Some more solutions to exercises in

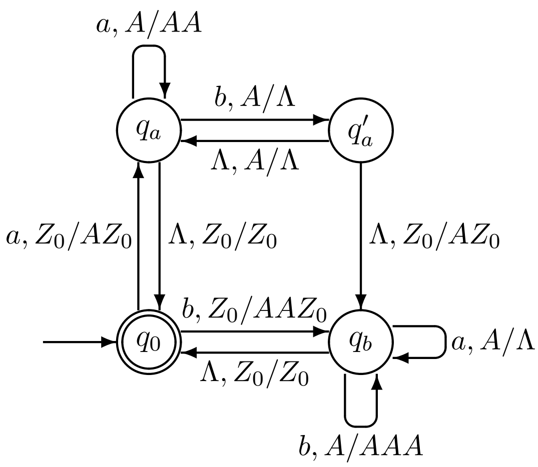
John C. Martin: Introduction to Languages and The Theory of Computation

fourth edition

5.25(b) In this language, every b counts for two a's.

A natural solution is to have the starting state q_0 as the only accepting state, to have a state q_a to count an excess in a's in the string (with A's on the stack) and to have a stare q_b to count an excess in b's in the string (with two A's on the stack for every extra b).

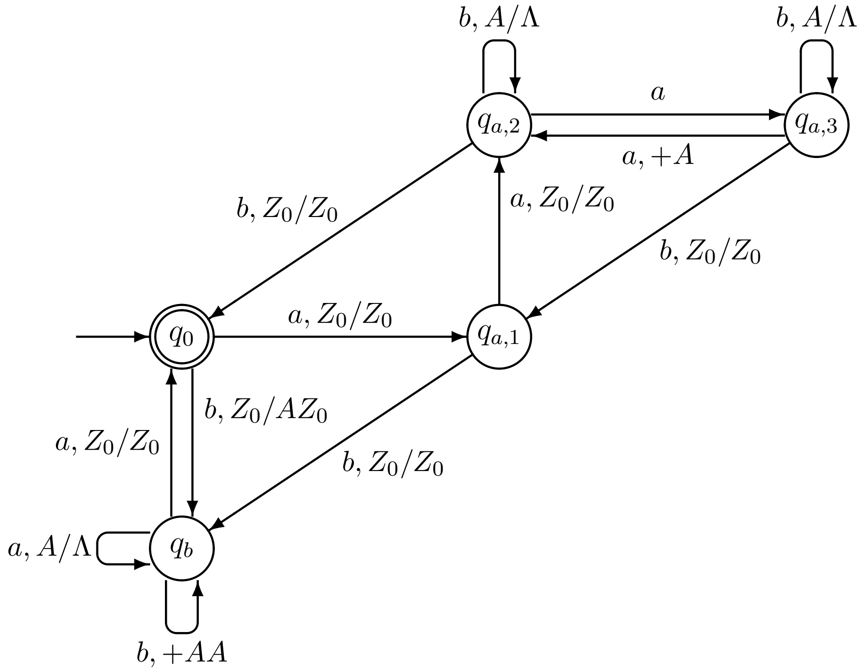
From q_a and q_b, we can return to the accepting state q_0 with a Λ-transition, when we see Z_0 on top of the stack. In that case, we do not have an excess of a's or b's anymore. When we are in q_a and read a b, we should remove two A's from the stack. We need to do this in two steps, with state q'_a as an intermediate state. If, in q'_a, we do not find on the stack the second A we wish to remove, we have an excess in b's and move to q_b, pushing only one A onto the stack. The result is:



The above counter automaton is perfectly deterministic, but we may still prefer an automaton without Λ-transitions. The Λ-transitions from q_a and q_b back to q_0 could be avoided by pushing one A less onto the stack on our way *from* q_0 *to* q_a and q_b. This does, however, not work for the Λ-transitions from the intermediate state q'_a to q_a and q_b.

In a more general pushdown automaton, we might try to use two different stack symbols: one to represent a single A and one to represent two A's. This is, however, not allowed in a counter automaton, because we only have one stack symbol (in addition to Z_0).

The solution is to split q_a in different states for an odd number of A's and an even number of A's, to push only one A onto the stack for two a's in the input, and to push one A less onto the stack (so we can easily recognize that an a or b we read restores the balance, because we see Z_0 on top of the stack). The result is:


$b, A/\Lambda$
$b, A/\Lambda$
a
$q_{a,2}$
$q_{a,3}$
$a, +A$
$b, Z_0/Z_0$
$a, Z_0/Z_0$
$b, Z_0/Z_0$
$a, Z_0/Z_0$
q_0
$q_{a,1}$
$b, Z_0/AZ_0$
$a, Z_0/Z_0$
$b, Z_0/Z_0$
$a, A/\Lambda$
q_b
$b, +AA$