Exercise 3.21. ♣

Consider the following transition table for an NFA with states 1–5, initial state 1 and input alphabet $\{a, b\}$. There are no Λ-transitions:

q	$\delta(q, a)$	$\delta(q, b)$
1	$\{1, 2\}$	$\{1\}$
2	$\{3\}$	$\{3\}$
3	$\{4\}$	$\{4\}$
4	$\{5\}$	$\emptyset$
5	$\emptyset$	$\{5\}$

a. Draw a transition diagram of the NFA (note that the accepting states are not specified).

b. Calculate $\delta^*(1, ab)$.
Hint: first calculate $\delta^*(1, \Lambda)$, then $\delta^*(1, a)$, then $\delta^*(1, ab)$.

c. Calculate $\delta^*(1, abaab)$.

Exercise 3.24. ♣

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA with no Λ-transitions. Show that for every $q \in Q$ and every $\sigma \in \Sigma$, $\delta^*(q, \sigma) = \delta(q, \sigma)$.

Exercise 3.33. ♣

Give an example of a regular language L containing Λ that cannot be accepted by any NFA having only one accepting state and no Λ-transitions, and show that your answer is correct.

Exercise 3.22.
A transition table is given for an NFA with seven states.

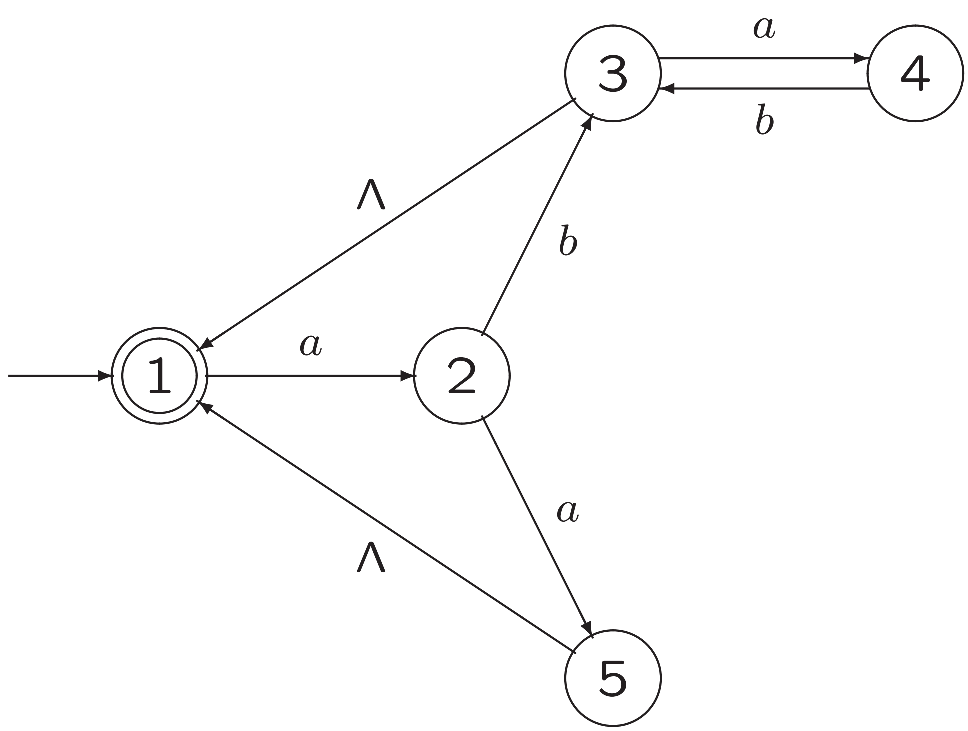
q	$\delta(q,a)$	$\delta(q,b)$	$\delta(q,\Lambda)$
1	$\emptyset$	$\emptyset$	$\{2\}$
2	$\{3\}$	$\emptyset$	$\{5\}$
3	$\emptyset$	$\{4\}$	$\emptyset$
4	$\{4\}$	$\emptyset$	$\{1\}$
5	$\emptyset$	$\{6,7\}$	$\emptyset$
6	$\{5\}$	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	$\{1\}$

Find:
d. ♣ $\delta^*(1, ba)$
Hint: first calculate $\delta^*(1,\Lambda)$, then $\delta^*(1,b)$, then $\delta^*(1,ba)$.
e. $\delta^*(1, ab)$
f. $\delta^*(1, ababa)$

Exercise 3.37.

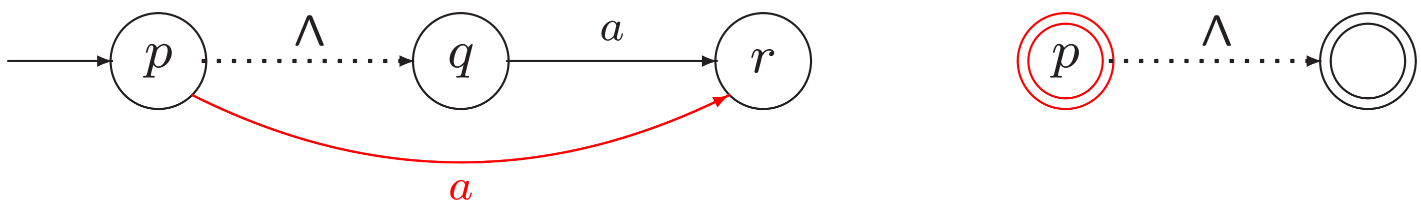
For each part below, use the algorithm from the lecture to draw an NFA with no Λ-transitions accepting the same language as the NFA pictured.

b.



Exercise.

Our construction:



Λ-removal

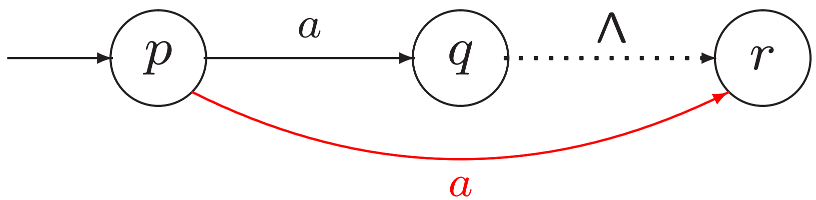
Given NFA $M = (Q, \Sigma, \delta, q_0, A)$,
construct NFA $M_1 = (Q, \Sigma, \delta_1, q_0, A_1)$ without Λ-transitions:

- whenever $q \in \Lambda_M(\{p\})$ and $r \in \delta(q, a)$, add r to $\delta_1(p, a)$
- whenever $\Lambda_M(\{p\}) \cap A \neq \emptyset$, add p to A_1.

continued on next slide. . .

Exercise. (ctd.)

Is it possible to invert the construction:



Λ-removal

Given NFA $M = (Q, \Sigma, \delta, q_0, A)$,

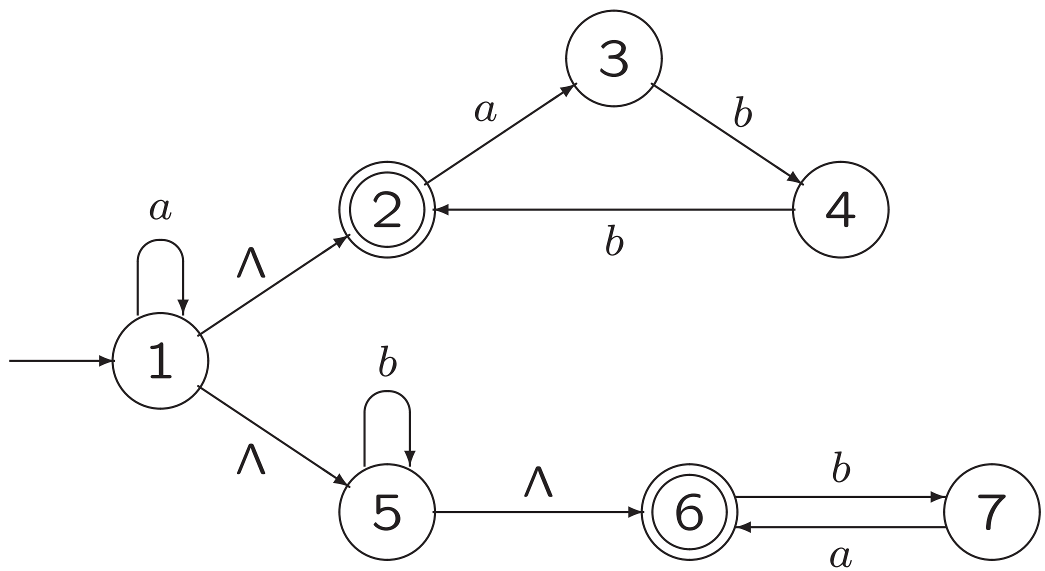
construct NFA $M_1 = (Q, \Sigma, \delta_1, q_0, A_1)$ without Λ-transitions:

- whenever $q \in \delta(p, a)$ and $r \in \Lambda_M(\{q\})$, add r to $\delta_1(p, a)$
- whenever $p \in A$ and $q \in \Lambda_M(\{p\})$, add q to A_1.

Exercise 3.40.

For each part below, draw an FA accepting the same language as the NFA shown.

a.



Exercise 3.32.

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA accepting a language L. Assume that there are no transitions to q_0, that A has only one element, q_f, and that there are no transitions from q_f.

a. Let M_1 be obtained from M by adding Λ-transitions from q_0 to every state that is reachable from q_0 in M.
(If p and q are states, q is reachable from p if there is a string $x \in \Sigma^*$ such that $q \in \delta^*(p, x)$.)
Describe (in terms of L) the language accepted by M_1.

b. Let M_2 be obtained from M by adding Λ-transitions to q_f from every state from which q_f is reachable in M.
Describe (in terms of L) the language accepted by M_2.

c. Let M_3 be obtained from M by adding both the Λ-transitions in (a) and those in (b).
Describe (in terms of L) the language accepted by M_3.

Exercise 3.7.

Find a regular expression corresponding to each of the following subsets of $\{a, b\}^*$.

a. ♣ The language of all strings containing exactly two a's.

c. ♣ The language of all strings that do not end with ab.

e. ♣ The language of all strings not containing the substring aa.

f. ♣ The language of all strings in which the number of a's is even.

g. ♣ The language of all strings containing no more than one occurrence of the string aa. (The string aaa should be viewed as containing two occurrences of aa.)

Exercise 3.7.

Find a regular expression corresponding to each of the following subsets of $\{a, b\}^*$.

i. The language of all strings containing both bb and aba as substrings.

j. The language of all strings not containing the substring aaa.

k. ♣ The language of all strings not containing the substring bba.

l. ♣ The language of all strings containing both aba and bab as substrings.

m. ♣ The language of all strings in which the number of a's is even and the number of b's is odd.

Exercise 3.1. ♣

In each case below, find a string of minimum length in $\{a, b\}^*$ not in the language corresponding to the given regular expression.

a. $b^*(ab)^*a^*$

b. $(a^* + b^*)(a^* + b^*)(a^* + b^*)$

Exercise 3.2. Consider the two regular expressions

$$r = a^* + b^* \qquad s = ab^* + ba^* + b^*a + (a^*b)^*$$

a. Find a string corresponding to r but not to s.
b. Find a string corresponding to s but not to r.
c. Find a string corresponding to both r and s.
d. Find a string in $\{a, b\}^*$ corresponding to neither r nor s.

Exercise 3.10. ♣

a. If L is the language corresponding to the regular expression $(aab + bbaba)^*baba$, find a regular expression corresponding to $L^r = \{x^r \mid x \in L\}$.

b. Using the example in part (a) as a model, give a recursive definition (based on Definition 3.1) of the reverse e^r of a regular expression e.

c. Show that for every regular expression e, if the language L corresponds to e, then L^r corresponds to e^r.

Exercise 3.41. For each of the following regular expressions, draw an NFA accepting the corresponding language, so that there is a recognizable correspondence between the regular expression and the transition diagram.

e. $(a^*bb)^* + bb^*a^*$

Exercise 3.42. For part (e) of Exercise 3.41, draw the NFA that is obtained by a literal application of Kleene's theorem, without any simplifications.

Exercise 3.51 (variant).

Use the algorithm of Brzozowski and McCluskey to find a regular expression corresponding to the FA below.

a.