the configuration at each step.


A/A,R
B/B,R
a/a,R
b/b,R
Δ/Δ,L
A/A,L
B/B,L
a/a,L
b/b,L
Δ/Δ,R
a/A,R
b/B,R
a/A,L
b/B,L
0
1
2
3
4
A/A,L
B/B,L
Δ/Δ,S
A/a,L
B/b,L
5
h_a
Δ/Δ,R
Δ/Δ,S
6
a/A,R
b/B,R
a/a,R
b/b,R
Δ/Δ,R
7
A/A,R
B/B,R
8
a/a,R
b/b,R
Δ/Δ,R
A/Δ,L
B/Δ,L
9
Δ/Δ,L
a/a,L
b/b,L

Exercise 7.2.

Below is a transition table for a TM with input alphabet $\{a, b\}$.

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	Δ	(q_1, Δ, R)	q_2	Δ	(h_a, Δ, R)	q_6	a	(q_6, a, R)
q_1	a	(q_1, a, R)	q_3	Δ	(q_4, a, R)	q_6	b	(q_6, b, R)
q_1	b	(q_1, b, R)	q_4	a	(q_4, a, R)	q_6	Δ	(q_7, b, L)
q_1	Δ	(q_2, Δ, L)	q_4	b	(q_4, b, R)	q_7	a	(q_7, a, L)
q_2	a	(q_3, Δ, R)	q_4	Δ	(q_7, a, L)	q_7	b	(q_7, b, L)
q_2	b	(q_5, Δ, R)	q_5	Δ	(q_6, b, R)	q_7	Δ	(q_2, Δ, L)

What is the final configuration if the TM starts with input string x?

Exercise 7.3.

Let $T = (Q, \Sigma, \Gamma, q_0, \delta)$ be a TM, and let s and t be the sizes of the sets Q and Γ, respectively.
How many distinct configurations of T could there possibly be in which all tape squares past square n are blank and T's tape head is on or to the left of square n? (The tape squares are numbered beginning with 0.)

Exercise 7.10.

We do not define Λ-transitions for a TM. Why not?
What features of a TM make it unnecessary or inappropriate to talk about Λ-transitions?

Exercise 7.17.

For each case below, draw a TM that computes the indicated function.

In the first four parts, the function is from $\mathbb{N}$ to $\mathbb{N}$. In each of these parts, assume that the TM uses unary notation — i.e., the natural number n is represented by the string 1^n.

a. $f(x) = x + 2$

b. $f(x) = 2x$

c. $f(x) = x^2$

e. $E : \{a,b\}^* \times \{a,b\}^* \to \{0,1\}$
defined by $E(x,y) = 1$ if $x = y$, $\qquad E(x,y) = 0$ otherwise.

Exercise.

Draw a TM that computes the function

$$f(x, y) = x + y$$

where x, y are integers ≥ 0.

Assume that the TM uses unary notation, both for its input and for its output.

Exercise.

Draw a TM that computes the function $f(x, y) = x \bmod y$

Hint: implement the following algorithm:

```
while (x >= y)
  x = x - y;
```

Exercise 7.12.

Suppose T is a TM that accepts a language L.
Describe how you would modify T to obtain another TM accepting L that never halts in the reject state h_r.

Exercise 7.16.

Does every TM compute a partial function? Explain.