1. **[0.5 point]** If $\Sigma = \{a, b, c\}$ and $L - \Sigma^* = \emptyset$, which of the following languages can be L?

   (a) $\emptyset$

   (b) $\{\Lambda, b^{100}\}$

   (c) Σ^*

   (d) $\{a^n b^n c^n \mid n > 0\}$

   all of the above

2. **[1 point]** Give a *regular expression* for each of the following languages:

   (a) $L = \{a^m b^n \mid n + m \text{ is even}\}$

   $(aa)^*(\Lambda + ab)(bb)^*$

   (b) $L = \{w \in \{0,1\}^* \mid w \text{ contains } \textit{exactly} \text{ one pair of consecutive 0's}\}$

   $(1 + 01)^*00(1 + 10)^*$

3. **[1 point]** Suppose language L represented by the *regular expression* $(00)^*01$. Give a maximal set S of pairwise L-distinguishable strings. From the set S, what can we say about the number of states of a DFA M that accepts L?

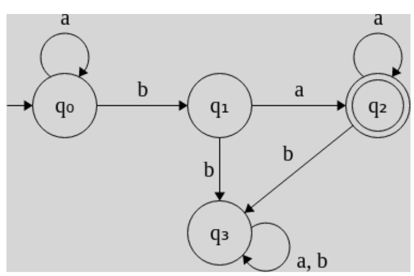
   There are infinite number of sets of pairwise L-distinguishable strings, e.g., $\{\Lambda, 0, 1, 01\}$ or $\{10, 00, 000, 0001\}$.

   From the above examples, the DFA M must have *at least* 4 states.

4. **[1 point]** Languages L_1 and L_2 are represented by a^*baa^* and ab^*, respectively. Construct a DFA that accepts $L_1 - L_2$.

   Since $L_1 \cap L_2 = \emptyset$, obviously $L_1 - L_2 = L_1$.

   Therefore, we only need to construct a DFA that accepts L_1.



5. **[1 points]** Give a context-free grammar G generating the following language L:

$$L = \{a^n b^m c^k \mid k = |n - m|\}$$

The final score is given by the sum of the points obtained.

$S \to A \mid B$
$A \to aAc \mid C$
$C \to aCb \mid \Lambda$
$B \to DE$
$D \to aDb \mid \Lambda$
$E \to bEc \mid \Lambda$

6. [**1 point**] Prove the following grammars are ambigious:

   (a)

$$S \to T \mid Sa \mid a$$
$$T \to ab \mid \lambda$$

   we show that for the string **a** there are more than one left-most derivation
   $S \Rightarrow Sa \Rightarrow Ta \Rightarrow a$
   $S \Rightarrow a$

   (b)

$$S \to bSc \mid bbSc \mid a$$

   we show that for the string **bbbacc** there are more than one left-most derivation
   $S \Rightarrow bSc \Rightarrow bbbScc \Rightarrow bbbacc$
   $S \Rightarrow bbSc \Rightarrow bbbScc \Rightarrow bbbacc$

7. [**2 points**] Suppose the following context-free grammar G:

$$S \to aSU \mid X$$
$$X \to bXU \mid \Lambda$$
$$U \to aY$$
$$Y \to aY \mid \Lambda$$

   (a) Give $L(G)$.
   $L = \{a^n b^m a^k \mid n + m \leq k\}$

   (b) Give a grammar in *Chomsky Normal Form* that generates $L(G) - \{\Lambda\}$.
   First, we eliminate Λ-productions, using nullable variables:
   $S \to aSU \mid aU \mid X$

$X \to bXU \mid bU$

$U \to aY \mid a$

$Y \to AY \mid a$

Then, we eliminate the unit production $S \to X$:

$S \to aSU \mid aU \mid bXU \mid bU$

$X \to bXU \mid bU$

$U \to aY \mid a$

$Y \to aY \mid a$

Finally, we use new variables Z, W and A to write the grammar in CNF:

$S \to AZ \mid AU \mid BW \mid BU$

$Z \to SU$

$W \to XU$

$X \to BW \mid BU$

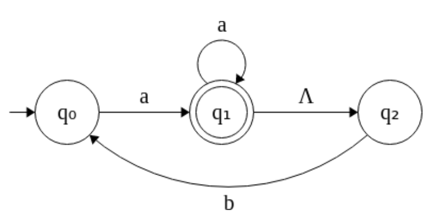
$U \to AY \mid a$

$Y \to AY \mid a$

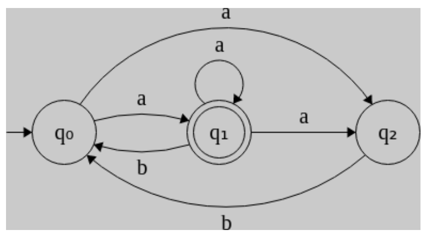
$A \to a$

Note that the above grammar can be further simplified.

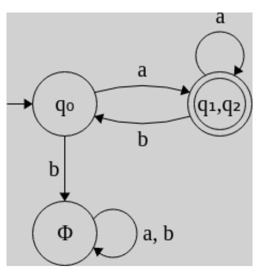
8. **[1 point]** Construct a DFA that accepts the same language as the following NFA. Explain the steps.



First, the Λ-transition is eliminated:



Then, the non-determinism is eliminated:



9. **[1.5 points]** Suppose the following PDA M, where q_0 is the initial state and q_2 is the accepting state:

$$\sigma(q_0, a, Z_0) = \{(q_1, AZ_0), (q_2, Z_0)\}$$
$$\sigma(q_1, b, A) = \{(q_1, B)\}$$
$$\sigma(q_1, b, B) = \{(q_1, B)\}$$
$$\sigma(q_1, a, B) = \{(q_2, \Lambda)\}$$

(a) give L(M).

$L(M) = \{ab^n a \mid n \geq 1\} \cup \{a\}$

(b) is M deterministic or not? Why?

M is non-deterministic, because there are more than one transition for the combination of the state q_0, the input symbol a and the stack symbol Z_0