Exercise 5.21.

Prove the converse of Theorem 5.28:
If there is a PDA $M = (Q, \Sigma, \Gamma, q_0, Z_0, A, \delta)$ accepting L by empty stack (that is, $x \in L$ if and only if $(q_0, x, Z_0) \vdash_M^* (q, \Lambda, \Lambda)$ for some state q),
then there is a PDA M_1 accepting L by final state (i.e., the ordinary way).

From exercise class 13:

Example 6.4. $XX = \{xx \mid x \in \{a, b\}^* \}$ is not context-free.
Use $u = a^n b^n a^n b^n$

Exercise 6.4.

In the proof given in Example 6.4 using the pumping lemma, the contradiction was obtained in each case by considering the string vw^0xy^0z.

Would it have been possible instead to use vw^2xy^2z in each case?
If so, give the proof in at least one case;
if not, explain why not.

From exercise class 13:

Example 6.4. $XX = \{xx \mid x \in \{a,b\}^* \}$ is not context-free.
Use $u = a^n b^n a^n b^n$

Exercise 6.3.

In the pumping-lemma proof in Example 6.4, give some examples of choices of strings $u \in L$ with $|u| \geq n$ that would not work.

Exercise 6.2.

In each case below, show using the pumping lemma that the given language is not a CFL.

a. ♣ $L = \{a^i b^j c^k \mid i < j < k\}$

b. $L = \{a^{2^i} \mid i \geq 0\}$

d. $L = \{a^i b^{2i} a^i \mid i \geq 0\}$

e. ♠ $L = \{s \in \{a, b, c\}^* \mid n_a(s) = \max\{n_b(s), n_c(s)\}\ \}$

g. ♣ $L = \{a^i b^j a^i b^{i+j} \mid i, j \geq 0\}$

Exercise 6.5. ♣

For each case below, decide whether the given language is a CFL, and prove your answer.

a. $L = \{a^i b^j a^j b^i \mid i, j \geq 0\}$

c. $L = \{scs \mid s \in \{a, b\}^*\}$

d. $L = \{sts \mid s, t \in \{a, b\}^*$ en $|s| \geq 1\}$

g. $L =$ the set of non-balanced strings of parentheses

Exercise.
(to demonstrate that in the application of the pumping lemma for context-free languages, one can often safely assume that $n \geq 2^2 = 4$)

Let $G = (V, \Sigma, S, P)$ be a context-free grammar in Chomsky normal form.

Suppose that $V = \{S\}$, i.e., that G has only one variable. What could $L(G)$ be in this case?

Exercise.

Give a context-free grammar for the language

$$\{a^i x\, y\, b^j \quad | \quad i, j \geq 0,\ x, y \in \{a, b\}^*,\ |x| = j \text{ and } |y| = i\}$$

Exercise.

Show using the pumping lemma that the language

$$L = \{a^i b^j a^i b^j \mid i, j \geq 0\}$$

is not context-free.

Exercise 6.6.

If L is a CFL, does it follow that $r(L) = \{x^r \mid x \in L\}$ is a CFL? Give a proof or a counterexample.

Exercise 6.9.

In each case below, show that the given language is a CFL but that its complement is not.

b. ♣ $\{a^i b^j c^k \quad | \quad i \neq j \text{ or } i \neq k\}$

a. ♣ ♠ $\{a^i b^j c^k \quad | \quad i \geq j \text{ or } i \geq k\}$

Exercise 6.12. ♣

a. Show that if L is a CFL and F is finite, $L - F$ is a CFL.

b. Show that if L is not a CFL and F is finite, $L - F$ is not a CFL.

c. Show that if L is not a CFL and F is finite, $L \cup F$ is not a CFL.

Exercise 6.13. ♣

For each part below, say whether the statement is true or false, and give reasons for your answer.

a. If L is a CFL and F is regular, then $L - F$ is a CFL.

b. If L is not a CFL and F is regular, then $L - F$ is not a CFL.

c. If L is not a CFL and F is regular, then $L \cup F$ is not a CFL.

Theorem 6.13.

If L_1 is a context-free language and L_2 is a regular language, then $L_1 \cap L_2$ is a CFL.

Exercise 6.8. ♣

Show that if L_1 is a DCFL and L_2 is regular, then $L_1 \cap L_2$ is a DCFL.