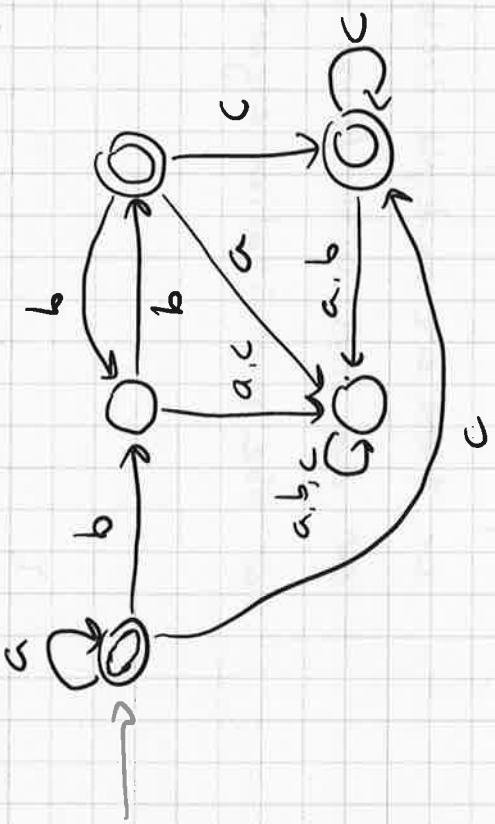


1.

2.) a.) i. 1

ii. 2

iii. 3

iv. 4

v. 5

b.) $[x_1] = \{ x \in \{a,b\} \mid x \text{ does not end in } b, |x| \text{ even} \}$

$[x_2] = \{ x \in \{a,b\} \mid |x| \text{ odd} \}$

$[x_3] = [x_2]$

$[x_4] = [x_1]$

$[x_5] = L$

c.) No, because there are multiple states corresponding to the same equivalence class;

or there are multiple states with the same future set;

or an FA with fewer states is provided;

or the minimization algorithm is performed.

3.) c.) i. yes
ii. no
iii. yes
iv. yes

b.)

4.) Suppose L is accepted by an automaton having n states, $n \geq 1$.

Note $x = a^n b^n \in L$, as $n_a(x) = n = n_b(x)$ and

$$n \geq n \geq n-2.$$

Moreover, $|x| = 2n \geq n$.

Let $x = uvw$ with $|uv| \leq n$ and $|v| \geq 1$ be arbitrary.

Then $u = a^i$, $v = a^j$, $i + j \leq n$, $j \geq 1$.

However, $uv^4w = a^{n+3j}b^n \notin L$,

because $n + 3j - 2 \geq n + 1 \not\leq n$.

Hence, L is not regular.

21.11

5) a) Yes, L(G) is regular. You can easily simulate all productions by regular productions.

21.13

b) Yes L(G) is regular. The language can be described by the regular expression $a^*(a+b)b^*$

21.14

c) No L(G) is not regular. The languge is $\{a^n\{a,aa,b\}b^n \mid n \geq 0\}$

21.19

d) Yes, L(G) is regular. G is a regular grammar.

21.20

6) a) The first six elements in the canonical (shortlex) order of L:
a, aa, aaa, aab, aba, baa

21.22 The next elements are: aaaa, aaab, aaba, abaa, baaa, aaaaa

b) G has startvariable S and the following transitions

$S \to S_1 \mid S_2 \mid S_3$ — S_1 for $j=0$ → a non-empty string of a's; S_2 for $1 \leq j < i$; S_3 for $j \geq i$, so $i \leq j < i+k$

$S_1 \to aS_1 \mid a$

$S_2 \to XA$ — X for $a^i b^j$ with $1 \leq j < i$; A for a^k with $k \geq 0$

$X \to aXb \mid aaAb$ — for every b also an a in a^i, and at least one a more

$A \to aA \mid \Lambda$

$S_3 \to YZ$

$Y \to aYb \mid \Lambda$ — for every a in a^i also a b.

$Z \to bZa \mid aA$ — for every extra b an a in a^k, and at least one a more.

21.40

A simple ambiguous grammar would be:

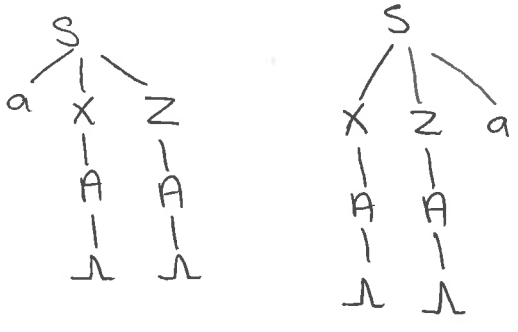
$S \to aXZ \mid XZa$ — X for b's that are matched by a's in a^i; Z for b's that are matched by a's in a^k at least one a more

$X \to aXb \mid A$

$Z \to bZa \mid A$

$A \to aA \mid \Lambda$

Two derivation trees for x = a



21.45

7)

a) Set of nullable variables: $\{B\}$

21.47

b) Add productions where occurrences (or an occurrence) of B are left out. G_2 has start variable S and the following productions

$S \to XBBb \mid XB \mid XBb \mid Xb \mid X$
$X \to aX \mid ab$
$B \to bB \mid b$

21.50

c) There is one unit production: $S \to X$
G_3 has start variable S and the following productions:

$S \to XBBb \mid XB \mid XBb \mid Xb \mid aX \mid ab$
$X \to aX \mid ab$
$B \to bB \mid b$

21.53

d) G_4 has start variable S and the following productions:

$S \to XBBZ_b \mid XB \mid XBZ_b \mid XZ_b \mid Z_aX \mid Z_aZ_b$
$X \to Z_aX \mid Z_aZ_b$
$B \to Z_bB \mid b$
$Z_a \to a$ (we use Z_a and Z_b, rather than X_a and X_b to avoid confusion with variable X)
$Z_b \to b$

21.56

e) Two productions are too long: $S \to XBBZ_b \mid XBZ_b$
G_5 has start variable S and the following productions:

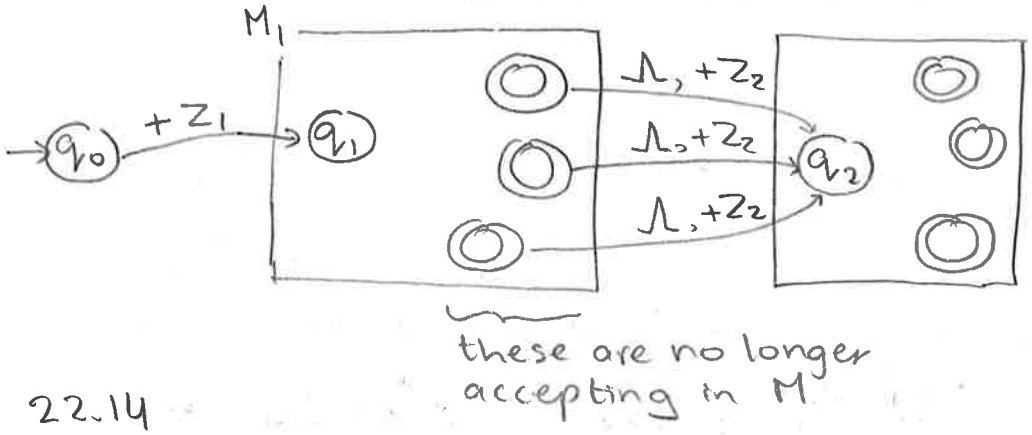
$S \to XY_1 \mid XB \mid XY_3 \mid XZ_b \mid Z_aX \mid Z_aZ_b$
$Y_1 \to BY_2$
$Y_2 \to BZ_b$
$Y_3 \to BZ_b$
$X \to Z_aX \mid Z_aZ_b$
$B \to Z_bB \mid b$
$Z_a \to a$
$Z_b \to b$

It is possible to combine Y_2 and Y_3 in a single variable.

22.01

8/ new start state q_0 and new initial stack symbol Z_0,
M starts by pushing Z_1 onto the stack, while moving from q_0 to q_1, the start state of M_1
M then performs a computation in M_1
If M arrives in an accepting state of M_1, it may take a Λ-transition (for any stack symbol in $\Gamma_1 \cup \{Z_0\}$) to q_2, the start state of M_2, while pushing Z_2 onto the stack.
M then performs a computation in M_2.
The (only) accepting states of M are the ones of M_2

In pictures:



22.14

9a) Occurrence 1: state 1, input a, stack symbol Z_0
(the notation $a, +A$ includes $a, Z_0/AZ_0$.)
Occurrence 2: state 2, input b, or Λ, stack symbol Z_0
(the notation $b, +B$ includes b, Z_0, BZ_0)

22.19
b) $Le(M_1) = \{ x \in \{a,b\}^* \mid n_a(x) = n_b(x)+1 \text{ and } x \text{ ends with } a \}$
M keeps track of the difference in the numbers of a's and b's read so far. In state 1, M counts an excess in a's by symbols A on the stack. In state 2, M counts an excess in b's by symbols B on the stack.
A's are pushed in state 1 on reading a's, and are popped in that state on reading b's. Similarly, B's are pushed in state 2 on reading b's, and are popped in that state on reading a's.
When the top stack symbol is Z_0, this is the only symbol on the stack, and the number of a's read so far is equal to the number of b's. If we are in state 2 then we can follow the Λ-transition to state 1. If we are in state 1 with top stack symbol Z_0, then we can pop Z_0 on reading an extra a, and accept the string, as the stack is completely empty.

1233