

## Exercise 6.4.

In the proof given in Example 6.4 using the pumping lemma, the contradiction was obtained in each case by considering the string  $vw^0xy^0z$ .

Would it have been possible instead to use  $vw^2xy^2z$  in each case? If so, give the proof in at least one case; if not, explain why not.

### Exercise 6.3.

In the pumping-lemma proof in Example 6.4, give some examples of choices of strings  $u \in L$  with  $|u| \geq n$  that would not work.

## Exercise 6.2.

In each case below, show using the pumping lemma that the given language is not a CFL.

a.  $L = \{a^i b^j c^k \mid i < j < k\}$

b.  $L = \{a^{2^i} \mid i \geq 0\}$

d.  $L = \{a^i b^{2^i} a^i \mid i \geq 0\}$

e.  $L = \{s \in \{a, b, c\}^* \mid n_a(s) = \max\{n_b(s), n_c(s)\}\}$

g.  $L = \{a^i b^j a^i b^{i+j} \mid i, j \geq 0\}$

## Exercise 6.5.

For each case below, decide whether the given language is a CFL, and prove your answer.

a.  $L = \{a^i b^j a^j b^i \mid i, j \geq 0\}$

c.  $L = \{scs \mid s \in \{a, b\}^*\}$

d.  $L = \{sts \mid s, t \in \{a, b\}^* \text{ en } |s| \geq 1\}$

g.  $L =$  the set of non-balanced strings of parentheses

### Exercise 6.12.

- a. Show that if  $L$  is a CFL and  $F$  is finite,  $L - F$  is a CFL.
- b. Show that if  $L$  is not a CFL and  $F$  is finite,  $L - F$  is not a CFL.
- c. Show that if  $L$  is not a CFL and  $F$  is finite,  $L \cup F$  is not a CFL.

### Exercise 6.13.

For each part below, say whether the statement is true or false, and give reasons for your answer.

- a. Show that if  $L$  is a CFL and  $F$  is regular,  $L - F$  is a CFL.
- b. Show that if  $L$  is not a CFL and  $F$  is regular,  $L - F$  is not a CFL.
- c. Show that if  $L$  is not a CFL and  $F$  is regular,  $L \cup F$  is not a CFL.

### **Theorem 6.13.**

If  $L_1$  is a context-free language and  $L_2$  is a regular language, then  $L_1 \cap L_2$  is a CFL.

### Exercise 6.8.

Show that if  $L_1$  is a DCFL and  $L_2$  is regular, then  $L_1 \cap L_2$  is a DCFL.