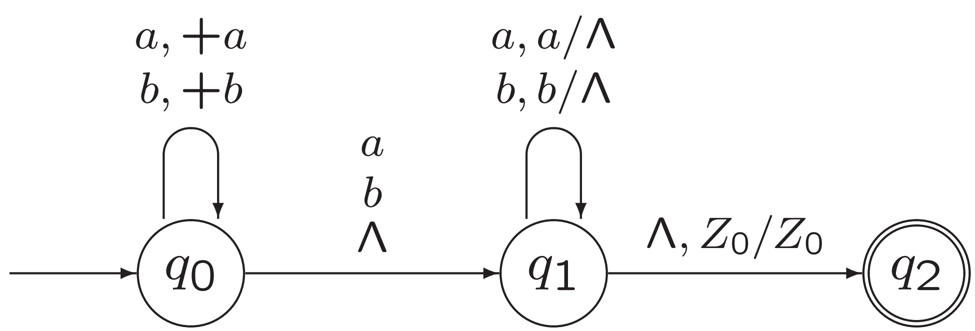
Exercise 5.2.

For the PDA below, trace every possible sequence of moves for the two input strings aba and aab.

Example 5.7. A Pushdown Automaton Accepting *Pal*

$$\textit{Pal} = \{y \in \{a,b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a,b\}^*\}$$



Exercise 5.4.

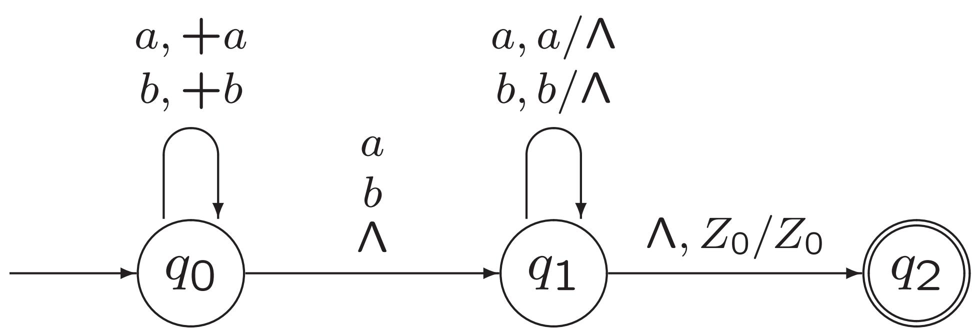
For each of the following languages over $\{a, b\}^*$, modify the PDA below to obtain a PDA accepting the language.

a. The language of even-length palindromes.

b. The language of odd-length palindromes.

Example 5.7. A Pushdown Automaton Accepting *Pal*

$$\textit{Pal} = \{y \in \{a,b\}^* \mid y = y^r\} = \{xx^r, xax^r, xbx^r \mid x \in \{a,b\}^*\}$$



Exercise 5.5.

Give transition diagrams for PDAs accepting each of the following languages.

a. The language of all odd-length strings over $\{a, b\}$ with middle symbol a.

b. $\{a^n x \mid n \geq 0, x \in \{a, b\}^*$ and $|x| \leq n\}$.

c. $\{a^i b^j c^k \mid i, j, k \geq 0$ and $j = i$ or $j = k\}$.

Exercise 5.6.

In both cases below (on the blackboard), a transition diagram is given for a PDA with intial state q_0 and accepting state q_2. Describe in each case the language that is accepted.

(first case treated)

Exercise.

Let $L_1 = \{a^i b^j c^k \mid \ i, j, k \geq 0 \text{ en } 2i > j\}$.

a. Give the first five elements of L_1 in the canonical order.

b. Give a DPDA M_1 such that $L(M_1) = L_1$.

Exercise 5.10.

Show that every regular language can be accepted by a (deterministic) PDA M with only two states in which there are no Λ-transitions and no symbols are ever removed from the stack.

Exercise 5.12.

Show that if L is accepted by a PDA
in which no symbols are ever removed from the stack,
then L is regular.

Exercise 5.18.

For each of the following languages, give a transition diagram for a deterministic PDA that accepts that language.

a. $\{x \in \{a,b\}^* \mid n_a(x) < n_b(x)\}$

b. $\{x \in \{a,b\}^* \mid n_a(x) \neq n_b(x)\}$

c. $\{x \in \{a,b\}^* \mid n_a(x) = 2n_b(x)\}$

d. $\{a^n b^{n+m} a^m \mid n, m \geq 0\}$

Exercise 5.16.

Show that if L is accepted by a PDA, then L is accepted by a PDA that never crashes (i.e., for which the stack never empties and no configuration is reached from which there is no move defined).

From lecture 9:

Stack in PDA contains symbols from certain alphabet.

Usual stack operations: pop, top, push

Extra possiblity: replace top element X by string α

$\alpha = \Lambda$	pop
$\alpha = X$	top
$\alpha = YX$	push
$\alpha = \beta X$	push*
$\alpha = \ldots$	

Top element X is required to do a move!

Exercise 5.17.

Show that if L is accepted by a PDA, then L is accepted by a PDA in which every move
$*$ either pops something from the stack (i.e., removes a stack symbol without putting anything else on the stack);
$*$ or pushes a single symbol onto the stack on top of the symbol that was previously on top;
$*$ or leaves the stack unchanged.

Hence, each action on the stack due to a move in the PDA has one of the following forms:
$*$ either X/Λ (with $X \in \Gamma$),
$*$ or X/YX (with $X, Y \in \Gamma$),
$*$ or X/X (with $X \in \Gamma$).