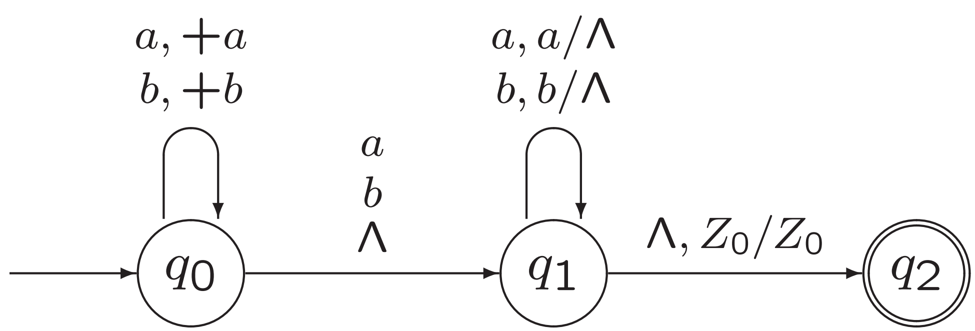
## Exercise 5.2.

For the PDA below, trace every possible sequence of moves for the two input strings $aba$ and $aab$.

**Example 5.7.** A Pushdown Automaton Accepting *Pal*

$$Pal = \{y \in \{a,b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a,b\}^*\}$$



**Exercise 5.4.**

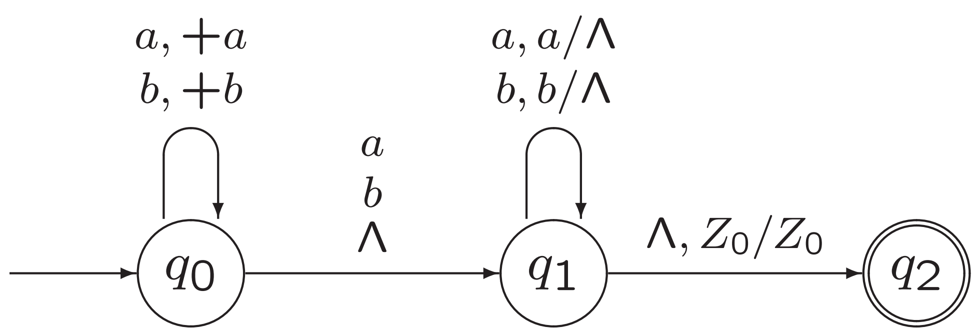
For each of the following languages over $\{a, b\}^*$, modify the PDA below to obtain a PDA accepting the language.

**a.** The language of even-length palindromes.

**b.** The language of odd-length palindromes.

**Example 5.7.** A Pushdown Automaton Accepting *Pal*

$$Pal = \{y \in \{a,b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a,b\}^*\}$$



**Exercise 5.5.**

Give transition diagrams for PDAs accepting each of the following languages.

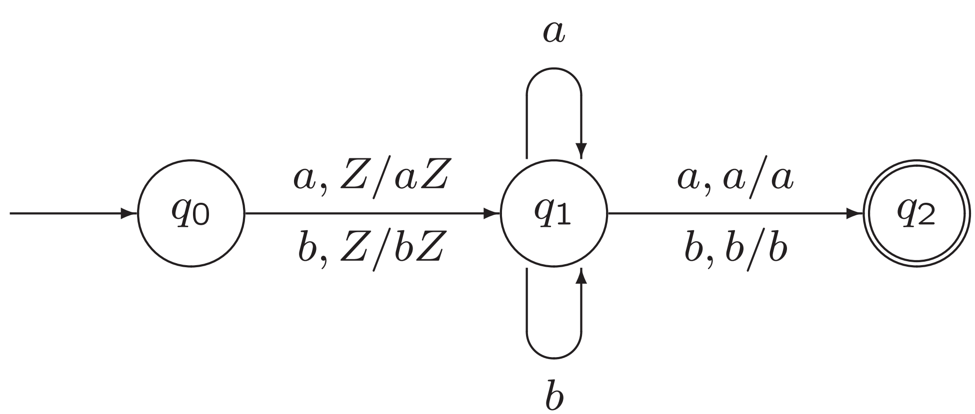
**a.** The language of all odd-length strings over $\{a, b\}$ with middle symbol $a$.

**b.** $\{a^n x \mid n \geq 0, x \in \{a, b\}^* \text{ and } |x| \leq n\}$.

**c.** $\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } j = i \text{ or } j = k\}$.

## Exercise 5.6.

Below, a transition diagram is given for a PDA with intial state $q_0$ and accepting state $q_2$. Describe the language that is accepted.



**Exercise.**

Let $L_1 = \{a^i b^j c^k \mid \ i, j, k \geq 0$ en $2i > j\}$.

**a.** Give the first five elements of $L_1$ in the canonical order.

**b.** Give a PDA $M_1$ such that $L(M_1) = L_1$.

**Exercise.**

Let $L_1 = \{a^i b^j c^k \mid \ i, j, k \geq 0$ en $2i > j\}$.

**a.** Give the first five elements of $L_1$ in the canonical order.

**b.** Give a DPDA $M_1$ such that $L(M_1) = L_1$.

**Exercise 5.10.**

Show that every regular language can be accepted by a (deterministic) PDA $M$ with only two states in which there are no $\Lambda$-transitions and no symbols are ever removed from the stack.

**Exercise 5.12.**

Show that if $L$ is accepted by a PDA
in which no symbols are ever removed from the stack,
then $L$ is regular.

**Exercise 5.18.**

For each of the following languages, give a transition diagram for a deterministic PDA that accepts that language.

**a.** $\{x \in \{a, b\}^* \mid n_a(x) < n_b(x)\}$

**b.** $\{x \in \{a, b\}^* \mid n_a(x) \neq n_b(x)\}$

**c.** $\{x \in \{a, b\}^* \mid n_a(x) = 2n_b(x)\}$

**d.** $\{a^n b^{n+m} a^m \mid n, m \geq 0\}$

**Exercise 5.16.**

Show that if $L$ is accepted by a PDA, then $L$ is accepted by a PDA that never crashes (i.e., for which the stack never empties and no configuration is reached from which there is no move defined).

*From lecture 10:*

Stack in PDA contains symbols from certain alphabet.

Usual stack operations: pop, top, push

Extra possiblity: replace top element $X$ by string $\alpha$

| | |
|---|---|
| $\alpha = \Lambda$ | pop |
| $\alpha = X$ | top |
| $\alpha = YX$ | push |
| $\alpha = \beta X$ | push* |
| $\alpha = \ldots$ | |

Top element $X$ is required to do a move!

**Exercise 5.17.**

Show that if $L$ is accepted by a PDA, then $L$ is accepted by a PDA in which every move
$*$ either pops something from the stack (i.e., removes a stack symbol without putting anything else on the stack);
$*$ or pushes a single symbol onto the stack on top of the symbol that was previously on top;
$*$ or leaves the stack unchanged.

Hence, each action on the stack due to a move in the PDA has one of the following forms:
$*$ either $X/\Lambda$ (with $X \in \Gamma$),
$*$ or $X/YX$ (with $X, Y \in \Gamma$),
$*$ or $X/X$ (with $X \in \Gamma$).