A slide from lecture 5:

Theorem 8.7. If L is a recursively enumerable language, and its complement L' is also recursively enumerable, then L is recursive
(and therefore, by Theorem 8.6, L' is recursive).

Proof. . .

Exercise 8.4.

Suppose $L_1, L_2, \ldots, L_k$ form a partition of Σ^*: in other words, their union is Σ^* and any two of them are disjoint.

Show that if each L_i is recursively enumerable, then each L_i is recursive.

Exercise 8.8.

Suppose L is recursively enumerable but not recursive. Show that if T is a TM accepting L, there must be infinitely many input strings for which T loops forever.

Exercise 8.17.

In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a, b, and c are terminals, and all other symbols are variables.

a.

$S \to ABCS \quad | \quad ABC$

$AB \to BA \quad AC \to CA \quad BC \to CB$

$BA \to AB \quad CA \to AC \quad CB \to BC$

$A \to a \quad B \to b \quad C \to c$

Exercise 8.17.

In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a, b, and c are terminals, and all other symbols are variables.

b.

$S \to LaR \quad L \to LD \mid LT \mid \Lambda \quad Da \to aaD \quad Ta \to aaaT$

$DR \to R \quad TR \to R \quad R \to \Lambda$

Exercise 8.17.

In each case below, describe the language generated by the unrestricted grammar with the given productions. The symbols a, b, and c are terminals, and all other symbols are variables.

c.

$S \to LaMR \quad L \to LT \mid E$

$Ta \to aT \quad TM \to aaMT \quad TR \to aMR$

$Ea \to aE \quad EM \to E \quad ER \to \Lambda$

Exercise 8.18.

Consider the unrestricted grammar with the following productions.

$S \to TD_1D_2 \quad T \to ABCT \mid \Lambda$

$AB \to BA \quad BA \to AB \quad CA \to AC \quad CB \to BC$

$CD_1 \to D_1C \quad CD_2 \to D_2a \quad BD_1 \to D_1b$

$A \to a \quad D_1 \to \Lambda \quad D_2 \to \Lambda$

a. Describe the language generated by this grammar.

b. Find a single production that could be substituted for $BD_1 \to D_1b$ so that the resulting language would be

$$\{xa^n \mid \ n \geq 0,\ |x| = 2n, \text{ and } n_a(x) = n_b(x) = n\}$$

Exercise 8.19.

For each of the following languages, find an unrestricted grammar that generates the language.

a. $\{a^n b^n a^n b^n \mid n \geq 0\}$

Exercise 8.19.

For each of the following languages, find an unrestricted grammar that generates the language.

c. $\{sss \mid s \in \{a,b\}^*\}$

d. $\{ss^r s \mid s \in \{a,b\}^*\}$

Exercise 8.20.

For each of the following languages, find an unrestricted grammar that generates the language.

a. $\{x \in \{a, b, c\}^* \mid \; n_a(x) < n_b(x) \text{ and } n_a(x) < n_c(x)\}$

c. $\{a^n \;\mid\; n = j(j+1)/2 \text{ for some } j \geq 1\}$
(Suggestion: if a string has j groups of a's, the ith group containing i a's, then you can create $j+1$ groups by adding an a to each of the j groups and adding a single extra a at the beginning.)

Exercise 8.21.

Suppose G is an unrestricted grammar with start symbol T that generate the language $L \subseteq \{a, b\}^*$. In each part below, another unrestricted grammar is described. Say (in terms of L) what language it generates.

a. The grammar containing all the variables and all the productions of G, two additional variables S (the start variable) and E, and the additional productions

$$S \to ET \qquad E \to \Lambda \qquad Ea \to E \qquad Eb \to E$$

Exercise 8.21.

Suppose G is an unrestricted grammar with start symbol T that generate the language $L \subseteq \{a, b\}^*$. In each part below, another unrestricted grammar is described. Say (in terms of L) what language it generates.

b. The grammar containing all the variables and all the productions of G, four additional variables S (the start variable), F, R, and E, and the additional productions

$$S \to FTR \quad Fa \to aF \quad Fb \to bF \quad F \to E$$
$$Ea \to E \quad Eb \to E \quad ER \to \Lambda$$

Exercise.

In lecture 5, we have discussed an implementation of phase 2 of the construction from Theorem 8.13, which simulates arbitrary derivations in an unrestricted grammar, in a nondeterministic Turing machine.

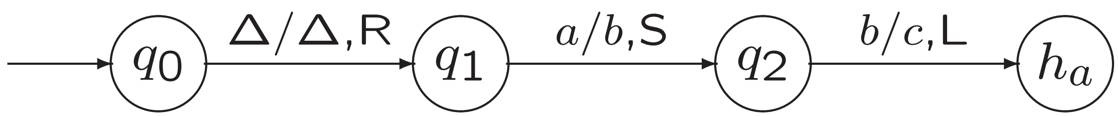
Draw the resulting NTM for the unrestricted grammar below:

$S \to LaR \quad L \to LD \mid \Lambda$
$Da \to aaD \quad DR \to R \quad R \to \Lambda$

N.B.: You may need components *Insert*(σ) and *Delete* in your answer.

Exercise.

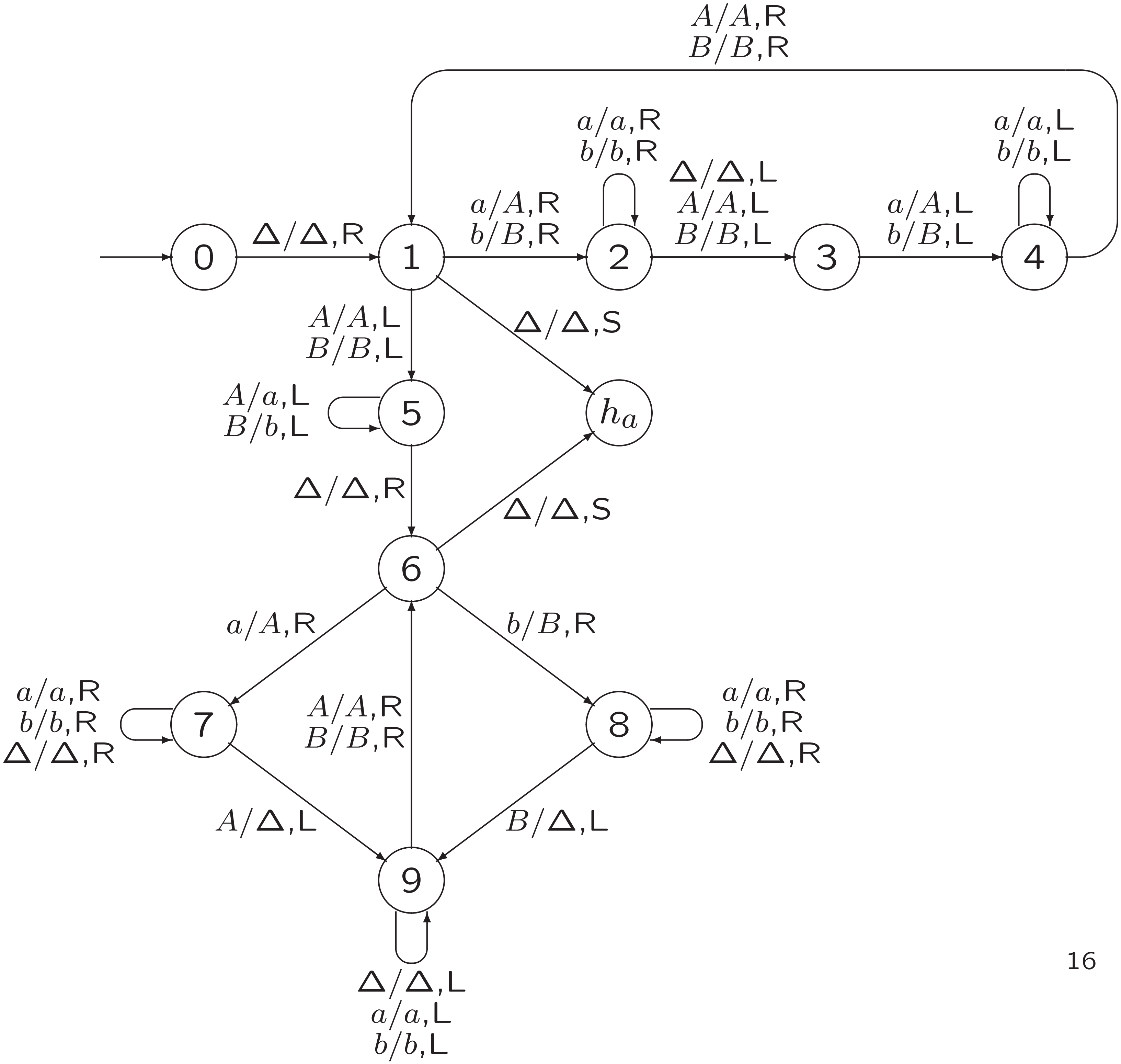
For the Turing machine below, give all productions of the second type (to simulate the computation of the Turing machine) that result from the construction in the proof of Theorem 8.14.



Exercise 8.22.

Figure 7.6 (see next slide) shows the transition diagram for a TM accepting $XX = \{xx \mid x \in \{a,b\}^*\}$.

In the grammar obtained from this TM as in the proof of Theorem 8.14, give a derivation for the string $abab$.



A/A,R
B/B,R
a/a,R
b/b,R
a/a,L
b/b,L
Δ/Δ,L
A/A,L
B/B,L
Δ/Δ,R
a/A,R
b/B,R
a/A,L
b/B,L
0
1
2
3
4
A/A,L
B/B,L
Δ/Δ,S
A/a,L
B/b,L
5
h_a
Δ/Δ,R
Δ/Δ,S
6
a/A,R
b/B,R
a/a,R
b/b,R
Δ/Δ,R
7
A/A,R
B/B,R
8
a/a,R
b/b,R
Δ/Δ,R
A/Δ,L
B/Δ,L
9
Δ/Δ,L
a/a,L
b/b,L

Exercise 8.27.

Show that if L is any recursively enumerable language, then L can be generated by a grammar in which the left side of every production is a string of one or more variables.