Huiswerk...
Vanmiddag: geen werkcollege
Vragenuur: dinsdag 21 december, 14.15-16.00, zaal 412 (+ Kaltura)
Tentamen: donderdag 23 december, 10.15-13.15

From lecture 2:

FA $M_i = (Q_i, \Sigma, q_i, A_i, \delta_i) \quad i = 1, 2$

Product construction

construct FA $M = (Q, \Sigma, q_0, A, \delta)$ such that

- $Q = Q_1 \times Q_2$
- $q_0 = (q_1, q_2)$
- $\delta(\ (p, q), \sigma) = (\ \delta_1(p, \sigma), \delta_2(q, \sigma)\ )$
- A as needed

Theorem (2.15 Parallel simulation)

- $A = \{(p, q) \mid p \in A_1$ *or* $q \in A_2\}$, *then* $L(M) = L(M_1) \cup L(M_2)$
- $A = \{(p, q) \mid p \in A_1$ *and* $q \in A_2\}$, *then* $L(M) = L(M_1) \cap L(M_2)$
- $A = \{(p, q) \mid p \in A_1$ *and* $q \notin A_2\}$, *then* $L(M) = L(M_1) - L(M_2)$

Proof...

[M] Sect 2.2

Theorem

If L_1 is a CFL, and L_2 in REG, then $L_1 \cap L_2$ is CFL.

[M] Thm 6.13

product construction

PDA $M_1 = (Q_1, \Sigma, \Gamma, q_1, Z_1, A_1, \delta_1)$
FA $M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$

$Q = Q_1 \times Q_2 \qquad q_0 = \langle q_1, q_2 \rangle \qquad A = A_1 \times A_2$

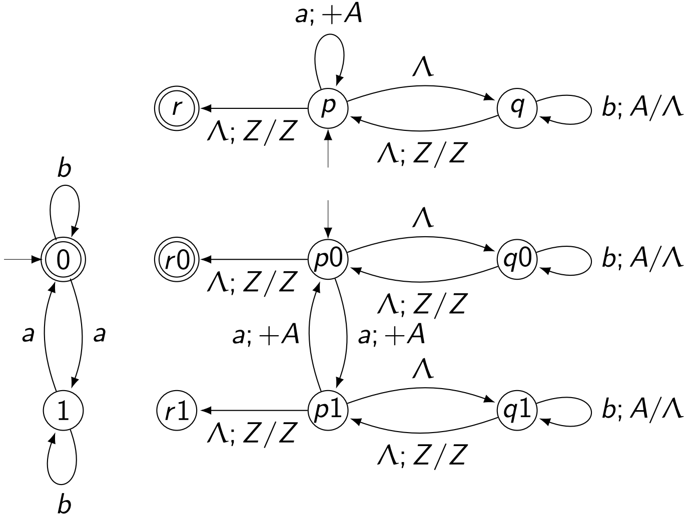
$\delta(\langle p_1, q_1 \rangle, \sigma, X) \ni (\langle p_2, q_2 \rangle, \alpha)$
whenever $\delta_1(p_1, \sigma, X) \ni (p_2, \alpha)$ and $\delta_2(q_1, \sigma) = q_2$

$\delta(\langle p_1, q \rangle, \Lambda, X) \ni (\langle p_2, q \rangle, \alpha)$
whenever $\delta_1(p_1, \Lambda, X) \ni (p_2, \alpha)$ and $q \in Q_2$

The inductive proof that this construction works does not have to be known for the exam.

Also CFG proof

$\{ a^n b^n \mid n \geqslant 1 \}^* \cap \{ w \in \{a, b\}^* \mid n_a(x) \text{ even} \}$



Non-determinism of PDA

- enables $L(M_1) \cup L(M_2)$
- 'prevents' $L(M_1)'$ (also Λ-transitions)

If L is accepted by DPDA without Λ-transitions, then so is L'

Even: if L is accepted by DPDA, then so is L'

Hence, if L is CFL and L' is not, then there is no DPDA for L
Not reversed (see *Pal*)

"given a CFL L, does it have property … ?" yes/no
input CFG G

Given CFG G [G_1 and G_2]
– and given a string x, is $x \in L(G)$? membership problem
convert G to ChNF, and try all derivations of length $2|x| - 1$
(special case if $x = \Lambda$)
Cocke, Younger, and Kasami (1967): n^3 (with DP)
Earley (1970): n^3 (and n^2 if G is unambiguous)

– is $L(G) \neq \varnothing$? non-emptiness
 is S useful?
 pumping lemma
– is $L(G)$ infinite?
 pumping lemma

– is $L(G_1) \cap L(G_2)$ nonempty? [M] Thm 9.20
– is $L(G) = \Sigma^*$? [M] Thm 9.23
– is $L(G_1) \subseteq L(G_2)$?

$$L(G) = \Sigma^*, \text{ if and only if } \Sigma^* \subseteq L(G)$$

All undecidable

Questions

Given context-free L and regular R

– is $R \subseteq L$?

– is $L \subseteq R$?

ABOVE

$R \subseteq L$?

Special case $R = \Sigma^*$

$\Sigma^* \subseteq L$ iff $L = \Sigma^*$ undecidable

$L \subseteq R$?

iff $L \cap R' = \varnothing$

regular languages are closed under complement

CFL closed under intersection with regular languages

emptiness context-free decidable

Section 7

Course Computability

7 Course Computability

- Turing machines
- Recursively enumerable languages / recursive languages
- Unrestricted grammars
- Undecidability

END.

Thanks to HJH for the slides