From exercise class 10:

Exercise 5.16.

Show that if L is accepted by a PDA, then L is accepted by a PDA that never crashes (i.e., for which the stack never empties and no configuration is reached from which there is no move defined).

From lecture 10:

Stack in PDA contains symbols from certain alphabet.

Usual stack operations: pop, top, push

Extra possiblity: replace top element X by string α

$\alpha = \Lambda$	pop
$\alpha = X$	top
$\alpha = YX$	push
$\alpha = \beta X$	push*
$\alpha = \ldots$	

Top element X is required to do a move!

From exercise class 10:

Exercise 5.17.

Show that if L is accepted by a PDA, then L is accepted by a PDA in which every move
$*$ either pops something from the stack (i.e., removes a stack symbol without putting anything else on the stack);
$*$ or pushes a single symbol onto the stack on top of the symbol that was previously on top;
$*$ or leaves the stack unchanged.

Hence, each action on the stack due to a move in the PDA has one of the following forms:
$*$ either X/Λ (with $X \in \Gamma$),
$*$ or X/YX (with $X, Y \in \Gamma$),
$*$ or X/X (with $X \in \Gamma$).

From lecture 7:

Theorem 4.9.

If L_1 and L_2 are context-free languages over an alphabet Σ, then

$$L_1 \cup L_2, \quad L_1 L_2 \quad \text{and } L_1^*$$

are also CFLs.

From exercise class 11:

Exercise 5.19.

Suppose M_1 and M_2 are PDAs accepting L_1 and L_2, respectively. For both the languages L_1L_2 and L_1^*, describe a procedure for constructing a PDA accepting the language.

In each case, nondeterminism will be necessary. Be sure to say precisely how the stack of the new machine works; no relationship is assumed between the stack alphabets of M_1 and M_2.

Answer begins with:
Let $M_1 = (Q_1, \Sigma, \Gamma_1, q_{01}, Z_{01}, A_1, \delta_1)$
and let $M_2 = (Q_2, \Sigma, \Gamma_2, q_{02}, Z_{02}, A_2, \delta_2)$.

Exercise 5.21.

Prove the converse of Theorem 5.28:
If there is a PDA $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ accepting L by empty stack (that is, $x \in L$ if and only if $(q_0, x, Z_0) \vdash_M^* (q, \Lambda, \Lambda)$ for some state q),
then there is a PDA M_1 accepting L by final state (i.e., the ordinary way).

Exercise 5.34.

In each case below, you are given a CFG G and a string x that it generates.
Draw the nondeterministic bottom-up PDA *NB*(G).

Trace a sequence of moves in *NB*(G) by which x is accepted, showing at each step the stack contents and the unread input. Show at the same time the corresponding rightmost derivation of x (in reverse order) in the grammar. See Example 5.24 for a guide.

a. The grammar has productions $S \to S[S] \mid \Lambda$ and $x = [][[]]$.

Exercise 5.30.

For a certain CFG G, the moves shown below are those by which the nondeterministic bottom-up PDA *NB*(G) accepts the input aabbab. Each occurrence of $\vdash^*$ indicates a sequence of moves constituting a reduction. Draw the derivation tree for $aabbab$ that corresponds to this sequence of moves.

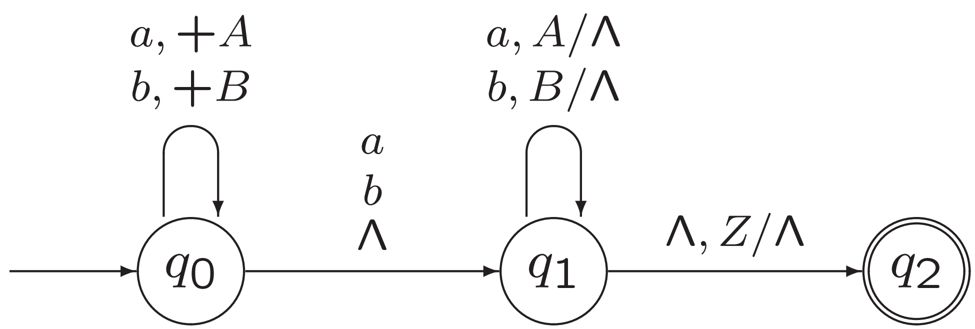
$$
\begin{aligned}
(q_0, aabbab, Z_0) \quad &\vdash \quad (q_0, abbab, aZ_0) \vdash (q_0, bbab, aaZ_0) \\
&\vdash \quad (q_0, bab, baaZ_0) \vdash^* (q_0, bab, SaZ_0) \\
&\vdash \quad (q_0, ab, bSaZ_0) \vdash^* (q_0, ab, SZ_0) \\
&\vdash \quad (q_0, b, aSZ_0) \vdash (q_0, \Lambda, baSZ_0) \\
&\vdash^* \quad (q_0, \Lambda, SSZ_0) \vdash^* (q_0, \Lambda, SZ_0) \\
&\vdash \quad (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)
\end{aligned}
$$

Exercise 5.32.

Lem M be the PDA below, accepting

$$\textit{Pal} = \{y \in \{a,b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a,b\}^*\}$$

(by empty stack). Let $x = ababa$. Find a sequence of moves of M by which x is accepted, and give the corresponding leftmost derivation in the CFG obtained from M as in Theorem 5.29.



Exercise 5.35.

Let M be the PDA on the blackboard, accepting *SimplePal* by empty stack. Consider the simplistic approach to obtaining a CFG described in the discussion preceding Theorem 5.29. The states of M are ignored, the variables of the grammar are the stack symbols of M, and for every move that reads σ and replaces A on the stack by $BC \ldots D$, we introduce the production $A \to \sigma BC \ldots D$.

a. Give all productions resulting from this approach.

b. Find a string $x \in \{a, b, c\}^*$ that is not accepted by M, but is generated by this CFG.