

### Exercise 1.36.

- (a) Consider the language  $L$  of all strings of  $a$ 's and  $b$ 's that do not end with  $b$  and do not contain the substring  $bb$ . Find a finite language  $S$  such that  $L = S^*$ .
- (b) Show that there is no language  $S$  such that  $S^*$  is the language of all strings of  $a$ 's and  $b$ 's that do not contain the substring  $bb$ .

## Exercise 1.32.

For a **finite** language  $L$ , let  $|L|$  denote the number of elements of  $L$ . For example,  $|\{\Lambda, a, ababb\}| = 3$ . This notation has nothing to do with the length  $|x|$  of a string  $x$ .

The statement  $|L_1L_2| = |L_1||L_2|$  says that the number of strings in the concatenation  $L_1L_2$  is the same as the product of the two numbers  $|L_1|$  and  $|L_2|$ .

Is this always true? If so, give reasons, and if not, find two finite languages  $L_1, L_2 \subseteq \{a, b\}^*$  such that  $|L_1L_2| \neq |L_1||L_2|$ .

**Exercise 1.33.**

Let  $L_1$  and  $L_2$  be subsets of  $\{a, b\}^*$ .

**(b)** Show that  $L_1^* \cup L_2^* \subseteq (L_1 \cup L_2)^*$ .

### Exercise 1.37.

Let  $L_1$ ,  $L_2$  and  $L_3$  be languages over some alphabet  $\Sigma$ . In each case below, two languages are given. Say what the relationship is between them. (Are they always equal? If not, is one always a subset of the other?) Give reasons for your answers, including counterexamples if appropriate.

(a)  $L_1(L_2 \cap L_3)$  and  $L_1L_2 \cap L_1L_3$

(b)  $L_1^* \cap L_2^*$  and  $(L_1 \cap L_2)^*$

(c)  $L_1^*L_2^*$  and  $(L_1L_2)^*$

**Exercise 2.1.** In each part below, draw an FA accepting the indicated language over  $\{a, b\}$ .

**a.** The language of all strings containing exactly two  $a$ 's.

**f.** The language of all strings in which the number of  $a$ 's is even.

**g.** The language of all strings in which both the number of  $a$ 's and the number of  $b$ 's is even.

**g2.** The language of all strings in which either the number of  $a$ 's or the number of  $b$ 's is odd (or both).

**j.** The language of all strings containing both  $bb$  and  $aba$  as substrings.

**k.** The language of all strings containing both  $aba$  and  $bab$  as substrings.

## Exercise 2.2.

For each of the FAs pictured in Fig. 2.43, give a simple verbal description of the language it accepts.

a. on the blackboard

d.

