From exercise class 12:

Exercise 5.28.

In each case below, you are given a CFG G and a string x that it generates.
Draw the nondeterministic top-down PDA $NT(G)$.

Trace a sequence of moves in $NT(G)$ by which x is accepted, showing at each step the state, the unread input, and the stack contents.
Show at the same time the corresponding leftmost derivation of x in the grammar. See Example 5.19 for a guide.

b. ♣ The grammar has productions $S \to S{+}S \mid S{*}S \mid [S] \mid a$, and $x = [a * a + a]$.

From exercise class 12:

Exercise 5.34.

In each case below, you are given a CFG G and a string x that it generates.
Draw the nondeterministic bottom-up PDA *NB*(G).

Trace a sequence of moves in *NB*(G) by which x is accepted, showing at each step the state, the (reversed) stack contents and the unread input.
Show at the same time the corresponding rightmost derivation of x (in reverse order) in the grammar. See Example 5.24 for a guide.

a. ♣ The grammar has productions $S \to S[S] \mid \Lambda$ and $x = [][[]]$.

From exercise class 12:

Exercise 5.30.

For a certain CFG G, the moves shown below are those by which the nondeterministic bottom-up PDA *NB*(G) accepts the input aabbab. Each occurrence of $\vdash^*$ indicates a sequence of moves constituting a reduction. Draw the derivation tree for $aabbab$ that corresponds to this sequence of moves.

$$\begin{array}{rcl}
(q_0, aabbab, Z_0) & \vdash & (q_0, abbab, aZ_0) \vdash (q_0, bbab, aaZ_0) \\
 & \vdash & (q_0, bab, baaZ_0) \vdash^* (q_0, bab, SaZ_0) \\
 & \vdash & (q_0, ab, bSaZ_0) \vdash^* (q_0, ab, SZ_0) \\
 & \vdash & (q_0, b, aSZ_0) \vdash (q_0, \Lambda, baSZ_0) \\
 & \vdash^* & (q_0, \Lambda, SSZ_0) \vdash^* (q_0, \Lambda, SZ_0) \\
 & \vdash & (q_1, \Lambda, Z_0) \vdash (q_2, \Lambda, Z_0)
\end{array}$$

Exercise 5.32.

Lem M be the PDA below, accepting

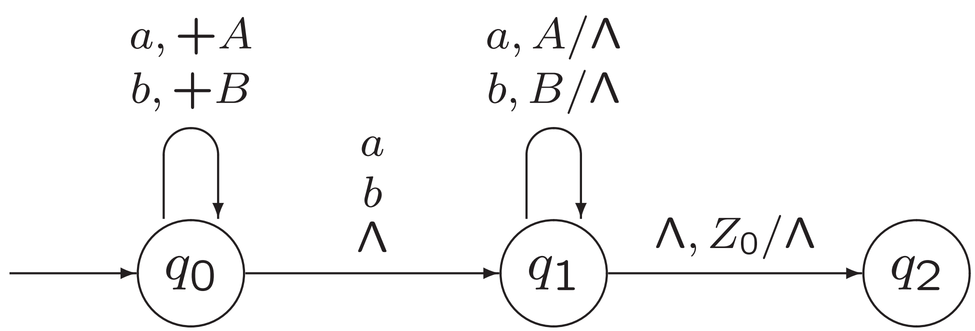
$$\textit{Pal} = \{y \in \{a,b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a,b\}^*\}$$

(by empty stack). Let $x = ababa$.

a. ♣ Find a sequence of moves of M by which x is accepted.

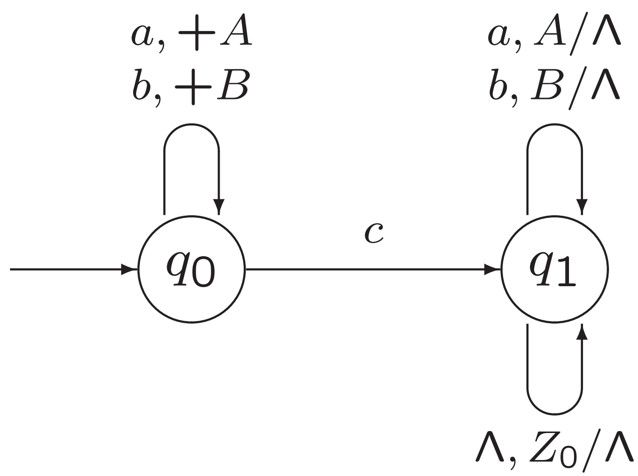
b. Let G be the CFG obtained from M as in Theorem 5.29. For each transition in M, give/describe the corresponding productions in G.

c. ♣ Give the leftmost derivation of x in G corresponding to the computation from part (a).



Exercise 5.35.

Let M be the PDA below, accepting *SimplePal* by empty stack.



Consider the simplistic approach to obtaining a CFG described in the discussion preceding Theorem 5.29. The states of M are ignored, the variables of the grammar are the stack symbols of M, and for every move that reads σ and replaces A on the stack by $BC \ldots D$, we introduce the production $A \to \sigma BC \ldots D$.

a. Give all productions resulting from this approach.

b. Find a string $x \in \{a, b, c\}^*$ that is generated by this CFG, but is not accepted by M.

Pumping Lemma for context-free languages:

For every context-free language L

there exists a constant $n \geq 2$ such that

for every $u \in L$ with $|u| \geq n$

there exists a decomposition $u = vwxyz$ such that

(1) $|wy| \geq 1$,

(2) $|wxy| \leq n$,

(3) for all $m \geq 0$, $vw^m xy^m z \in L$

Example 6.4. $XX = \{xx \mid x \in \{a,b\}^* \}$ is not context-free.
Use $u = a^n b^n a^n b^n$

Exercise 6.4.

In the proof given in Example 6.4 using the pumping lemma, the contradiction was obtained in each case by considering the string vw^0xy^0z.

Would it have been possible instead to use vw^2xy^2z in each case?
If so, give the proof in at least one case;
if not, explain why not.

Example 6.4. $XX = \{xx \mid x \in \{a, b\}^* \}$ is not context-free.
Use $u = a^n b^n a^n b^n$

Exercise 6.3.

In the pumping-lemma proof in Example 6.4, give some examples of choices of strings $u \in L$ with $|u| \geq n$ that would not work.